

# Modeling Reflection and Transmission of Elastodiffusive Wave Sata Semiconductor Interface

A. A. Sharma, B. J. N. Sharma

**Abstract**—This paper deals with the study of reflection and transmission characteristics of acoustic waves at the interface of a semiconductor half-space and elastic solid. The amplitude ratios (reflection and transmission coefficients) of reflected and transmitted waves to that of incident wave varying with the incident angles have been examined for the case of quasi-longitudinal wave. The special cases of normal and grazing incidence have also been derived with the help of Gauss elimination method. The mathematical model consisting of governing partial differential equations of motion and charge carriers' diffusion of n-type semiconductors and elastic solid has been solved both analytically and numerically in the study. The numerical computations of reflection and transmission coefficients has been carried out by using MATLAB programming software for silicon (Si) semiconductor and copper elastic solid. The computer simulated results have been plotted graphically for Si semiconductors. The study may be useful in semiconductors, geology, and seismology in addition to surface acoustic wave (SAW) devices.

**Keywords**—Quasilongitudinal, reflection and transmission, semiconductors, acoustics.

## I. INTRODUCTION

GUTENBERG [1] studied the amplitudes of elastic waves passing through layers of the earth, either in earthquakes or from artificial explosions, which requires knowledge of the distribution of energy at points of reflection or refraction, and also of the ratio between the amplitudes arriving at the surface of the earth and the displacements of the ground there. Deresiewicz [2] calculated the amplitude ratios for the four reflected and transmitted waves generated on the interface between two thermally conducting solids by an incident dilatational wave. Sinha and Elsibai [3] studied the reflection and refraction of thermoelastic waves at an interface of two semi-infinite media in contact with two relaxation times and carried out analytical expressions for the partition of energy.

The detailed discussion and investigations on waves and vibrations in elastic solids can be found in the monumental books by Graff [4], Achenbach [5], and Love [6]. Maruszewski [7] derived the coupled evolution equations of charge carriers, momentum, energy and the evolution equations of heat flux and charge carrier flux in deformable, extrinsic semiconductor in the framework of extended irreversible thermodynamics.

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## II. FORMULATION OF THE PROBLEM

The origin of rectangular Cartesian co-ordinate system  $OXYZ$  has been taken at a fixed point on the interface of the ( $n$ -type) semiconductor-elastic solid half-spaces with positive  $Z$  - axis directed normally into the elastic solid medium and  $X$  -axis has been taken along the direction of propagation of waves. The  $Y$  -axis has been taken in the direction of the line of intersection of the plane wave front with the plane surface as shown in Fig. 1. We restrict our analysis to plain strain in the  $XZ$  -plane, so that the entire field variables may be taken as function of  $X$ ,  $Z$  and  $\tau$  only.

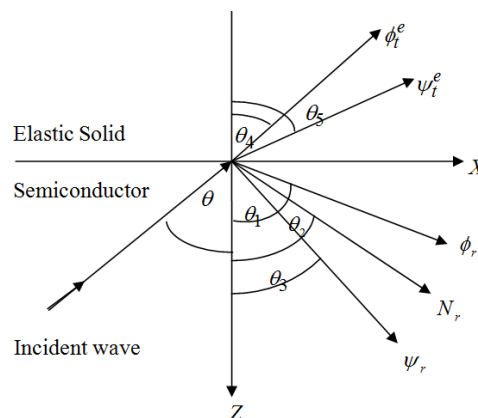


Fig. 1 Geometry of the Problem

The basic governing equations for the present case in non-dimensional form are given by:

$$\delta^2 \nabla^2 \mathbf{U} + (1 - \delta^2) \nabla \nabla \cdot \mathbf{U} - \nabla N^* = \ddot{\mathbf{U}} \quad (1)$$

$$\nabla^2 N^* - \left[ -\frac{1}{\tau_n^+} + \left( 1 - \frac{\tau^n}{\tau_n^+} \right) \frac{\partial}{\partial t} + \tau^n \frac{\partial^2}{\partial t^2} \right] N^* - \varepsilon_n \nabla \cdot \mathbf{U} = 0$$

$$\sigma_{ZZ} = (1 - 2\delta^2) \frac{\partial U}{\partial X} + \frac{\partial W}{\partial Z} - N^* \quad (2)$$

$$\sigma_{XZ} = \delta^2 \left( \frac{\partial U}{\partial Z} + \frac{\partial W}{\partial X} \right)$$

where

$$\omega_n^* = \frac{c_1^2}{D^n}, c_1^2 = \frac{\lambda + 2\mu}{\rho}, c_2^2 = \frac{\mu}{\rho}, c_L^2 = \frac{\lambda L}{\rho L} \quad (3)$$

are respectively, the elastodiffusive characteristic frequency, velocities of longitudinal and shear waves in  $n$ -type semiconductor and velocity of sound in liquid.

In view of the geometry of the problem given in Fig. 1, the formal solutions in case of ( $n$ -type) semiconductor and elastic solid media become:

$$\phi_i(X, Z, \tau) = A_i \exp\{ik_1(X \sin \theta_1 - Z \cos \theta_1 - v_1 \tau)\} \quad (4)$$

$$\begin{aligned} \phi_r(X, Z, \tau) = & A_1 \exp\{ik_1(X \sin \theta_1 + Z \cos \theta_1 - v_1 \tau)\} \\ & + A_2 \exp\{ik_2(X \sin \theta_2 + Z \cos \theta_2 - v_2 \tau)\} \end{aligned} \quad (5)$$

$$N_i(X, Z, \tau) = S_1 A_i \exp\{ik_1(X \sin \theta_1 - Z \cos \theta_1 - v_1 \tau)\} \quad (6)$$

$$N_r(X, Z, \tau) = \sum_{j=1}^2 S_j A_j \exp\{ik_j(X \sin \theta_j + Z \cos \theta_j - v_j \tau)\} \quad (7)$$

$$\psi_r(X, Z, \tau) = A_3 \exp\{ik_3(X \sin \theta_3 + Z \cos \theta_3 - v_3 \tau)\} \quad (8)$$

$$\phi_i^e(X, Z, \tau) = A_4 \exp\{ik_4(X \sin \theta_4 - Z \cos \theta_4 - v_4 \tau)\} \quad (9)$$

$$\psi_i^e(X, Z, \tau) = A_5 \exp\{ik_5(X \sin \theta_5 - Z \cos \theta_5 - v_5 \tau)\} \quad (10)$$

where the quantities  $S_j$  ( $j=1,2$ ) are defined by  $S_j = \Omega^2(1 - a_j^2) = \frac{\kappa \Omega \varepsilon_n a_j^2}{\alpha_n - a_j^2}$ , ( $j=1, 2$ ) and other non-dimensional quantities have been defined by:  
*n-type semiconductor:*

$$\begin{aligned} X &= \frac{\omega_n^*}{c_1} x, \quad Z = \frac{\omega_n^*}{c_1} z, \quad \tau = \omega_n^* t, \quad N^* = \frac{N}{n_0}, \\ \sigma_{ZZ}^n &= \frac{\tau_{zz}^n}{\lambda^n n_0}, \quad \sigma_{XZ}^n = \frac{\tau_{xz}^n}{\lambda^n n_0}, \quad \sigma_{ZZ}^L = \frac{\tau_{zz}^L}{\lambda^n n_0}, \quad \sigma_{XZ}^L = \frac{\tau_{xz}^L}{\lambda^n n_0} \\ \mathbf{U} &= \frac{\rho \omega_n^* c_1}{\lambda^n n_0} \mathbf{u}, \quad \mathbf{U}_L = \frac{\rho \omega_n^* c_1}{\lambda^n n_0} \mathbf{u}_L, \quad \tau_n^+ = t_n^+ \omega_n^*, \quad \tau_n = t_n \omega_n^*, \\ \delta_L^2 &= \frac{c_L^2}{c_1^2}, \quad \delta^2 = \frac{c_2^2}{c_1^2}, \quad \varepsilon_n = \frac{a_2^n T_0 \lambda^T \lambda^n}{\rho(\lambda + 2\mu)} \end{aligned} \quad (11)$$

### III. BOUNDARY CONDITIONS

The interface ( $Z=0$ ) of the ( $n$ -type) semiconductor and elastic solid is subjected to the following conditions:

- (i) The stresses appearing at the interface ( $Z=0$ ) of the semiconductor-elastic solid must balance the effect of each other. This in non-dimensional form requires that:

$$\sigma_{ZZ} = \sigma_{ZZ}^e,$$

$$\sigma_{XZ} = \sigma_{XZ}^e \quad (12)$$

- (ii) The requirement of the continuity of displacement components at the interface ( $Z=0$ ) leads to the boundary conditions in non-dimensional form:

$$\begin{aligned} U &= U^e, \\ W &= W^e \end{aligned} \quad (13)$$

- (iii) The electron concentration satisfies the condition at ( $Z=0$ ) as:

$$\frac{\partial N^*}{\partial Z} + h_n \left( 1 + \tau^n \frac{\partial}{\partial \tau} \right) N^* = 0 \quad (14)$$

where  $h_n = \frac{s^n}{c_1}$ ,  $s^n$  surface recombination velocity of electrons, respectively.

### IV. SOLUTION OF THE PROBLEM

Consider the cases of wave incidence at the interface of ( $n$ -type) semiconductor and elastic solid half-spaces namely:

#### A. Case I: Quasi-Longitudinal ( $q_L$ ) Wave Incidence

The similar cases of wave incidence can be discussed at the interface of ( $p$ -type) semiconductor and elastic solid half-spaces. Here, we discuss the reflection and transmission of waves at the interface of the ( $n$ -type) semiconductor and elastic solid half-spaces for the above case.

The functions  $\phi$ ,  $\psi$ ,  $N$ ,  $\phi_i^e$ , and  $\psi_i^e$  for of incidence and reflected waves can be written from (1) to (7) as:

$$\begin{aligned} \phi &= \phi_i + \phi_r = A_i \exp\{ik_1(X \sin \theta_1 - Z \cos \theta_1 - v_1 \tau)\} \\ &+ \sum_{j=1}^2 A_j \exp\{ik_j(X \sin \theta_j + Z \cos \theta_j - v_j \tau)\} \end{aligned} \quad (15)$$

$$\begin{aligned} N^* &= N_i^* + N_r^* = S_1 A_i \exp\{ik_1(X \sin \theta - Z \cos \theta - v_1 \tau)\} \\ &+ \sum_{j=1}^2 S_j A_j \exp\{ik_j(X \sin \theta_j + Z \cos \theta_j - v_j \tau)\} \end{aligned} \quad (16)$$

$$\psi = \psi_r = A_3 \exp\{ik_3(X \sin \theta_3 + Z \cos \theta_3 - v_3 \tau)\} \quad (17)$$

$$\phi^e = \phi_i^e = A_4 \exp\{ik_4(X \sin \theta_4 - Z \cos \theta_4 - v_4 \tau)\} \quad (18)$$

$$\psi^e = \psi_i^e = A_5 \exp\{ik_5(X \sin \theta_5 - Z \cos \theta_5 - v_5 \tau)\} \quad (19)$$

Upon using the above solutions (15)-(19) in the boundary conditions (12)-(14) one obtains a system of five coupled linear algebraic equations:

$$\begin{aligned} & (2\delta^2 k_1^2 \sin^2 \theta - \Omega^2) A_1 \exp\{ik_1 (X \sin \theta)\} \\ & + \sum_{j=1}^2 (2\delta^2 k_j^2 \sin^2 \theta_j - \Omega^2) A_j \exp\{ik_j (X \sin \theta_j)\} + \\ & (\delta^2 k_3^2 \sin 2\theta_3) A_3 \exp\{ik_3 (X \sin \theta_3)\} \\ & - (2\delta_e^2 k_4^2 \sin^2 \theta_4 - \Omega^2) A_4 \exp\{ik_4 (X \sin \theta_4)\} \\ & + (\delta_e^2 k_5^2 \sin 2\theta_5) A_5 \exp\{ik_5 (X \sin \theta_5)\} = 0 \quad (19a) \end{aligned}$$

$$\begin{aligned} & (\delta^2 k_1^2 \sin 2\theta) A_1 \exp\{ik_1 (X \sin \theta)\} \\ & + \sum_{j=1}^2 (\delta^2 k_j^2 \sin 2\theta_j) A_j \exp\{ik_j (X \sin \theta_j)\} \\ & - (2\delta^2 k_3^2 \sin^2 \theta_3 - \Omega^2) A_3 \exp\{ik_3 (X \sin \theta_3)\} \\ & - (\delta_e^2 k_4^2 \sin 2\theta_4) A_4 \exp\{ik_4 (X \sin \theta_4)\} \\ & - (2\delta_e^2 k_5^2 \sin 2\theta_5 - \Omega^2) A_5 \exp\{ik_5 (X \sin \theta_5)\} = 0 \quad (19b) \end{aligned}$$

$$\begin{aligned} & k_1 \sin \theta A_1 \exp\{ik_1 (X \sin \theta)\} \\ & + \sum_{j=1}^2 k_j \sin \theta_j A_j \exp\{ik_j (X \sin \theta_j)\} \\ & + k_3 \cos \theta_3 A_3 \exp\{ik_3 (X \sin \theta_3)\} \\ & - k_4 \sin \theta_4 A_4 \exp\{ik_4 (X \sin \theta_4)\} \\ & + k_5 \cos \theta_5 A_5 \exp\{ik_5 (X \sin \theta_5)\} = 0 \quad (19c) \end{aligned}$$

$$\begin{aligned} & -k_1 \cos \theta A_1 \exp\{ik_1 (X \sin \theta)\} \\ & + \sum_{j=1}^2 k_j \cos \theta_j A_j \exp\{ik_j (X \sin \theta_j)\} \\ & - k_3 \sin \theta_3 A_3 \exp\{ik_3 (X \sin \theta_3)\} \\ & + k_4 \cos \theta_4 A_4 \exp\{ik_4 (X \sin \theta_4)\} \\ & + k_5 \sin \theta_5 A_5 \exp\{ik_5 (X \sin \theta_5)\} = 0 \quad (19d) \end{aligned}$$

$$\begin{aligned} & -S_1 k_1 \cos \theta A_1 \exp\{ik_1 (X \sin \theta)\} \\ & + \sum_{j=1}^2 S_j k_j \cos \theta_j A_j \exp\{ik_j (X \sin \theta_j)\} = 0 \quad (19e) \end{aligned}$$

Assuming that all the waves; incident, reflected or transmitted; must be in phase at the surface  $Z = 0$ , for all values of  $X$  and  $\tau$ , one has:

$$k_1 \sin \theta = k_1 \sin \theta_1 = k_2 \sin \theta_2 = k_3 \sin \theta_3 = k_4 \sin \theta_4 \quad (20)$$

Upon using relation (20) in the above stated system of equations in the amplitude ratios of reflected and transmitted waves, one obtains:

$$\begin{aligned} & \cos 2\theta_3 R_1^{qL} + \cos 2\theta_3 R_2^{qL} - \sin 2\theta_3 R_3^{qL} + (2\delta_e^2 \sin^2 \theta_4 - 1) T_1^{qL} \\ & - \sin 2\theta_5 T_2^{qL} = -\cos 2\theta_3 \quad (21) \end{aligned}$$

$$\begin{aligned} & a_1^2 \delta^2 \sin 2\theta_1 R_1^{qL} + a_2^2 \delta^2 \sin 2\theta_2 R_2^{qL} + \cos 2\theta_3 R_3^{qL} \\ & - \delta_e^2 \sin 2\theta_4 T_1^{qL} + \cos 2\theta_5 T_2^{qL} = -a_1^2 \delta^2 \sin 2\theta_1 \quad (22) \end{aligned}$$

$$\sin \theta_1 R_1^{qL} + \frac{a_2}{a_1} \sin \theta_2 R_2^{qL} + \frac{1}{a_1 \delta} \cos \theta_3 R_3^{qL} - \frac{\sin \theta_4}{a_1} T_1^{qL} + \frac{\cos \theta_5}{a_1 \delta_e} T_2^{qL} = -\sin \theta_1 \quad (23)$$

$$\begin{aligned} & \cos \theta_1 R_1^{qL} + \frac{a_2}{a_1} \cos \theta_2 R_2^{qL} - \frac{1}{a_1 \delta} \sin \theta_3 R_3^{qL} + \frac{\cos \theta_4}{a_1} T_1^{qL} + \frac{\sin \theta_5}{a_1 \delta_e} T_2^{qL} \\ & = \cos \theta_1 \quad (24) \end{aligned}$$

$$\begin{aligned} & a_1 S_1 \cos \theta_1 R_1^{qL} + a_2 S_2 \cos \theta_2 R_2^{qL} + 0 R_3^{qL} + 0 T_1^{qL} + 0 T_2^{qL} \\ & = a_1 S_1 \cos \theta_1 \quad (25) \end{aligned}$$

The above system of coupled algebraic equations (21)-(25) can be rewritten in matrix form as:

$$\mathbf{M} \mathbf{R}^{qL} = \mathbf{P}^{qL} \quad (26)$$

where:

$$\mathbf{M} = \begin{bmatrix} \cos 2\theta_3 & \cos 2\theta_3 & -\sin 2\theta_3 & (2\delta_e^2 \sin^2 \theta_4 - 1) & -\sin 2\theta_5 \\ a_1^2 \delta^2 \sin 2\theta_1 & a_2^2 \delta^2 \sin 2\theta_2 & -\cos 2\theta_3 & -\delta_e^2 \sin 2\theta_4 & \cos 2\theta_3 \\ \sin \theta_1 & \frac{a_2}{a_1} \sin \theta_2 & \frac{1}{a_1 \delta} \cos \theta_3 & -\frac{\sin \theta_4}{a_1} & \frac{\cos \theta_5}{a_1 \delta_e} \\ \cos \theta_1 & \frac{a_2}{a_1} \cos \theta_2 & -\frac{1}{a_1 \delta} \sin \theta_3 & \frac{\cos \theta_4}{a_1} & \frac{\sin \theta_5}{a_1 \delta_e} \\ a_1 S_1 \cos \theta_1 & a_2 S_2 \cos \theta_2 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{R}^{qL} = \begin{bmatrix} R_1^{qL} \\ R_2^{qL} \\ R_3^{qL} \\ T_1^{qL} \\ T_2^{qL} \end{bmatrix}$$

$$\mathbf{P}^{qL} = \begin{bmatrix} -\cos 2\theta_3 \\ -a_1^2 \delta^2 \sin 2\theta_1 \\ -\sin \theta_1 \\ \cos \theta_1 \\ a_1 S_1 \cos \theta_1 \end{bmatrix} \quad (27)$$

Solving the system of equations (26) the amplitude ratios of reflected waves  $R_k^{qL}$  ( $k = 1, 2, 3$ ) and transmitted waves  $T_i^{qL}$  ( $i = 1, 2$ ) have been obtained as:

$$R_1^{qL} = \frac{|\mathbf{M}_1|}{|\mathbf{M}|} \quad R_2^{qL} = \frac{|\mathbf{M}_2|}{|\mathbf{M}|} \quad R_3^{qL} = \frac{|\mathbf{M}_3|}{|\mathbf{M}|} \quad T_1^{qL} = \frac{|\mathbf{M}_4|}{|\mathbf{M}|} \quad T_2^{qL} = \frac{|\mathbf{M}_5|}{|\mathbf{M}|} \quad (28)$$

where

$$|\mathbf{M}| = \begin{vmatrix} \cos 2\theta_3 & \cos 2\theta_3 & -\sin 2\theta_3 & (2\delta_e^2 \sin^2 \theta_4 - 1) & -\sin 2\theta_5 \\ a_1^2 \delta^2 \sin 2\theta_1 & a_2^2 \delta^2 \sin 2\theta_2 & -\cos 2\theta_3 & -\delta_e^2 \sin 2\theta_4 & \cos 2\theta_5 \\ \sin \theta_1 & \frac{a_2 \sin \theta_2}{a_1} & \frac{\cos \theta_3}{a_1 \delta} & \frac{-\sin \theta_4}{a_1} & \frac{\cos \theta_5}{a_1 \delta_e} \\ \cos \theta_1 & \frac{a_2 \cos \theta_2}{a_1} & \frac{-\sin \theta_3}{a_1 \delta} & \frac{\cos \theta_4}{a_1} & \frac{\sin \theta_5}{a_1 \delta_e} \\ a_1 S_1 \cos \theta_1 & a_2 S_2 \cos \theta_2 & 0 & 0 & 0 \end{vmatrix}$$

$$|\mathbf{M}_1| = \begin{vmatrix} -\cos 2\theta_3 & \cos 2\theta_3 & -\sin 2\theta_3 & (2\delta_e^2 \sin^2 \theta_4 - 1) & -\sin 2\theta_5 \\ -a_1^2 \delta^2 \sin 2\theta_1 & a_2^2 \delta^2 \sin 2\theta_2 & -\cos 2\theta_3 & -\delta_e^2 \sin 2\theta_4 & \cos 2\theta_5 \\ -\sin \theta_1 & \frac{a_2 \sin \theta_2}{a_1} & \frac{\cos \theta_3}{a_1 \delta} & \frac{-\sin \theta_4}{a_1} & \frac{\cos \theta_5}{a_1 \delta_e} \\ \cos \theta_1 & \frac{a_2 \cos \theta_2}{a_1} & \frac{-\sin \theta_3}{a_1 \delta} & \frac{\cos \theta_4}{a_1} & \frac{\sin \theta_5}{a_1 \delta_e} \\ a_1 S_1 \cos \theta_1 & a_2 S_2 \cos \theta_2 & 0 & 0 & 0 \end{vmatrix}$$

$$|\mathbf{M}_2| = \begin{vmatrix} \cos 2\theta_3 & -\cos 2\theta_3 & -\sin 2\theta_3 & (2\delta_e^2 \sin^2 \theta_4 - 1) & -\sin 2\theta_5 \\ a_1^2 \delta^2 \sin 2\theta_1 & a_1^2 \delta^2 \sin 2\theta_1 & -\cos 2\theta_3 & -\delta_e^2 \sin 2\theta_4 & \cos 2\theta_5 \\ \sin \theta_1 & -\sin \theta_1 & \frac{\cos \theta_3}{a_1 \delta} & \frac{-\sin \theta_4}{a_1} & \frac{\cos \theta_5}{a_1 \delta_e} \\ \cos \theta_1 & \cos \theta_1 & \frac{-\sin \theta_3}{a_1 \delta} & \frac{\cos \theta_4}{a_1} & \frac{\sin \theta_5}{a_1 \delta_e} \\ a_1 S_1 \cos \theta_1 & a_1 S_1 \cos \theta_1 & 0 & 0 & 0 \end{vmatrix}$$

$$|\mathbf{M}_3| = \begin{vmatrix} \cos 2\theta_3 & \cos 2\theta_3 & -\cos 2\theta_1 & (2\delta_e^2 \sin^2 \theta_4 - 1) & -\sin 2\theta_5 \\ a_1^2 \delta^2 \sin 2\theta_1 & a_2^2 \delta^2 \sin 2\theta_2 & -a_1^2 \delta^2 \sin 2\theta_1 & -\delta_e^2 \sin 2\theta_4 & \cos 2\theta_5 \\ \sin \theta_1 & \frac{a_2 \sin \theta_2}{a_1} & -\sin \theta_1 & \frac{-\sin \theta_4}{a_1} & \frac{\cos \theta_5}{a_1 \delta_e} \\ \cos \theta_1 & \frac{a_2 \cos \theta_2}{a_1} & \cos \theta_1 & \frac{\cos \theta_4}{a_1} & \frac{\sin \theta_5}{a_1 \delta_e} \\ a_1 S_1 \cos \theta_1 & a_2 S_2 \cos \theta_2 & a_1 S_1 \cos \theta_1 & 0 & 0 \end{vmatrix}$$

$$|\mathbf{M}_4| = \begin{vmatrix} \cos 2\theta_3 & \cos 2\theta_3 & -\sin 2\theta_3 & -\cos 2\theta_3 & -\sin 2\theta_5 \\ a_1^2 \delta^2 \sin 2\theta_1 & a_2^2 \delta^2 \sin 2\theta_2 & -\cos 2\theta_3 & -a_1^2 \delta^2 \sin 2\theta_1 & \cos 2\theta_5 \\ \sin \theta_1 & \frac{a_2 \sin \theta_2}{a_1} & \frac{\cos \theta_3}{a_1 \delta} & -\sin \theta_1 & \frac{\cos \theta_5}{a_1 \delta_e} \\ \cos \theta_1 & \frac{a_2 \cos \theta_2}{a_1} & \frac{-\sin \theta_3}{a_1 \delta} & \cos \theta_1 & \frac{\sin \theta_5}{a_1 \delta_e} \\ a_1 S_1 \cos \theta_1 & a_2 S_2 \cos \theta_2 & 0 & a_1 S_1 \cos \theta_1 & 0 \end{vmatrix}$$

$$|\mathbf{M}_5| = \begin{vmatrix} \cos 2\theta_3 & \cos 2\theta_3 & -\sin 2\theta_3 & (2\delta_e^2 \sin^2 \theta_4 - 1) & -\cos 2\theta_3 \\ a_1^2 \delta^2 \sin 2\theta_1 & a_2^2 \delta^2 \sin 2\theta_2 & -\cos 2\theta_3 & -\delta_e^2 \sin 2\theta_4 & a_1^2 \delta^2 \sin 2\theta_1 \\ \sin \theta_1 & \frac{a_2 \sin \theta_2}{a_1} & \frac{\cos \theta_3}{a_1 \delta} & \frac{-\sin \theta_4}{a_1} & -\sin \theta_1 \\ \cos \theta_1 & \frac{a_2 \cos \theta_2}{a_1} & \frac{-\sin \theta_3}{a_1 \delta} & \frac{\cos \theta_4}{a_1} & \cos \theta_1 \\ a_1 S_1 \cos \theta_1 & a_2 S_2 \cos \theta_2 & 0 & 0 & a_1 S_1 \cos \theta_1 \end{vmatrix}$$

Thus the energy will be distributed among the reflected and transmitted waves.

### B. Grazing Incidence

For grazing incidence ( $\theta = \frac{\pi}{2} = \theta_1 = \theta_4$ ), so that the determinants (29) become:

$$\begin{aligned} |\mathbf{M}_1| &= -|\mathbf{M}| \\ |\mathbf{M}_2| &= 0 = |\mathbf{M}_3| \\ |\mathbf{M}_4| &= 0 = |\mathbf{M}_5| \end{aligned}$$

Consequently, the expressions for the reflection and transmission coefficients (28) reduce to:

$$R_1^{qL} = -1, \quad R_2^{qL} = 0, \quad R_3^{qL} = 0, \quad T_1^{qL} = 0, \quad T_2^{qL} = 0$$

Thus in the general case, the  $qL$  wave is again being  $180^\circ$  out of phase with the incidence wave and annihilates itself. Other waves neither reflect nor transmit through the interface.

### C. Normal Incidence

For normal incidence ( $\theta = 0^\circ = \theta_1 = \theta_4$ ), the reflection and transmission coefficients in (28) become:

$$\begin{aligned} R_1^{qL} &= 1 + \frac{2a_2 S_2 \cos \theta_2 \cos 2\theta_3}{a_1 \delta \delta_e |\hat{\mathbf{M}}|} [\delta \cos 2\theta_3 \cos \theta_5 + \delta_e \cos \theta_3 \cos 2\theta_5] \\ R_2^{qL} &= -\frac{2S_1 \cos 2\theta_3}{a_1 \delta \delta_e |\hat{\mathbf{M}}|} [\delta \cos 2\theta_3 \cos \theta_5 + \delta_e \cos \theta_3 \cos 2\theta_5] \end{aligned}$$

$$R_3^{qL} = -\frac{2a_2S_1 \cos 2\theta_3}{a_1\delta_e|\hat{\mathbf{M}}|} [a_2\delta^2 \sin 2\theta_2 \cos \theta_3 - \delta_e \sin \theta_2 \cos 2\theta_3]$$

$$T_1^{qL} = \frac{2a_2 \cos 2\theta_3}{a_1|\hat{\mathbf{M}}|} \begin{vmatrix} a_2\delta^2 \sin 2\theta_2 & -\cos 2\theta_3 & 0 & \cos 2\theta_3 \\ \sin \theta_2 & \frac{\cos \theta_3}{\delta} & 0 & \frac{\cos \theta_3}{\delta_e} \\ \cos \theta_2 & \frac{-\sin \theta_3}{\delta} & 1 & \frac{\sin \theta_3}{\delta_e} \\ a_1S_2 \cos \theta_2 & 0 & a_1S_1 & 0 \end{vmatrix}$$

$$T_2^{qL} = \frac{2a_2S_1 \cos 2\theta_3}{a_1|\hat{\mathbf{M}}|} [a_2\delta \sin 2\theta_2 \cos \theta_3 + \sin \theta_2 \cos 2\theta_3] \quad (31)$$

where:

$$|\hat{\mathbf{M}}| = \frac{1}{a_1^2} \begin{vmatrix} \cos 2\theta_3 & \cos 2\theta_3 & -\sin 2\theta_3 & 1 & -\sin 2\theta_3 \\ 0 & a_2^2\delta^2 \sin 2\theta_2 & -\cos 2\theta_3 & 0 & \cos 2\theta_3 \\ 0 & a_2 \sin \theta_2 & \frac{\cos \theta_3}{\delta} & 0 & \frac{\cos \theta_3}{\delta_e} \\ a_1 & a_2 \cos \theta_2 & \frac{-\sin \theta_3}{\delta} & 1 & \frac{\sin \theta_3}{\delta_e} \\ a_1S_1 & a_2S_2 \cos \theta_2 & 0 & 0 & 0 \end{vmatrix} \quad (32)$$

### V. NUMERICAL RESULTS AND DISCUSSION

In this section, the reflection and transmission coefficients given by (28) have been computed numerically for silicon (Si) material under the assumption of relaxation type semiconductor (*n*-type or *p*-type) so that  $t_n$ ,  $t_n^+$  and  $t_p$ ,  $t_p^+$  become comparable to each other in their values such that  $t_n = t_n^+$  and  $t_p = t_p^+$ . Here the elastic solid chosen for the purpose of numerical calculations is copper, for which the physical data is given by  $\lambda = 8.2 \times 10^{10} \text{ N/m}^2$ ,  $\mu = 4.2 \times 10^{10} \text{ N/m}^2$  and density is  $\rho_e = 8.95 \times 10^3 \text{ kg/m}^3$

The values of reflection coefficients  $R_k^{qL}$  ( $k=1, 2, 3$ ),  $R_k^{qT}$  ( $k=1, 2, 3$ ) and transmission coefficients  $T_1^{qL}$ ,  $T_2^{qL}$  and  $T_1^{qT}$ ,  $T_2^{qT}$  for incident  $qL$  and  $qT$  waves have been computed from (28) for various values of the angle of incidence ( $\theta$ ) lying between  $0^\circ \leq \theta \leq 90^\circ$ .

From Fig. 2, it is noticed that the magnitude of reflection coefficient  $R_1^{qL}$  sharply decreases in the range between  $0^\circ \leq \theta \leq 30^\circ$  and after which it starts increasing slowly and steadily to regain its original value at  $\theta = 90^\circ$ . This shows that  $qL$  wave loses power to other reflected/transmitted waves in the former range and regains its power for  $\theta \geq 30^\circ$  to recover it fully in the neighborhood of grazing incidence. The magnitude of the reflection coefficient ( $R_2^{qL}$ ) increases in the

range  $0^\circ \leq \theta \leq 22^\circ 36'$  and attains its maximum value at  $\theta = 22^\circ 36'$  due to resonance phenomenon in this case. It is noticed that the loss of energy by the reflected quasi-longitudinal elastic wave, ( $R_1^{qL}$ ) is gained by quasi-longitudinal electron ( $R_2^{qL}$ ) wave in the above range. This shows that there is a complete transfer of mechanical energy into electronic energy in this range of incidence angle which is observed to be a new phenomenon here. The magnitude of reflection coefficient ( $R_3^{qL}$ ) decreases sharply initially and then steadily for  $\theta \geq 22^\circ 36'$  to vanish at  $\theta = 90^\circ$ . It is also noticed that a meager amount of energy is associated with ( $R_3^{qL}$ ) and then wave virtually dies out at  $\theta = 45^\circ$  and  $\theta = 90^\circ$ . In contrast to this, the transmission coefficients ( $T_1^{qL}$ ) and ( $T_2^{qL}$ ) attain maximum values at  $\theta = 22^\circ 36'$  and  $\theta = 37^\circ$  respectively. Thus some part of energy is carried by both transmission coefficients ( $T_1^{qL}$ ) and ( $T_2^{qL}$ ) before they also vanish at  $\theta = 90^\circ$ . Therefore, it is noticed that maximum energy has been taken by electron wave in this case as compared to other reflected and transmitted waves.

Thus, the major portion of energy is carried by electron wave in comparison to reflected and transmitted waves in case of  $qT$  wave incidence at the semiconductor-elastic solid interface, that is there is complete conversion of mechanical energy into the electronic energy in the present case.

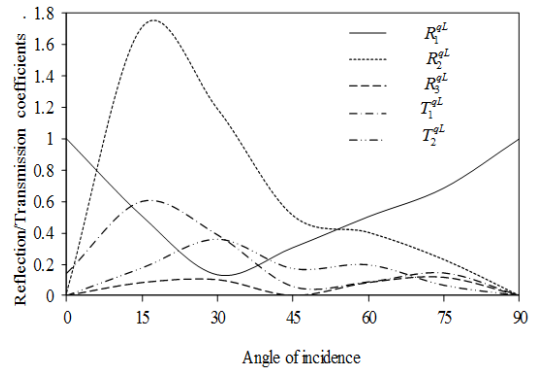


Fig. 2  $qL$  wave incidence at semiconductor-elastic solid interface

### VI. CONCLUSIONS

It is observed that the loss of energy by the reflected quasi-longitudinal ( $qL$ ) wave is acquired by quasi-longitudinal ( $N$ ) wave and hence there is a complete transfer of mechanical energy into an electronic energy. The maximum energy is carried by the electron waves in contrast to reflected and transmitted waves. At the grazing incidence the incident wave annihilates itself being  $180^\circ$  out of phase and no other wave is either reflected or transmitted. The incident wave get reflected and transmitted as it is in case of normal incidence. The

reflected wave is in phase and transmitted wave is  $180^\circ$  out of phase with incidence wave.

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