Estimation of Stress Intensity Factors from Near Crack Tip Field

Zhuang He, Andrei Kotousov

Abstract—All current experimental methods for determination of stress intensity factors are based on the assumption that the state of stress near the crack tip is plane stress. Therefore, these methods rely on strain and displacement measurements made outside the near crack tip region affected by the three-dimensional effects or by process zone. In this paper, we develop and validate an experimental procedure for the evaluation of stress intensity factors from the measurements of the out-of-plane displacements in the surface area controlled by 3D effects. The evaluation of stress intensity factors is possible when the process zone is sufficiently small, and the displacement field generated by the 3D effects is fully encapsulated by K-dominance region.

Keywords—Digital image correlation, stress intensity factors, three-dimensional effects, transverse displacement.

I. INTRODUCTION

STRESS intensity factors play a pivotal role in fracture assessment of structural components. It is not surprising that various analytical, numerical and experimental approaches were developed in the past, to numerically, theoretically or experimentally evaluate the values of the stress intensity factors. For relatively simple geometries and loading conditions, these values can be determined using many theoretical solutions developed in the past [1]-[3]. For more complicated geometries or loading conditions, they are often obtained from a careful finite element analysis [4]-[7]. However, these solutions and calculations normally require a proper validation before we can use it for practical fracture assessment. This validation can be achieved with various experimental techniques, which also were a subject of intensive studies over the past fifty years [8].

The experimental technique for the evaluation of the stress intensity factors can rely on the measurement of the displacement and strain fields around the crack tip region with strain gauges [9]-[11] or laser interferometry [12]-[14]. The optical method of caustics has extensively been used for the determination of stress intensity factors in crack problems [15]-[17] as well as the method utilising photoelasticity effects [18], [19]. The latter method is based on the property of birefringence, as exhibited by certain transparent materials. Over the past two decades the digital image correlation (DIC) method has been increasing in popularity in light of the

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advances in computer power, digital imaging and processing. Compared with above mentioned experimental techniques, digital image correlation has following advantages: 1, full field measurement compared to strain gages, which is limited to finite points; 2, applicable to any type of materials (both opaque and transparent materials); 3, needs no or very little surface preparation, experimental setup is less complex compared with caustics and photoelasticity. Moreover, it is suitable for measurements at various scale lengths [20], [21]. This method will be utilised in the current paper as well.
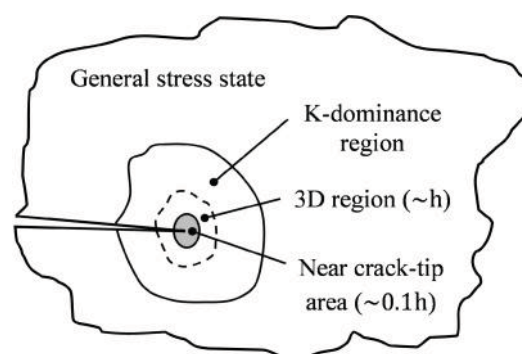

Fig. 1 Characteristic regions around the tip of a through-the-thickness crack

All current techniques and methods for the experimental evaluation of stress intensity factors are based on the assumption that the state of stress near the crack tip is plane stress [20], [22]-[24]. The collected measurements from the appropriate data extraction region is typically analysed based on the theoretical asymptotic expansion of plane strain or displacement field near the crack tip. However, many experimental and analytical solutions have demonstrated that the state of stress changes from plane stress to the three-dimensional near the crack tip [25]-[27]. This 3D region of the three-dimensional stress state is confined to approximately one-half of the plate thickness around the crack tip [28]-[31] as illustrated in Fig. 1. Therefore, this region is normally excluded from the current experimental procedures.

For sufficiently thick fracture specimens or geometries, depending on the type of loading and the material, K-dominance region, where the stress and strain fields are dominated by the leading singular term of the asymptotic expansion, may be very small or even disappear (immerge into the three-dimensional region). In these cases, additional higher order terms in the series expansion of displacement (or strain) fields have to be considered in order to improve the accuracy of the experimental evaluations. This situation has been

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widely discussed in many papers. For example, [32] identified the minimum required number of asymptotic terms needed for an accurate evaluation of stress intensity factors for different types of fracture specimens and loading conditions. Similarly, [33] analysed the influence of the radius of the data extraction region in conjunction with the minimum number of terms in the series expansion. It was concluded that more terms of the series expansion have to be taken into the evaluation procedure for a large radius of the data extraction region.

In this paper, we attempt to develop a new experimental procedure for the evaluation of the stress intensity factors, K_I (mode I) and K_{II} (mode II), by extracting and analysing the out-of-plane (transverse) displacement very near the crack tip, specifically from the surface area encapsulating the crack tip with a radius less than five percent of the plate thickness (see Fig. 1). This area, which is controlled by three-dimensional effects, including plate thickness [34], [35], coupled mode effects [36]-[38] and 3D corner singularities [39]-[41], does not follow either plane stress or plane strain assumptions; and, it is normally excluded from the data extraction region. The experimental estimation is possible when the material is sufficiently brittle and the process zone or zone of plastic deformations is smaller than 0.1h, where h is the half thickness of the specimen. In the case when the near crack area is fully encapsulated by K-dominance region, see Fig. 1, the intensity of the out-of-plane displacement in this surface area ($r < 0.1h$) is a linear function of the applied stress intensity factors, which can be expressed as [40]:

$$\widehat{U}_z = \frac{u_z(r, \phi, \pm h)E}{K_I^\infty \sqrt{h}} \approx -1.34 \cdot \nu, \tag{1}$$

for pure mode I loading, and

$$\widetilde{U}_z = -\frac{u_z(r, \phi, \pm h)E}{\sin\left(\frac{\phi}{2}\right) K_{II}^\infty \sqrt{h}} = \alpha(\nu) \cdot \left(\frac{r}{h}\right)^{\lambda_O} + \beta(\nu) \cdot \left(\frac{r}{h}\right)^{\lambda_{uII}} \tag{2}$$

for pure mode II loading.

Here, u_z is the out-of-plane displacement, r is the distance from the crack tip, ϕ is the angle measured with respect to the crack bisector line, K_I^∞, K_{II}^∞ are remotely applied mode I and mode II stress intensity factors, respectively. λ_O is the strength of the out-of-plane singularity ($\lambda_O = 1/2$ for sharp cracks [42]) and λ_{uII} is the strength of corner singularity, which as well as the coefficients α and β from (2) can be expressed in the form of [40]:

$$\lambda_{uII}(\nu) = 0.4\nu^2 - 0.46\nu + 0.497 \tag{3}$$

$$\alpha(\nu) = -3.125\nu^2 + 0.874\nu - 1.355 \tag{4}$$

$$\beta(\nu) = 2.75\nu^2 + 0.529\nu + 1.34 \tag{5}$$

Above (1)-(5) enable the experimental evaluation of K_I and K_{II}. In addition, the higher order non-singular terms in the series expansion can be ignored in the present formulation,

since these terms produce zero transverse displacement (due to their non-singular nature) at the crack tip or negligible in the near crack tip area. These terms need to be included for the analysis of the experimental data utilising the plane stress assumption because these techniques exclude the near crack tip region.

The paper is structured as follows: firstly, we conduct a careful experimental study, which considers edge cracked semi-circular specimens (made of PMMA) subjected to symmetric/antisymmetric three-point bend loading. Digital image correlation (DIC) method is then applied to extract the out-of-plane displacement field from the near crack tip region. Finally, the stress intensity factors are determined from the near crack tip field using (1)-(5) and compared with previously published theoretical solutions for K_I and K_{II}. Main outcomes of this work are summarised in the conclusion.

II. SPECIMEN CONFIGURATION AND LOADING SET UP

The specimens were manufactured from 50 mm thick sheets of PMMA with Young's modulus E = 2.97 GPa and Poisson's ratio ν = 0.35. The dimensions of the fabricated specimens (see Fig. 2) were chosen as: radius of the semi-circular specimen R = 150 mm, crack length a = 75 mm, specimen thickness 2h = 50 mm, which is thick enough to guarantee that the size of the process zone is sufficiently smaller than the characteristic size of the near crack tip area (0.1h). The straight cracks were inclined at 90° with respect to the flat edge of the specimens. In order to obtain a sharp crack tip, in the beginning a saw blade of 1mm thickness was used to cut a notch with the depth of 70 mm from the flat edge, which was further sharpened with a very thin fret saw blade of thickness 0.3 mm to produce the final length of the crack of 75 mm.
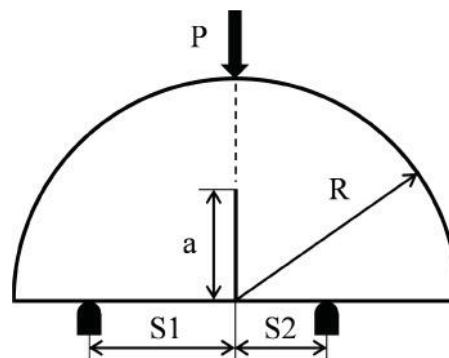

Fig. 2 Semi-circular specimen subjected to three-point bend loading

The loading set up is also shown in Fig. 2. The location of the first bottom support S1 was set at a fixed value of 100 mm. Two fracture modes were achieved by choosing the appropriate length of the second bottom-support S2 as follows: S2 = 100 mm for pure mode I condition (see Fig. 3 (a)), and S2 = 10.15 mm (suggested in [43]) for pure mode II (see Fig. 3 (b)). The specimens for digital image correlation experiment were prepared by painting the surface with a thin layer of white paint and then spraying black dots, which created a random speckle pattern to facilitate the image

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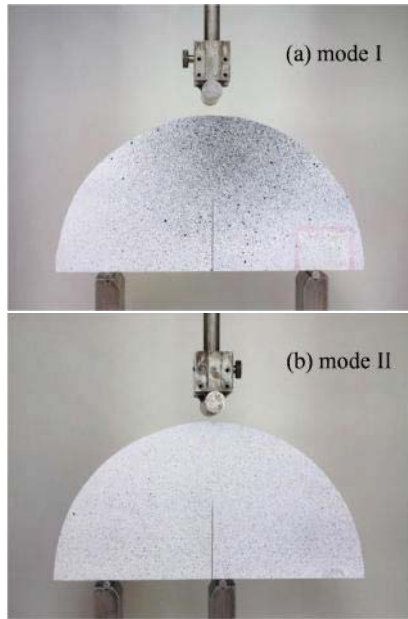correlation process for the calculation of surface displacement and strain fields.

Fig. 3 Loading set up for specimens in (a) pure mode I and (b) pure mode II conditions

III. DIC MEASUREMENT OF TRANSVERSE DISPLACEMENT

The experimental setup is shown in Fig. 4. The tests were performed using a screw-driven (INSTRON) tensile testing machine with a maximum loading capacity of 100 kN. The maximum loads applied on the top of the specimens were limited to $P = 3.5$ kN (for mode I) and 6.2 kN (for mode II) to avoid brittle fracture of the specimens, which are expected at $P_{max} = 9.4$ kN (for mode I) and 30.3 kN (for mode II). These values correspond to the average fracture toughness (for PMMA) of $K_C = 1.6$ MPa m$^{1/2}$.

Fig. 4 Vic-3D (Correlated Solutions, Inc.) set up for the measurement of out-of-plane surface displacement

The Vic-3D system (Correlated Solutions, Inc., Columbia, USA) was employed in this work to evaluate the out-of-plane (transverse) displacement field at the free surface of the specimen facing the cameras. The 3D system incorporates the image correlation software [44] and two digital cameras with a pair of lenses with adjustable focal lengths to achieve the required resolution and accuracy. A standard LED lighting system was also utilised to illuminate the sample surfaces. One hundred images acquired by each of the digital cameras in a short period immediately after loading were averaged to reduce the statistical error and correlated with the corresponding averaged image of the unloaded sample to obtain the three-dimensional displacement fields of the specified region.

IV. RESULTS AND DISCUSSION

Figs. 5 (a) and (b) show the contour plot of the transverse surface displacement u_z around the crack tip for semi-circular specimens stressed in mode I and mode II, respectively. The presented displacements had eliminated rigid body motion effects (displacement and rotation).
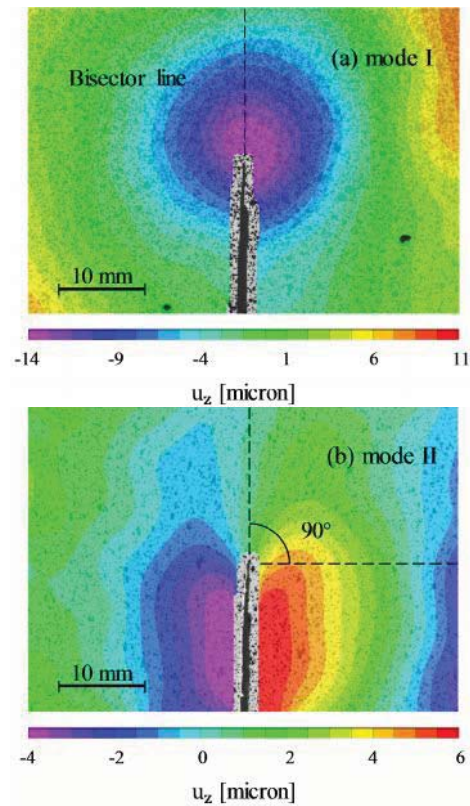

Fig. 5 Contour plot of the out-of-plane surface displacement u_z around the crack tip for specimens stressed in (a) mode I and (b) mode II

Further, we extracted the out-of-plane surface displacement component u_z along the bisector line for mode I loading (see dashed line in Fig. 5 (a)). For mode II loading, the transverse displacement component u_z was extracted along a line perpendicular to the bisector line (see Fig. 5 (b)). These experimentally obtained displacements were compared with the theoretical predictions, which can be derived by substituting the values of the applied stress intensity factors into (1), (2). K_I^{∞} and K_{II}^{∞} for the considered fracture specimen geometry and loading conditions can be calculated as given in [43]:

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$$K_I^{\infty} = \frac{P}{4Rh}\sqrt{\pi a}Y_I(a/R, S1/R, S2/R) \qquad (6)$$

$$K_{II}^{\infty} = \frac{P}{4Rh}\sqrt{\pi a}Y_{II}(a/R, S1/R, S2/R) \qquad (7)$$

Here, P is the applied load, Y_I and Y_{II} are dimensionless geometry factors, which equal to 5.23 and 1.63, respectively, for the present geometry of the specimens and loading types.
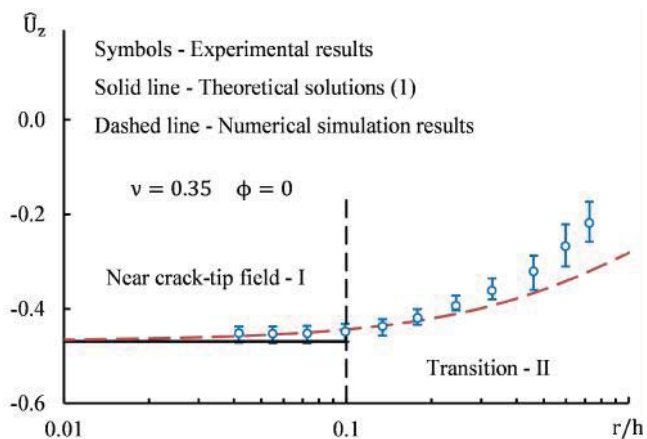

Fig. 6 Distribution of $\tilde{U}_z$ near mode I crack tip as a function of r/h, for the selected Poisson's ratio $\nu = 0.35$
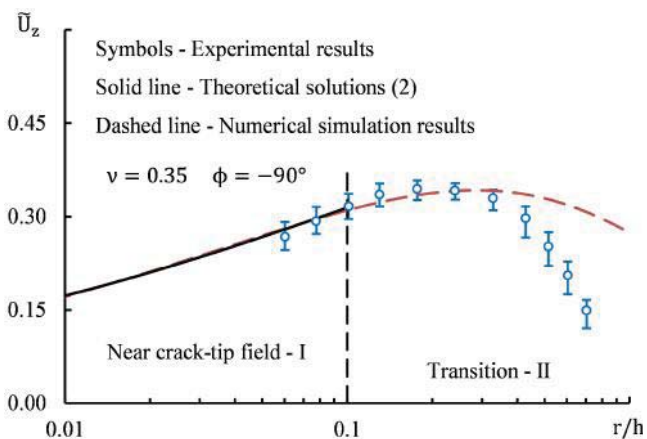

Fig. 7 Distribution of $\tilde{U}_z$ near mode II crack tip as a function of r/h, for the selected Poisson's ratio $\nu = 0.35$

Comparison among experimental, theoretical as well as the corresponding numerical simulation results derived by [40] are summarised in Fig. 6 (for mode I) and Fig. 7 (for mode II). It can be seen from both figures that the experimental data agrees very well with those obtained from the numerical simulations, especially in the near crack tip field. Some discrepancies between numerical and experimental results relatively far from the crack tip can be observed. This is due to the ignorance of higher order non-singular terms in the numerical simulation. However, the excellent correlation between experimental and theoretical results in the near crack tip field did confirm the possibility, effectiveness and accuracy of the proposed experimental technique for the evaluation of stress intensity factors.

V. CONCLUSION

In this paper, we presented a new experimental technique for the evaluation of stress intensity factors from the transverse displacement measurements in the area adjoined to the crack tip, which is subjected to 3D stress state. For validation, the measured experimental data were compared with those obtained from theoretical solutions and finite element computations. A good correlation is obtained, which confirmed the reliability of the determination of stress intensity factors from the surface measurement of the transverse displacements within the near crack tip surface area.

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