Synthesis of Filtering in Stochastic Systems on Continuous-Time Memory Observations in the Presence of Anomalous Noises

S. Rozhkova, O. Rozhkova, A. Harlova, V. Lasukov

Abstract—We have conducted the optimal synthesis of root-mean-squared objective filter to estimate the state vector in the case if within the observation channel with memory the anomalous noises with unknown mathematical expectation are complement in the function of the regular noises. The synthesis has been carried out for linear stochastic systems of continuous - time.

Keywords—Mathematical expectation, filtration, anomalous noise, memory.

I. INTRODUCTION

KALMAN filtering theory [1] is the basis in designing modern control systems, navigation, transmission and processing of information, as well as the processing of trajectory changes [2], [3]. Practically, this is a tool applied in the case of inaccurate mathematical model tasks or the disruption in the normal functioning mode of a system [4]. Within the framework of this problem, we consider the problem of estimating the state vector of Kalman type system if: 1) the observation channel has a memory for the values of the state vector occurring either in the presence of inertial measuring devices or in the presence of delays in the channels of information transmission [5], [6]; 2) besides the regular noises in the observation channel there are functioning anomalous noises, whereas not all the components of the observation vector are used; 3) the abnormal noise is non-stationary, the mathematical expectation of which is an unknown function of time.

Further: $P\{\cdot\}$ – probability of the event; $M\{\cdot\}$ – mathematical expectation; $tr[\cdot]$ – trace of the matrix, "T" and "$+$" – transposition and pseudoinversion of the matrix, if they are right superscripts; $\delta(\cdot)$ – Dirac delta-function; 0 – zero vector of the appropriate size; O and I_k – zero matrix of the appropriate size and identity $(k \times k)$ matrix. $A > 0 (\geq 0)$ – positively (nonnegatively) determined matrix.

II. PROBLEM FORMULATION

Observed n - dimensional process $x(t)$ is a Gaussian Markov process and is determined by

$$\dot{x}(t) = F(t)x(t) + w(t), \tag{1}$$

where $\omega(t)$ - white Gaussian process with the intensity matrix $Q(t)$. Observed l - dimensional process $z(t)$ has the form

$$z(t) = H_0(t)x(t) + H_1(t)x(\tau) + \upsilon(t) + Cf(t), \\ 0 \leq \tau < t < t_1, \tag{2}$$

where $\upsilon(t)$ - white Gaussian process with the intensity matrix $R(t)$, which is a regular noise, $f(t)$ - r -dimensional $(r \leq l)$ white Gaussian process with an unknown mean $f_0(t)$ and intensity matrix $\Theta(t)$, which is anomalous noise, and matrix C determining the structure of the effect of anomalous noise components on the components of the observed process, is constructed by the technique [7]. It is supposed: 1) x_0 has normal distribution with parameters μ_0 and Γ_0; 2) x_0, $\omega(t)$, $\upsilon(t)$, $f(t)$ are independent in the aggregate; 3) matrices $\Gamma(t)$, $Q(t)$, $R(t)$, $\Theta(t)$ are positively determined; 4) $f_0(t)$ is unknown. The problem is formulated as follows: based on the observations of random process $z(t)$, it is necessary to find the optimal in the mean-square unbaised estimate of the filtering $\mu(t)$ and interpolation $\mu(\tau, t)$.

III. FILTER STRUCTURE

Statement: If $f_0(t) = 0$, then the optimal Bayesian filter in terms of mean-square is determined by

$$\dot{\mu}(t) = F(t)\mu(t) + \tilde{H}_0^T(t)\tilde{R}^{-1}(t)\tilde{z}(t), \tag{3}$$

$$\dot{\mu}(\tau, t) = \tilde{H}_1^T(t)\tilde{R}^{-1}(t)\tilde{z}(t), \tag{4}$$

$$\dot{\Gamma}(t) = F(t)\Gamma(t) + \Gamma(t)F^T(t) + Q(t) - \tilde{H}_0^T(t)\tilde{R}^{-1}(t)\tilde{H}_0(t), \tag{5}$$

$$\dot{\Gamma}_{11}(\tau, t) = -\tilde{H}_1^T(t)\tilde{R}^{-1}(t)\tilde{H}_1(t), \tag{6}$$

$$\dot{\Gamma}_{01}(\tau, t) = F(t)\Gamma_{01}(\tau_k, t) - \tilde{H}_0^T(t)\tilde{R}^{-1}(t)\tilde{H}_1(t), \tag{7}$$

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where

$$\tilde{z}(t) = z(t) - [H_0(t)\mu(t) + H_1(t)\mu(\tau, t)], \qquad (8)$$

$$\tilde{H}_0(t) = H_0(t)\Gamma(t) + H_1(t)\Gamma_{01}^T(\tau, t), \qquad (9)$$

$$\tilde{H}_1(t) = H_0(t)\Gamma_{01}(\tau, t) + H_1(t)\Gamma_{11}^T(\tau, t), \qquad (10)$$

$$\tilde{R}(t) = R(t) + C\Theta(t)C^T. \qquad (11)$$

Validity of this statement follows from [8], including the limitations 2 of the formulated problem.

Let us introduce the following notation

$$\tilde{\Gamma}(\tau, t) = \begin{bmatrix} \Gamma(t) & \Gamma_{01}(\tau, t) \\ \Gamma_{01}^T(\tau, t) & \Gamma_{11}(\tau, t) \end{bmatrix} = M\{\tilde{\mu}^0(\tau, t), \tilde{\mu}^{0^T}(\tau, t)\}, \qquad (12)$$

$$\tilde{\mu}(\tau, t) = \begin{bmatrix} \mu(t) \\ \mu(\tau, t) \end{bmatrix}, \tilde{x}(\tau, t) = \begin{bmatrix} x(t) \\ x(\tau) \end{bmatrix}, \tilde{\mu}^0(\tau, t) = \tilde{x}(\tau, t) - \tilde{\mu}(\tau, t), \qquad (13)$$

$$\tilde{F}(t) = \begin{bmatrix} F(t) & 0 \\ 0 & 0 \end{bmatrix}, \quad K(t) = \begin{bmatrix} \tilde{H}_0^T(t)\tilde{R}^{-1}(t) \\ \tilde{H}_1^T(t)\tilde{R}^{-1}(t) \end{bmatrix} = \begin{bmatrix} K_0(t) \\ K_1(t) \end{bmatrix}, \qquad (14)$$

$$H(t) = [H_0(t) \mid H_1(t)], \quad \overline{\omega}(t) = \begin{bmatrix} \omega(t) \\ \hline 0 \end{bmatrix}, \quad \tilde{Q}(t) = \begin{bmatrix} Q(t) & \mid 0 \\ \hline 0 & \mid 0 \end{bmatrix}. \qquad (15)$$

From (8) and taking (2), (13), (15) into account, we obtain

$$\tilde{z}(t) = H(t)\tilde{\mu}^0(\tau, t) + \tilde{\upsilon}(t), \qquad (16)$$

where $\tilde{\upsilon}(t) = \upsilon(t) + Cf(t)$. From (3), (4), (13), (14) it follows that

$$\dot{\tilde{\mu}}(\tau, t) = \tilde{F}(t)\tilde{\mu}(\tau, t) + K(t)\tilde{z}(t). \qquad (17)$$

Using the sequence (17), (13) (14) (15) (16), we obtain

$$\dot{\tilde{\mu}}^0(\tau, t) = \overline{F}(t)\tilde{\mu}^0(\tau, t) + \tilde{\omega}(t) - K(t)\tilde{\upsilon}(t), \qquad (18)$$

where $\overline{F}(t) = \tilde{F}(t) - K(t)H(t)$. Let $\Phi(t, \sigma)$ - transition matrix corresponding to the matrix $\overline{F}(t)$. Then the solution of (18) will be written as

$$\tilde{\mu}^0(\tau, t) = \Phi(t, t_0)\tilde{\mu}^0(\tau, t_0) + \int_0^t \Phi(t, \sigma)[\tilde{\omega}(\sigma) - K(\sigma)\tilde{\upsilon}(\sigma)]d\sigma. \qquad (19)$$

Suppose $f_0(t) \neq 0$. Then from (19) it follows that

$$M\{\tilde{\mu}^0(\tau, t)\} = -\int_0^t \Phi(t, \sigma)K(\sigma)Cf_0(\sigma)d\sigma, \qquad (20)$$

i.e. the estimation is biased. Since $M\{\tilde{z}(t)\} = Cf_0(t)$ at $f_0(t) \neq 0$, then to eliminate bias, it is necessary to use $\tilde{\tilde{z}}(t) = \tilde{z}(t) - Cf_0(t)$, instead of $\tilde{z}(t)$ in (17), so

$$\dot{\tilde{\mu}}(\tau, t) = \tilde{F}(t)\tilde{\mu}(\tau, t) + K(t)\tilde{\tilde{z}}(t). \qquad (21)$$

$$\dot{\tilde{\mu}}^0(\tau, t) = \overline{F}(t)\tilde{\mu}^0(\tau, t) + \tilde{\omega}(t) - K(t)[\tilde{\upsilon}(t) - Cf_0(t)]. \qquad (22)$$

Similar to (19) we obtain the solution of (22) in the form

$$\tilde{\mu}^0(\tau, t) = \Phi(t, t_0)\tilde{\mu}^0(\tau, t_0) + \\ + \int_0^t \Phi(t, \sigma)[\tilde{\omega}(\sigma) - K(\sigma)[\tilde{\upsilon}(t) - Cf_0(\sigma)]]d\sigma. \qquad (23)$$

Since $M\{\tilde{\upsilon}(\sigma)\} = Cf_0(\sigma)$, then $M\{\tilde{\mu}^0(\tau, t)\} = 0$, i.e. the estimation determined by (21) is unbiased.

According to the formulation of problem, $f_0(t)$ is unknown, then, it is supposed that in $\tilde{\tilde{z}}(t)$ instead of $f_0(t)$, the estimation $\hat{f}(t)$ can be used as linear transformation of the process $\tilde{z}(t)$, i.e.

$$\hat{f}(t) = Y(t)\tilde{z}(t), \qquad (24)$$

where $Y(t)$ - matrix, which will be determined by the condition of the unbiasedness of estimate. Using $\hat{f}(t)$ instead of $f_0(t)$ gives $\tilde{\tilde{z}}(t) = \tilde{Y}(t)\tilde{z}(t)$, where $\tilde{Y}(t) = I - CY(t)$. Then (3), (4) take the form

$$\dot{\mu}(t) = F(t)\mu(t) + \tilde{H}_0^T(t)\tilde{R}^{-1}(t)\tilde{Y}(t)\tilde{z}(t), \qquad (25)$$

$$\mu(\tau, t) = \tilde{H}_1^T\tilde{R}^{-1}(t)\tilde{Y}(t)\tilde{z}(t). \qquad (26)$$

From (25), (26) and (13), (14) it follows that

$$\dot{\tilde{\mu}}(\tau, t) = \tilde{F}(t)\tilde{\mu}(\tau, t) + K(t)\tilde{Y}(t)\tilde{z}(t). \qquad (27)$$

Using (1), (13), (16), (27) we have

$$\dot{\tilde{\mu}}^0(\tau, t) = \overline{F}_0(t)\tilde{\mu}^0(\tau, t) + \tilde{\omega}(t) - K(t)\tilde{Y}(t)\tilde{\upsilon}(t), \qquad (28)$$

where $\overline{F}_0(t) = \tilde{F}(t) - K(t)\tilde{Y}(t)H(t)$. Suppose $\overline{\Phi}(t, \sigma)$ - transition matrix corresponding to the matrix $\overline{F}_0(t)$, then the solution of (28) has the form

$$\tilde{\mu}^0(\tau, t) = \overline{\Phi}(t, t_0)\tilde{\mu}^0(\tau, t_0) + \int_0^t \overline{\Phi}(t, \sigma)[\tilde{\omega}(\sigma) - K(\sigma)\tilde{Y}(\sigma)\tilde{\upsilon}(\sigma)]d\sigma. \qquad (29)$$

Since $M\{\tilde{\upsilon}(t)\} = Cf_0(t)$, then

$$M\{\tilde{\mu}^0(\tau, t)\} = -\int_0^t \overline{\Phi}(t, \sigma)K(\sigma)\tilde{Y}(\sigma)Cf_0d\sigma. \qquad (30)$$

Thus, from (30) for arbitrary $K(\sigma)$ and $f_0(t)$ the condition of unbiasedness of estimate $\mu(t)$ is

$$\tilde{Y}(t)C = 0. \qquad (31)$$

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So, we have found estimates in the class of linear filters of (27), where matrix $K(t)$ must be determined from optimality condition $\tilde{\mu}(\tau,t)$ in the mean-square, and the matrix $Y(t)$ can be found within unbiasedness.

$$Y(t) = C^+ + A - ACC^+. \qquad (32)$$

Thus, C is a matrix with linearly-independent columns, then $C^+C = I_r$ [9]. From (32) the following condition must be satisfied by matrix $Y(t)$, providing the unbiasedness of estimate $\tilde{\mu}(\tau,t)$:

$$Y(t)C = I_r. \qquad (33)$$

IV. SYNTHESIS OF FILTER

Let us find the equation for $\tilde{\Gamma}(\tau,t) = M\{\tilde{\mu}^0(\tau,t)\tilde{\mu}^{0^T}(\tau,t)\}$. From (29) considering the limitations

$$\tilde{\Gamma}(\tau,t) = \overline{\Phi}(t,t_0)M\{\tilde{\mu}^0(\tau,t_0)\tilde{\mu}^{0^T}(\tau,t_0)\}\overline{\Phi}^T(t,t_0) + \\ + \int_0^t\int \overline{\Phi}(t,\sigma)\begin{bmatrix} M\{\tilde{\omega}(\sigma)\tilde{\omega}^T(\xi)\} + \\ + K(\sigma)\tilde{Y}(\sigma)M\{\tilde{\upsilon}(\sigma)\tilde{\upsilon}^T(\xi)\}\tilde{Y}^T(\xi)K^T(\xi) \end{bmatrix} d\sigma d\xi. \qquad (34)$$

Direct calculations using the condition of unbiasedness $\tilde{Y}(t)C = 0$ and properties of δ - Dirac function show that

$$\tilde{\Gamma}(\tau,t) = \overline{\Phi}(t,t_0)\tilde{\Gamma}_0\overline{\Phi}^T(t,t_0) + \int_0^t \overline{\Phi}(t,\sigma)\overline{Q}(\sigma)\overline{\Phi}^T(t,\sigma)d\sigma, \qquad (35)$$

where

$$\overline{Q}(\sigma) = \tilde{Q}(\sigma) + K(\sigma)\tilde{Y}(\sigma)\tilde{R}(\sigma)\tilde{Y}^T(\sigma)K^T(\sigma). \qquad (36)$$

Differentiating (35) over t, we obtain

$$\dot{\tilde{\Gamma}}(\tau,t) = \overline{F}_0(t)\tilde{\Gamma}(\tau,t) + \tilde{\Gamma}(\tau,t)\overline{F}_0^T(t) + \\ + K(t)\tilde{Y}(t)\tilde{R}(t)\tilde{Y}^T(t)K^T(t) + \tilde{Q}(t). \qquad (37)$$

Thus, we have the problem: in class of filters (27) to find $(2n \times l)$ - matrix $K(t)$, providing minimum functional $J = tr[A\tilde{\Gamma}[\tau,t]], A > 0$, on the trajectories of the matrix differential (36), under condition of (33).

Theorem. Optimal, in terms of the mean-square value, unbiased filter in the class of linear filters of (27) is determined by

$$\dot{\mu}(t) = F(t)\mu(t) + \tilde{K}_0(t)\tilde{z}(t), \qquad (38)$$

$$\dot{\mu}(\tau,t) = \tilde{K}_1(t)\tilde{z}(t), \qquad (39)$$

$$\dot{\Gamma}(t) = F(t)\Gamma(t) + \Gamma(t)F^T(t) + Q(t) - \tilde{K}_0(t)\tilde{H}_0(t), \qquad (40)$$

$$\dot{\Gamma}_{01}(\tau,t) = F(t)\Gamma_{01}(\tau,t) - \tilde{K}_0(t)\tilde{H}_1(t), \qquad (41)$$

$$\dot{\Gamma}_{11}(\tau,t) = -\tilde{K}_1(t)\tilde{H}_1(t), \qquad (42)$$

$$\tilde{K}_0(t) = K_0(t)[I - CY(t)], \quad \tilde{K}_1(t) = K_1(t)[I - CY(t)], \qquad (43)$$

$$K_0(t) = \tilde{H}_0^T(t)\tilde{R}^{-1}(t), \quad K_1(t) = \tilde{H}_1^T(t)\tilde{R}^{-1}(t), \qquad (44)$$

$$Y(t) = [C^T\tilde{R}^{-1}(t)C]^{-1}C^T\tilde{R}^{-1}(t), \qquad (45)$$

and $\tilde{z}(t)$, $\tilde{H}_0(t)$, $\tilde{H}_1(t)$, $\tilde{R}(t)$ are determined by (8)-(10).

Proof. In accordance with matrix variation of the Pontryagin maximum principle [10] the Hamiltonian $\mathbf{H}(t) = H[\tilde{\Gamma}(\tau,t), K(t), \Lambda(t)]$ according to (36) is determined by

$$\mathbf{H}(t) = tr[\overline{F}_0(t)\tilde{\Gamma}(\tau,t)\Lambda^T(t)] + tr[\tilde{\Gamma}(\tau,t)\overline{F}_0^T(t)\Lambda^T(t)] + \\ + tr[K(t)\tilde{Y}(t)\tilde{R}(t)\tilde{Y}^T(t)K^T(t)\Lambda^T(t)] + tr[\tilde{Q}(t)\Lambda^T(t)], \qquad (46)$$

where $\Lambda(t)$ - $(2n \times 2n)$ matrix of costate variables, for which the equation and boundary conditions are

$$\Lambda(t) = -\frac{\partial\mathbf{H}(t)}{\partial\tilde{\Gamma}}, \quad \Lambda(t_1) = \frac{\partial J}{\partial\tilde{\Gamma}}. \qquad (47)$$

Direct calculations show that

$$\dot{\Lambda}(t) = -\Lambda(t)\overline{F}_0(t) - \overline{F}_0^T(t)\Lambda(t), \quad \Lambda(t_1) = A^T. \qquad (48)$$

Necessary optimality condition $\partial\mathbf{H}/\partial K = 0$ using (46), (36), symmetry $\tilde{\Gamma}(\tau,t)$ and rules of vector-matrix differentiation [10] leads to

$$-\Lambda(t)\tilde{\Gamma}(\tau,t)H^T(t)\tilde{Y}^T(t) - \Lambda^T(t)\tilde{\Gamma}(\tau,t)H^T(t)\tilde{Y}^T(t) + \\ \Lambda(t)K(t)\tilde{Y}(t)\tilde{R}(t)\tilde{Y}^T(t) + \Lambda^T(t)K(t)\tilde{Y}(t)\tilde{R}(t)\tilde{Y}^T = 0. \qquad (49)$$

Similar to [11], it can be seen that $\Lambda(t)$, satisfying boundary-value problem (48), is symmetric positive definite matrix. Then from (49) the final form of the relation follows, which is satisfied by the optimal matrix $K(t)$:

$$K(t)\tilde{Y}(t)\tilde{R}(t)\tilde{Y}^T(t) = \tilde{\Gamma}(\tau,t)H^T(t)\tilde{Y}^T(t). \qquad (50)$$

The solution of (49) exists if and only if [9] ([.]⁺ – pseudoinversion of matrix [.])

$$\tilde{\Gamma}(\tau,t)H^T(t)\tilde{Y}^T(t)[\tilde{Y}(t)\tilde{R}(t)\tilde{Y}^T(t)]^+[\tilde{Y}(t)\tilde{R}(t)\tilde{Y}^T(t)] = \\ = \tilde{\Gamma}(\tau,t)H^T(t)\tilde{Y}^T(t). \qquad (51)$$

Validity of (51) follows from [9], [12]. Then the general solution of (50) has the form [3]

$$K(t) = \tilde{\Gamma}(\tau,t)H^T(t)\tilde{Y}^T(t)[\tilde{R}(t)\tilde{Y}^T(t)]^+ - \\ - B(t)\tilde{R}(t)\tilde{Y}^T(t)[\tilde{R}(t)\tilde{Y}^T(t)]^+ + B(t). \qquad (60)$$

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Let us choose arbitrary matrix $B(t)$ from the condition

$$\tilde{\Gamma}(\tau,t)H^T(t) = B(t)\tilde{R}^{-1}(t). \tag{61}$$

Using (61) in (60) leads to

$$K(t) = \tilde{\Gamma}(\tau,t)H^T(t)\tilde{R}^{-1}(t). \tag{62}$$

Writing out (62), (27), (37) with regard to (12), (14), (15), (27) leads to (37)-(43). Writing out (27) with regard to (13) (14) leads to (37), (38). Using (62) provides

$$\tilde{\Gamma}(\tau,t) = \tilde{F}(t)\tilde{\Gamma}(\tau,t) + \tilde{\Gamma}(\tau,t)\tilde{F}^T(t) + \tilde{Q}(t) - \\ - \tilde{\Gamma}(\tau,t)H^T(t)\begin{bmatrix} \tilde{R}^{-1}(t)\tilde{Y}(t) + \tilde{Y}^T(t)\tilde{R}^{-1}(t) - \\ - \tilde{R}^{-1}(t)\tilde{Y}(t)\tilde{R}(t)\tilde{Y}^T(t)\tilde{R}^{-1}(t) \end{bmatrix} H(t)\tilde{\Gamma}(\tau,t). \tag{63}$$

Using in sequence $\tilde{Y}(t) = I - CY(t)$, and then (57), we have

$$\tilde{R}^{-1}(t)\tilde{Y}(t)\tilde{R}(t)\tilde{Y}^T(t) = \tilde{R}^{-1}(t)[I - CY(t)]\tilde{R}(t)\tilde{Y}^T(t) = \\ = \tilde{R}^{-1}(t)[\tilde{R}(t)\tilde{Y}^T(t) - CY(t)\tilde{R}(t)\tilde{Y}^T(t)] = \tilde{Y}^T(t). \tag{64}$$

Substituting (64) in (63), we have

$$\tilde{\Gamma}(\tau,t) = \tilde{F}(t)\tilde{F}(\tau,t) + \tilde{\Gamma}(\tau,t)\tilde{F}^T(t) + \tilde{Q}(t) - \\ - \tilde{F}(\tau,t)H^T(t)\tilde{R}^{-1}(t)\tilde{Y}(t)H(t)\tilde{\Gamma}(\tau,t). \tag{65}$$

Writing out (65) with regard to (12), (14), (15), (35), leads to (40)-(42). Thus, all the relations of the theorem have been obtained and the proof has been completed by establishing the fact that the matrix of the filter does not depend on arbitrary matrix $B(t)$. From (56) we have

$$\tilde{K}(t) = \tilde{\Gamma}(\tau,t)H^T(t)\tilde{Y}^T(t)[\tilde{Y}(t)\tilde{R}(t)\tilde{Y}^T(t)]^+\tilde{Y}(t) - \\ - B(t)[\tilde{Y}(t)\tilde{R}(t)\tilde{Y}^T(t)][\tilde{Y}(t)\tilde{R}(t)\tilde{Y}^T(t)]^+\tilde{Y}(t) + B(t)\tilde{Y}(t). \tag{66}$$

Let us denote the second term in the right side of (66) through $\Phi(t)$. Then similarly to derivation of (54), we obtain

$$\Phi(t) = B(t)[\tilde{Y}(t)L(t)][\tilde{Y}(t)L(t)]^+\tilde{Y}(t). \tag{67}$$

Since [3]

$$[\tilde{Y}(t)L(t)][\tilde{Y}(t)L(t)]^+ = \tilde{Y}(t)\tilde{Y}^+(t), \tag{68}$$

then using the theorem for characterization of the pseudoinversion matrix [3], we have $\Phi(t) = B(t)\tilde{Y}(t)$. Using this formula in (66) shows

$$\tilde{K}(t) = \tilde{\Gamma}(\tau,t)H^T(t)\tilde{Y}^T(t)[\tilde{Y}(t)\tilde{R}(t)\tilde{Y}^T(t)]^+\tilde{Y}(t). \tag{69}$$

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