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Segmentation of Noisy Digital Images with Stochastic Gradient Kernel

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Abstract**—**Image segmentation and edge detection is a fundamental section in image processing. In case of noisy images Edge Detection is very less effective if we use conventional Spatial Filters like Sobel, Prewitt, LOG, Laplacian etc. To overcome this problem we have proposed the use of Stochastic Gradient Mask instead of Spatial Filters for generating gradient images. The present study has shown that the resultant images obtained by applying Stochastic Gradient Masks appear to be much clearer and sharper as per Edge detection is considered.

Keywords**—**Image segmentation, edge Detection, noisy images, spatialfilters, stochastic gradient kernel.

I. Introduction

IMAGE Segmentation [1]-[6], [9] is a very important image analysis process in the field of Digital Image Processing. It is the process of partitioning a digital image into multiple segments. The goal of segmentation is to simplify and change the representation of an image into something that is more meaningful and easy to analyze. It is used to fragment the image into regions of statistical co-relation as well as human visual similarity.

Edge detection [6], [8] is the name for a set of mathematical methods which are used for identifying points in a digital image at which the image brightness changes sharply or, more precisely, has discontinuities. Edge detection identifies the boundaries within images. Edges typically occur on the boundary between two different regions of an image. The purpose of detecting sharp changes in image brightness is to capture important events and changes in properties of the world. It can be shown that under rather general assumptions for an image formation model, discontinuities in image brightness are likely to correspond to discontinuities in depth, discontinuities in surface orientation, changes in material properties and variations in scene illumination.

There are four steps in Edge Detection: Smoothing, Enhancement, Detection and Localization. Smoothing is the process of suppressing as much noise as possible, without destroying the true edges. In the Enhancement step, a filter is applied to enhance the quality of the edges in an image i.e. sharpening the image. The detection step determines which edge pixels should be discarded as noise and which should be retained (usually, thresholding provides the criterion used for detection). Finally the Localization step determines the exact location of an edge (*sub-pixel* resolution might be required for some applications, that is, estimate the location of an edge to better than the spacing between pixels). Edge thinning and linking are usually required in this step.

Since digital images are mostly noisy, the use of Stochastic Gradient Mask over conventional filters like Sobel, Prewitt, Laplacian or LOG, is more proficient for Edge Detection. The efficiency of the proposed methodology has been explained by experimental results and statistical measurements.

The research paper has been arranged in the following manner. Section I is the Introduction to the paper. In Section II we have discussed about Noise in Digital Images. Section III contains discussion about Gradient of an image. Next, in Section IV we have discussed about spatial filters and in the following Section V, the Stochastic Gradient Masks have been discussed in details. In Section VI we have described the proposed methodology for this paper. In Section VII the results obtained have been thoroughly discussed. Section VIII is the conclusion of the paper, followed by Acknowledgement and References

II. Noise in Digital Images

Image noise [3], [6] is random variation of brightness or color information in images, and is usually an aspect of electronic noise. Digital noise usually occurs when we take low light photos or use very slow shutter speeds or very high sensitivity modes. When taking pictures with a digital camera an electronic sensor (also known as a CCD) built from many tiny pixels is used to measure the light for each pixel. As with any other electronic sensor the CCD is not perfect and includes some noise. Noise can also be introduced in a digital image during transmission.

A. Gaussian Noise or Amplifier Noise

This noise has a probability density function (pdf) of the normal distribution. It is also known as Gaussian distribution. It is a major part of the read noise of an image sensor that is of the constant level of noise in the dark areas of the image.

Gaussian Noise:

$$p(z) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(z-\mu)^2/2\sigma^2} \quad (1)$$

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B. Salt and Pepper Noise

The salt-and-pepper noise are also called shot noise, impulse noise or spike noise that is usually caused by faulty memory locations, malfunctioning pixel elements in the camera sensors, or there can be timing errors in the process of digitization .In the salt and pepper noise there are only two possible values exists that is a and b and the probability of each is less than 0.2.If the numbers greater than this numbers the noise will swamp out image. For 8-bit image the typical value for 255 for salt-noise and pepper noise is 0.

III. Gradient

An image gradient [3]-[5], [7] is a directional change in the intensity or color of an image. Image gradients may be used to extract information from images. In digital image processing the term gradient is used for a gradual blend of color which can be considered as even gradation from low to high values, as used from white to black in the images to the right.The gradient of a two variable function at each image point is a 2D vector with the components given by the components given by the derivatives in the horizontal and vertical directions. At each image point, the gradient vector points in the direction of largest possible intensity increase, and the length of the gradient vector corresponds to the rate of change in that direction .

Gradient of an image is given by the formula:

$$\nabla f = \begin{pmatrix} \frac{df}{dx} \\ \frac{df}{dy} \end{pmatrix} \tag{2}$$

$$magn(\nabla f) = \sqrt{\left(\frac{df}{dx}\right)^2 + \left(\frac{df}{dy}\right)^2} = \sqrt{{M_x}^2 + {M_y}^2} \tag{3}$$

$$dir(\nabla f) = \tan^{-1}\left(\frac{M_y}{M_x}\right) \tag{4}$$

where:$\frac{df}{dy}$is the gradient in the x direction.$\frac{df}{dx}$ is the gradient in the y direction. Gradient direction is calculated by:

$$\theta = a.tan2\left(\frac{df}{dy}, \frac{df}{dx}\right) \tag{5}$$

IV. Spatial Filtering

Filtering is an important part of image processing systems, in particular when it comes to image enhancement and restoration. Image filtering is useful for many applications, including smoothing, sharpening, removing noise, and edge detection. A filter is defined by a kernel, which is a small array applied to each pixel and its neighbors within an image.
Spatial filters [1]-[2], [9] are designed to highlight or suppress features in an image based on spatial frequency, such as suppressing noise or highlighting specific image characteristics. In this study we have used Sobel, Prewitt, Laplacian and LOG filters.

A. Sobel

The Sobel operator is named after Irwin Sobel, who presented the idea of an "Isotropic 3x3 Image gradient operator" at a talk at the Stanford Artificial Intelligence Project in 1968. Technically it is a discrete differentiation operator, computing an approximation of the gradient of the image intensity function. The operator uses two 3×3 kernels which are convolved with the original image to calculate approximations of the derivatives - one for horizontal changes, and one for vertical. If we define **A** as the source image, and $\mathbf{G}_x$ and $\mathbf{G}_y$ are two images which at each point contain the horizontal and vertical derivative approximations, the computations are as:

$$G_{x=}\begin{bmatrix} -1 & 0 & +1 \\ -2 & 0 & +2 \\ -1 & 0 & +1 \end{bmatrix} * A \tag{6}$$

and

$$G_y = \begin{bmatrix} +1 & +2 & +1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix} * A \tag{7}$$

where$*$ here denotes the 2-dimensional convolution operation.

B. Prewitt

The Prewitt operator is based on convolving the image with a small, separable, and integer valued filter in horizontal and vertical directions and is therefore relatively inexpensive in terms of computations. At each point in the image, the result of the Prewitt operator is either the corresponding gradient vector or the norm of this vector. Mathematically, the operator uses two 3×3 kernels which are convolved with the original image to calculate approximations of the derivatives - one for horizontal changes, and one for vertical. If we define A as the source image, and Gx and Gy are two images which at each point contain the horizontal and vertical derivative approximations, the latter are computed as:

$$G_{x=}\begin{bmatrix} -1 & 0 & +1 \\ -1 & 0 & +1 \\ -1 & 0 & +1 \end{bmatrix} * A \tag{8}$$

and

$$G_y = \begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ +1 & +1 & +1 \end{bmatrix} * A \tag{9}$$

where$*$ here denotes the 2-dimensional convolution operation.

C. Laplacian

The Laplacian is a 2-D isotropic measure of the 2nd spatial derivative of an image. The Laplacian of an image highlights regions of rapid intensity change and is therefore often used for edge detection.

The Laplacian *L(x,y)* of an image with pixel intensity values *I(x,y)* is given by:

$$\mathrm{L(x,y)} = \frac{\partial^2 \mathrm{I}}{\partial \mathrm{x}^2} + \frac{\partial^2 \mathrm{I}}{\partial \mathrm{y}^2} \tag{10}$$

This can be calculated using a convolution filter.

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Since the input image is represented as a set of discrete pixels, we have to find a discrete convolution kernel that can approximate the second derivatives in the definition of the Laplacian. Two commonly used small kernels are shown below-

0	−1	0
−1	4	−1
0	−1	0

−1	−1	−1
−1	8	−1
−1	−1	−1

Fig. 1 Two commonly used discrete approximations to the Laplacian filter

D. Laplacian of Gaussian (LOG)

The two kernels used in Laplacian are approximating a second derivative measurement on the image and they are very sensitive to noise. To counter this, the image is often Gaussian smoothed before applying the Laplacian filter. This pre-processing step reduces the high frequency noise components prior to the differentiation step.

The LOG filter is a convolution filter which first applies a Gaussian blur, then applies the Laplacian filter and finally checks for zero crossing, i.e. when the resulting value goes from negative to positive.

The 2-D LoG function centered on zero and with Gaussian standard deviationσhas the form:

$$LoG(x,y) = \frac{1}{\pi\sigma^4}\left[1 - \frac{x^2+y^2}{2\sigma^2}\right] e^{-\frac{x^2+y^2}{2\sigma^2}} \qquad (11)$$

0	0	-1	0	0
0	-1	-2	-1	0
-1	-2	16	-2	-1
0	-1	-2	-1	0
0	0	-1	0	0

Fig. 2 A conventional 5x5 LOG surround Filter

V. Stochastic Gradients

The gradient masks of standard high pass spatial filters perform poorly in noisy images. A better alternative is to design some masks, which take into the presence of noise in a controlled manner. The stochastic gradient operator [6]-[9] can be obtained using definition in (12)

$$g_1(m,n) \triangleq u_f(m,n-1) - u_b(m,n+1) \qquad (12)$$

Stochastic gradient operators with different masks are shown in Figs. 1-6. It is notable that for high signal noise ratio the filter waits decay rapidly.

0.776	0	-0.776
1.00	0	-1.00
0.776	0	-0.776

Fig. 3 3x3 Stochastic gradient mask, SNR=9

0.267	0.364	0	-0.364	-0.267
0.373	0.562	0	-0.562	-0.373
0.463	1.00	0	-1.00	-0.463
0.373	0.562	0	-0.562	-0.373
0.267	0.364	0	-0.364	-0.267

Fig. 4 5x5 Stochastic gradient mask, SNR=9.

0.073	0.240	0.283	0	-0.283	-0.240	-0.073
0.104	0.213	0.348	0	-0.348	-0.213	-0.104
0.165	0.354	0.579	0	-0.579	-0.354	-0.165
0.195	0.463	1.00	0	-1.00	-0.463	-0.195
0.165	0.354	0.579	0	-0.579	-0.354	-0.165
0.104	0.213	0.348	0	-0.348	-0.213	-0.104
0.073	0.240	0.283	0	-0.283	-0.240	-0.073

Fig. 5 7x7 Stochastic gradient mask, SNR=9

0.97	0	-0.97
1.00	0	-1.00
0.97	0	-0.97

Fig. 6 3x3 Stochastic gradient mask, SNR=1.

0.802	0.836	0	-0.836	-0.802
0.845	0.897	0	-0.897	-0.845
0.870	1.00	0	-1.00	-0.870
0.845	0.897	0	-0.897	-0.845
0.802	0.836	0	-0.836	-0.802

Fig. 7 5x5 Stochastic gradient mask, SNR=1.

0.641	0.672	0.719	0	-0.719	-0.672	-0.641
0.656	0.719	0.781	0	-0.781	-0.719	-0.656
0.688	0.781	0.875	0	-0.875	-0.781	-0.688
0.703	0.831	1.00	0	-1.00	-0.831	-0.703
0.688	0.781	0.875	0	-0.875	-0.781	-0.688
0.656	0.719	0.781	0	-0.781	-0.719	-0.656
0.641	0.672	0.719	0	-0.719	-0.672	-0.641

Fig. 8 7x7 Stochastic gradient mask, SNR=1.

VI. Proposed Methodology

Stochastic gradient masks are an effective method for Image Edge Detection of noisy images. The advantages of Stochastic Gradient masks over conventional spatial filters are clearly reflected in case of noisy images. The images taken for the study have been photographed using cell phone cameras.

First of all noise is added to the images using MATLAB. Salt & Pepper, speckle and Gaussian noise have been added to the different images. The image is then changed to grayscale. The different gradient masks are then applied to the images and the results are compared. The standard high pass filters- Sobel, Prewitt, Laplacian and LOG- are applied first, followed

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by the proposed stochastic gradients. In this proposed methodology 3x3, 5x5 and 7x7 Stochastic gradients of SNR 1 and 9 are used for gradient images as these gradient kernels are capable ofremoving noise much better than standard high pass filters.

Flow diagram is as shown below:

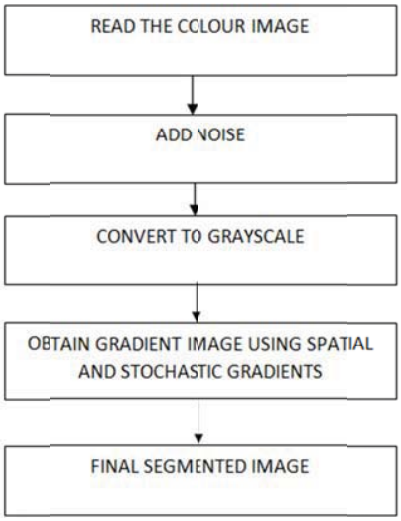
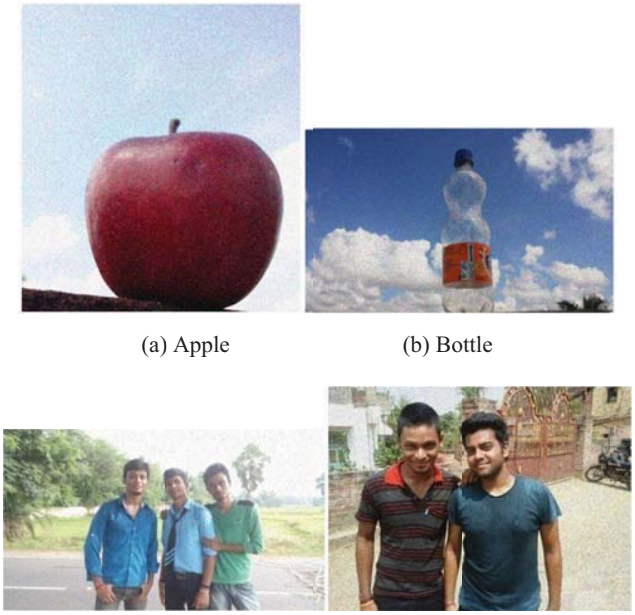


Fig. 9 Flow Diagram of the proposed methodology

VII. RESULTS AND DISCUSSIONS

In the present study four color images have been acquired from the real life. The images are of people, fruits, and objects of varying contrasts. They have been photographed using Cell phone cameras. Shown in Fig. 10 are the images taken for the study:

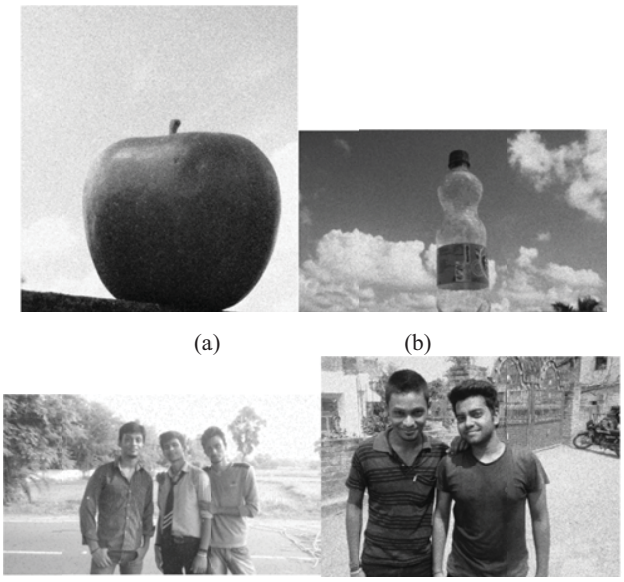


(a) Apple (b) Bottle

(c) Trio (d) Duo

Fig. 10 The images used for study

The purpose of the study is to show the effect of stochastic gradient masks on noisy images, so noise is added to the images using MATLAB. The images are converted to grayscale for the execution of the Algorithm. The grayscale images have been shown in Fig. 11.



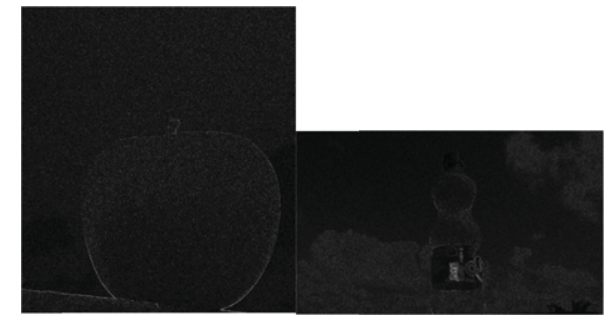
(a) (b)

(c) (d)

Fig. 11 (a)-(d) Images converted to Grayscale

The first image, “Apple” has been selected as a standard image. Edge detection can be prominently seen. Gaussian noise has been added to this image.The second image, “Bottle”, has been chosen because the Bottle is transparent and when edge detecting masks are applied the results have to be very perfect for detecting the boundary between the background and the object. The third image, named “Three”, has been selected as it has any boundaries to detect, and many edges to highlight.The fourth image, named “Two” has been selected because it has been taken it bright conditions i.e. in broad daylight.

Next the Standard High Pass filters (Spatial filters) namely: Sobel, Prewitt, LOG, and Laplacian, are applied to the images. The results have been shown in Figs. 12-15for Sobel, Laplacian, LoG and Prewitt accordingly.



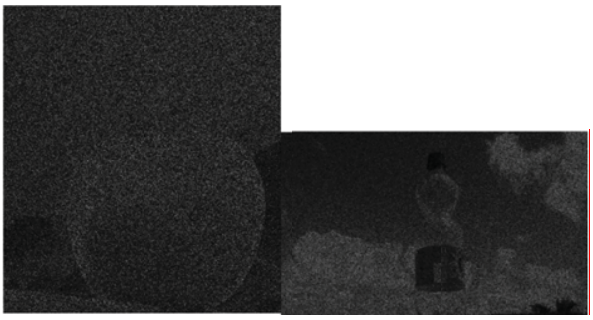
(a) (b)

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(c) (d)

Fig. 12 (a)-(d) Results obtained by Sobel Mask

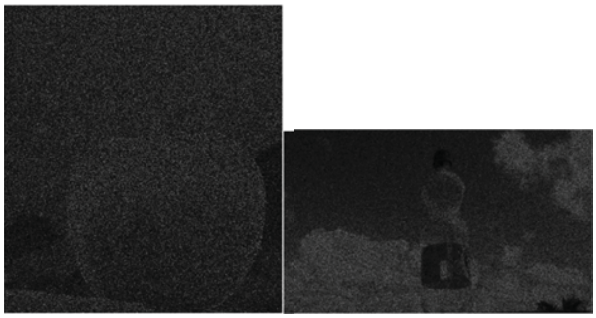


(a) (b)



(c) (d)

Fig. 13 (a)-(d) Results obtained by Laplacian mask

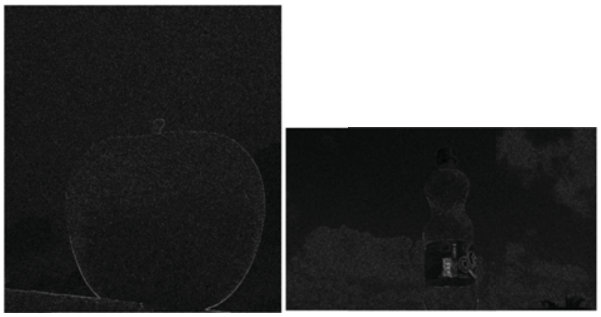


(a) (b)



(c) (d)

Fig. 14 (a)-(d) Results obtained by LOG mask



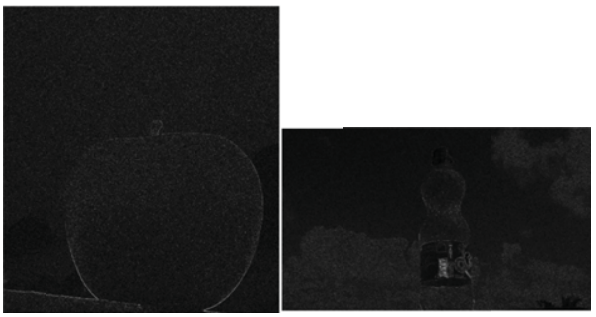
(a) (b)



(c) (d)

Fig. 15 (a)-(d) Results obtained by Prewitt mask

Next the Stochastic Gradient Masks of SNR=1 and SNR=9 are applied on the grayscale images and the results are compared with the stochastic gradients having mask of3x3, 5x5 and 7x7. The results are shown in Figs. 16-18 for SNR=9 and 19, 20, 21 for SNR=1.



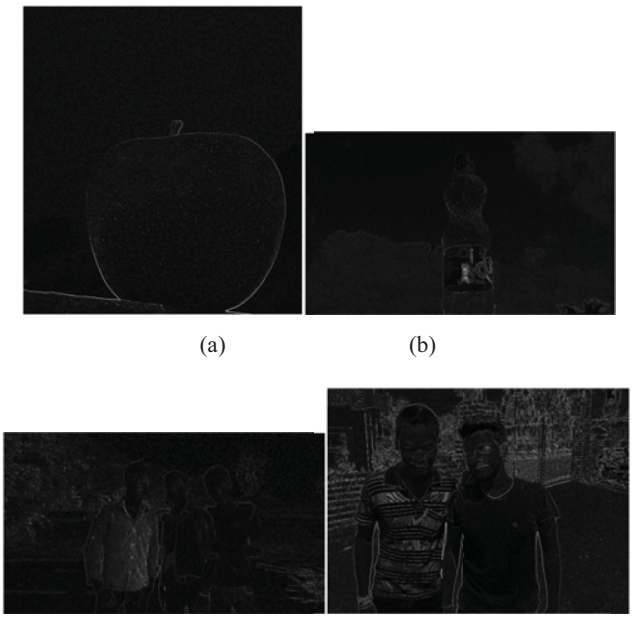
(a) (b)



(c) (d)

Fig. 16 (a)-(d) Results obtained by 3x3 Stochastic Gradient mask SNR=9

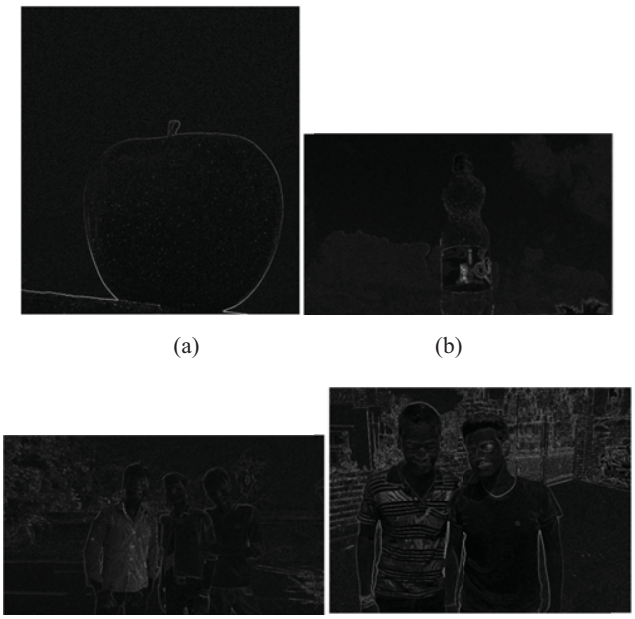
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(a) (b)

(c) (d)

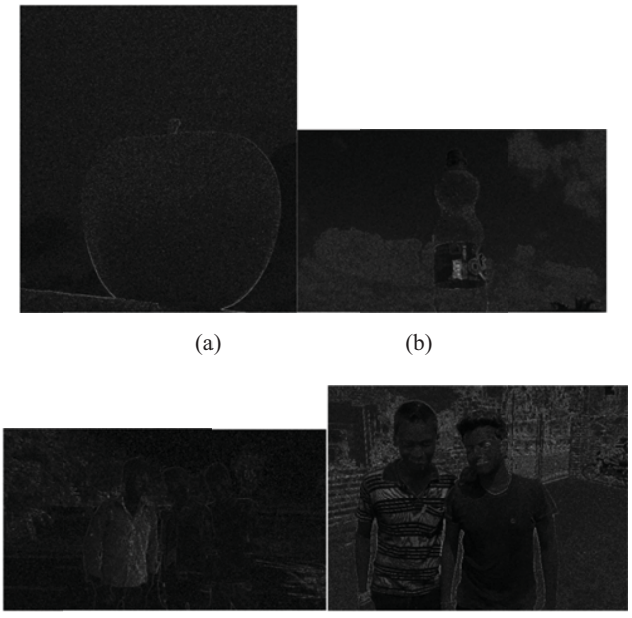
Fig. 17 (a)-(d) Results obtained by 5x5 Stochastic Gradient mask SNR=9



(a) (b)

(c) (d)

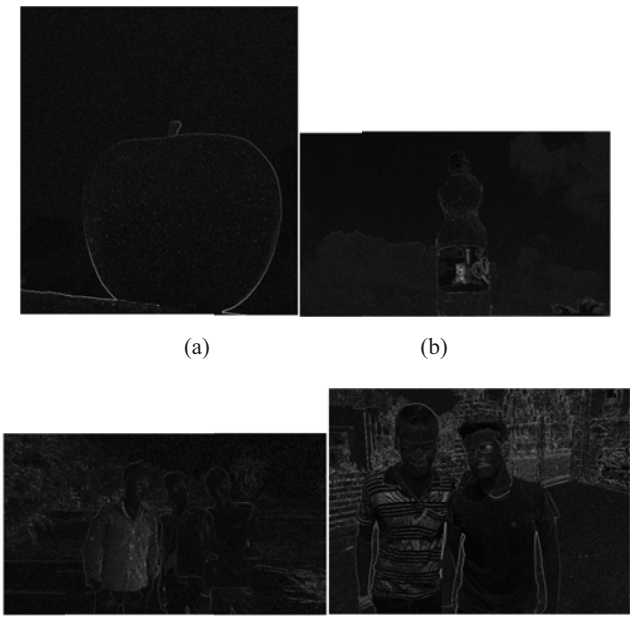
Fig. 18 (a)-(d) Results obtained by 7x7 Stochastic Gradient mask SNR=9



(a) (b)

(c) (d)

Fig. 19 (a)-(d) Results obtained by 3x3 Stochastic Gradient mask SNR=1

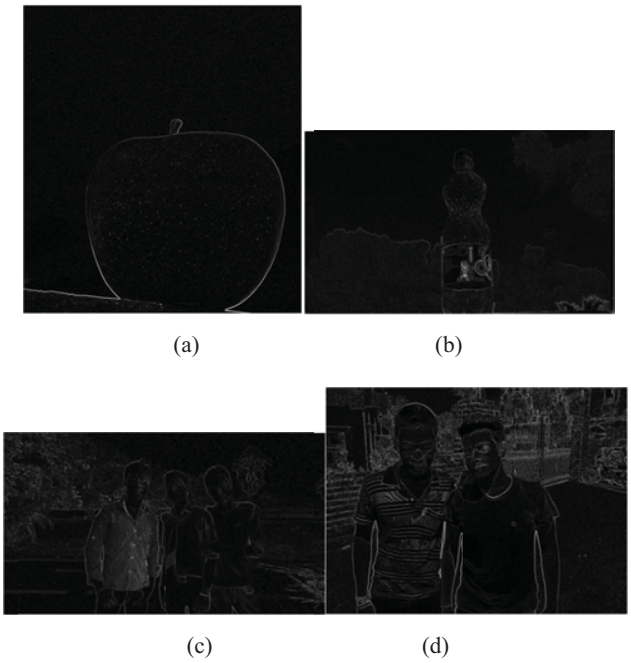


(a) (b)

(c) (d)

Fig. 20 (a)-(d) Results obtained by 5x5 Stochastic Gradient mask SNR=1

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(a) (b)

(c) (d)

Fig. 21 (a)-(d) Results obtained by 7x7 Stochastic Gradient mask SNR=1

As we can see in the above images that Laplacian and LOG filters give unsatisfactory results of Image Edge detection in every case. In image "Trio" these two masks give completely dark images as result where nothing is visible. Sobel and Prewitt give a better result, but still the edges are not very prominent. The 3x3 Stochastic Gradient masks o SNR 1 and 9 give a little better result than Sobel. The best results are given in 7x7 stochastic gradient masks. The edges are very prominently seen in these two images. The MSE, PSNR and Entropy of the different images are also found using MATLAB or statistical comparison. The statistical results are shown below.

TABLE I
STATISTICAL RESULTS FOR FIRST IMAGE

Image	Entropy	PSNR	MSE
Apple(Sobel)	4.6670	5.8819	1.6784e+004
Apple(Laplacian)	5.1734	6.3407	1.5101e+004
Apple(LOG)	5.1186	6.2627	1.5375e+004
Apple(Prewitt)	4.6843	5.8967	1.6727e+004
Apple (3x3 stochastic gradient mask,SNR=9)	4.6834	5.8969	1.6726e+004
Apple (5x5 stochastic gradient mask,SNR=9)	4.1498	5.4807	1.8408e+004
Apple (7x7 stochastic gradient mask,SNR=9)	4.1075	5.5376	1.8168e+004
Apple(3x3 stochastic gradient mask,SNR=1)	4.6371	5.8555	1.6886e+004
Apple (5x5stochastic gradient mask,SNR=1)	4.0411	5.4187	1.8673e+004
Apple (7x7 stochastic gradient mask,SNR=1)	3.7661	5.2513	1.9407e+004

Peak signal-to-noise ratio, often abbreviated **PSNR**, is an engineering term for the ratio between the maximum possible power of a signal and the power of corrupting noise that affects the fidelity of its representation. Because many signals have a very wide dynamic range, PSNR is usually expressed in terms of the logarithmic decibel scale.

TABLE II
STATISTICAL RESULTS FOR SECOND IMAGE

Image	Entropy	PSNR	MSE
Bottle(Sobel)	5.0251	8.3331	9.5448e+003
Bottle(Laplacian)	5.6638	9.4207	7.4304e+003
Bottle(LOG)	5.6211	9.3180	7.6081e+003
Bottle (Prewitt)	5.0359	8.3499	9.5190e+003
Bottle(3x3 stochastic gradient mask,SNR=9)	5.0287	8.3307	9.5502e+003
Bottle(5x5 stochastic gradient mask,SNR=9)	4.6275	7.8041	1.0781e+004
Bottle(7x7 stochastic gradient mask,SNR=9)	4.6045	7.7724	1.0860+004
Bottle(3x3 stochastic gradient mask,SNR=1)	5.1309	8.4896	9.2071e+003
Bottle(5x5stochastic gradient mask,SNR=1)	4.6326	7.7979	1.0797e+004
Bottle(7x7 stochastic gradient mask,SNR=1)	4.4768	7.6120	1.1269e+004

TABLE III
STATISTICAL RESULTS FOR THIRD IMAGE

Image	Entropy	PSNR	MSE
Trio(Sobel)	4.9800	4.7373	2.1845e+004
Trio(Laplacian)	3.8694	4.2684	2.4335e+004
Trio(LOG)	3.8713	4.2614	2.4375e+004
Trio(Prewitt)	4.9517	4.7157	2.1954e+004
Trio(3x3 stochastic gradient mask,SNR=9)	4.9378	4.7082	2.1992e+004
Trio (5x5 stochastic gradient mask,SNR=9)	4.9425	4.6417	2.2331e+004
Trio (7x7 stochastic gradient mask,SNR=9)	4.9513	4.7627	2.1717e+004
Trio(3x3 stochastic gradient mask,SNR=1)	4.9531	4.7171	2.1947e+004
Trio (5x5stochastic gradient mask,SNR=1)	4.9510	4.6381	2.2349e+004
Trio (7x7 stochastic gradient mask,SNR=1)	5.0009	4.6603	2.2236e+004

TABLE IV
STATISTICAL RESULTS FOR FOURTH IMAGE

Image	Entropy	PSNR	MSE
Duo(Sobel)	5.2921	8.1320	9.9972e+003
Duo(Laplacian)	5.4790	8.6967	8.7782e+003
Duo (LOG)	5.3489	8.4020	9.3947e+003
Duo (Prewitt)	5.3048	8.1345	9.9915e+003
Duo (3x3 stochastic gradient mask,SNR=9)	5.3216	8.1642	9.9233e+003
Duo (5x5 stochastic gradient mask,SNR=9)	5.1320	7.7275	1.0973e+004
Duo (7x7 stochastic gradient mask,SNR=9)	5.0426	7.6154	1.1260e+004
Duo (3x3 stochastic gradient mask,SNR=1)	5.2643	8.0671	1.0148e+004
Duo (5x5stochastic gradient mask,SNR=1)	5.0966	7.6681	1.1124e+004
Duo (7x7 stochastic gradient mask,SNR=1)	5.0004	7.5112	1.1533e+004

PSNR is most commonly used to measure the quality of reconstruction of lossy compression codecs (e.g., for image compression). The signal in this case is the original data, and the noise is the error introduced by compression. When comparing compression codecs, PSNR is an *approximation* to human perception of reconstruction quality.

Although a higher PSNR generally indicates that the reconstruction is of higher quality, in some cases it may not. One has to be extremely careful with the range of validity of this metric; it is only conclusively valid when it is used to compare results from the same codec (or codec type) and same content.

PSNR is most easily defined via the **mean squared error (*MSE*)**. Given a noise-free ***m*** x ***n*** monochrome image ***I*** and its noisy approximation ***K***, *MSE* is defined as:

$$\mathrm{MSE} = \frac{1}{mn}\sum_{i=0}^{m-1}\sum_{j=0}^{n-1}[I(i,j) - K(i,j)]^2 \qquad (13)$$

The PSNR (in dB) is defined as:

$$PSNR = 10.\log_{10}\left(\frac{{MAX_I}^2}{MSE}\right) = 20.\log_{10}\left(\frac{MAX_I}{\sqrt{MSE}}\right)$$
$$= 20\log_{10}(MAX_I) - 10.\log_{10}(MSE) \quad (14)$$

The images obtained as a result of applying 7x7 Stochastic gradient mask of SNR=1 and SNR=9 produce lesser entropy and PSNR with higher MSE. The images from applying 3x3 Stochastic mask give higher Entropy and PSNR with less MSE. It can also be noticed that Stochastic mask with SNR =1 gives slightly better result than SNR=9.

VIII. Conclusion

Image edge detection is an essential process for most subsequent image analysis tasks. In this research paper, an effective methodology for edge detection of noisy images has been publicized using Stochastic gradient masks. The proposed methodology has been successfully tested on four images. The results demonstrate that for noisy images, Stochastic Gradient yields much better results than conventional spatial filters. The experimental results and statistical measurements confirm the efficiency of the proposed methodology.

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