Assessment of Collapse Potential of Degrading SDOF Systems

Muzaffer Borekci, Murat S. Kirçil

***Abstract*—**Predicting the collapse potential of a structure during earthquakes is an important issue in earthquake engineering. Many researchers proposed different methods to assess the collapse potential of structures under the effect of strong ground motions. However most of them did not consider degradation and softening effect in hysteretic behavior. In this study, collapse potential of SDOF systems caused by dynamic instability with stiffness and strength degradation has been investigated. An equation was proposed for the estimation of collapse period of SDOF system which is a limit value of period for dynamic instability. If period of the considered SDOF system is shorter than the collapse period then the relevant system exhibits dynamic instability and collapse occurs.

***Keywords*—**Collapse, degradation, dynamic instability, seismic response.

I. Introduction

THE major source of injuries, mortalities and financial losses is the collapse of buildings during earthquakes, and therefore one of main goals of earthquake engineering is to predict the collapse potential of different types of structures [1]. In earthquake engineering, collapse refers to the loss of ability of a structural system to resist gravity loads in the presence of seismic effects [2]. Collapse may be either local or global. The fail of a component of a structure or connection refers to local collapse. The spread of local failure from element to element, excessive displacement of an individual story or deterioration of structural components subjected to cyclic loading may result in global collapse. There are many studies for the assessment of the seismic response of the RC structures and most of them interest in the RC structures using the elastic-perfectly plastic hysteretic behavior without degradation because of its simplicity; however, experimental studies showed that all materials degrade and hysteretic response of the RC structures under cyclic loadings does not match with the bilinear hysteretic behavior but has a good fit with the peak-oriented model [3]–[5]. Thus, it is important to use a hysteretic model which is similar to the real seismic response of the RC structure. Furthermore, not only degradation effect but also softening in the hysteretic behavior should be considered. The softening branch is the branch of skeleton curve of the hysteretic behavior which has a negative stiffness. This negative stiffness can be observed because of the P-delta effect or the strength degradation (described as in-cycle degradation in FEMA 440 [6]) occurred after reaching the maximum strength of the system. The branch which has a negative stiffness is also called post-capping branch as is seen in Fig. 1. On condition that the post-capping branch reaches a residual strength, dynamic instability occurs and system collapses. The structure subjected to a certain input is stable if small increase in the magnitude of the excitation results in small changes in the response [7]. Otherwise, structure will not be stable and it is called dynamic instability. Same assumption is also made by [8]. Several methods have been suggested or used in research studies in which structures are modeled as SDOF system to directly assess their collapse capacity. Bernal [9] developed a simplified method to check the safety against dynamic instability of equivalent single-degree of-freedom (SDOF) system of 2D buildings considering stiffness degrading elastic-perfectly plastic behavior. The method was based on the derivation of statistical expressions to correlate the required minimum base shear to prevent instability. MacRae [10] proposed a method for considering P-Δ effect with different post-yield stiffness ratios and observed that post-yield stiffness ratio is a major parameter that affects the system's stability. Miranda and Akkar [11] proposed an equation as a function of natural period and post-yield stiffness to estimate the lateral strength that is required to prevent collapse by dynamic instability of SDOF systems. Adam et al. [12] proposed a procedure for the determination of the collapse capacity of a MDOF structure through the use of equivalent SDOF system. References [7], [9], [11], [13] have shown that study of the dynamic instability of SDOF systems provides significant insight to the assessment of collapse of multi-degree-of-freedom (MDOF) structures subjected to earthquakes. Chenouda and Ayoub [14] investigated inelastic displacement ratio considering energy based stiffness and strength degrading hysteretic behavior (same model used in this study). They showed that all degrading systems with a period less than a certain value collapse and that collapse occurs because of the dynamic instability. However, their study includes only limited degradation cases. If a hysteretic model with degrading effect and softening branch is used to consider a more realistic behavior in the estimation of seismic response, dynamic instability should be considered and checked. In this study an equation is proposed for the estimation of a limit period at which a SDOF system exhibits dynamic instability. This limit value is called as collapse period (T_{col}) of the relevant SDOF system. If period of the considered structure is shorter than the collapse period ($T<T_{col}$) then the relevant structure exhibits dynamic instability and collapse occurs. In this study,

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nonlinear time history analyses were performed for strength reduction factor R_y = 1.5, 2, 3, 4, 5, 6 and post-yield stiffness ratio α_s = 0%, 3%, 5% considering 5% damping ratio. 53 natural vibration periods were used ranging from T = 0.1 s. to T = 3 s. (T = 0.1:0.02:0.2, 0.22:0.03:1, 1.1:0.1:3). The main objective of this study is to determine *collapse period* (T_{col}) which is a limit value for the dynamic instability and to propose an equation for the estimation of the collapse period.

II. Ground Motion Records

A total of 160 earthquake acceleration time histories were used in this study. 80 records were considered with two horizontal components at each station and magnitude of the records ranges from 6 to 7.9. Although there are different limitations on the fault distance defined in the literature to describe the near fault effect, minimum fault distance considered in this study is 30 km, so that near fault effect can be eliminated. The earthquake acceleration time histories were divided into four groups according to local soil conditions at the recording station. Each group consisted of 40 ground motions. Locations of stations in the first group correspond to site class A, second group corresponds to site class B, third group corresponds to site class C and the last group corresponds to site class D according to USGS [15] classification.

III. Hysteretic Model

A. Peak-Oriented Hysteretic Model

Modified-Clough model with energy based stiffness and strength degradation was used as hysteretic model in this study. Although it considers the pinching effect in addition to degradation; pinching effect was neglected for this study since [2] showed that the collapse capacities of peak- oriented model with and without pinching is very close.

This model keeps basic hysteretic rules proposed by [16] and later modified by [17], but the backbone curve was modified by [3] to include strength capping and residual strength as shown in Fig. 1 [3].

The basic idea of the model is that the reloading path always targets the previous maximum displacement where K_e is the elastic (initial) stiffness, f_y is the yield strength, f_r is the residual strength, f_c is the maximum strength, K_s is the post – yield stiffness, u_y is the yield displacement, u_c is the beginning of a softening branch which is called cap displacement, K_c is the post – capping stiffness which usually has a negative value.

When the loading path reaches the horizontal axis, the loading goes through reloading path. The basic idea of the peak-oriented model is that the reloading path always targets the previous maximum displacement. The basic rules of Peak-Oriented Model can be seen in Fig. 2.

Rahnama and Krawinkler [18] adopted a rule in the Modified-Clough model to account for degradation effects. Four different deterioration modes can occur after the loading path reaches the yielding point at least in one direction. These deterioration modes are *basic strength deterioration, post – capping deterioration, unloading stiffness degradation* and *reloading stiffness degradation.*

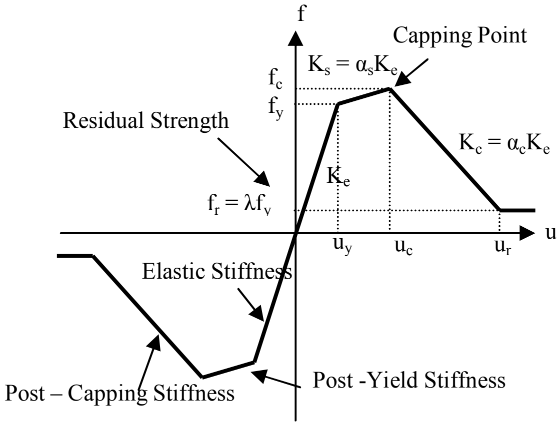

Fig. 1 Backbone curve of deteriorating hysteretic model [3]

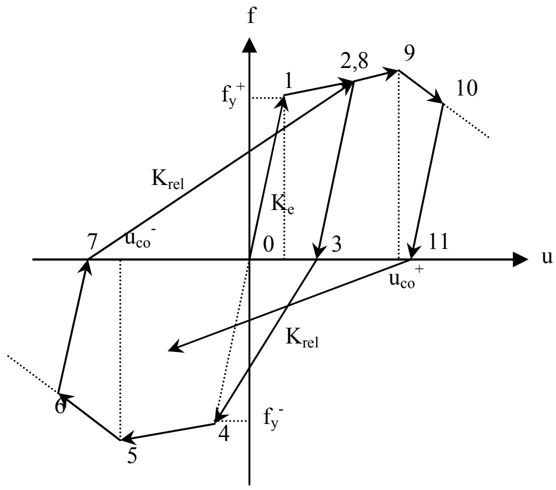

Fig. 2 Basic rules of peak-oriented hysteretic model [3]

It is assumed that the deterioration in excursion *i* is defined by a deterioration parameter β_i.

$$\beta_i = \left(\frac{E_i}{E_t - \sum_{j=1}^{i} E_j}\right)^c \tag{1}$$

E_i is the hysteretic energy dissipated in excursion i, E_t is the hysteretic energy dissipation capacity, ΣE_j is the hysteretic energy dissipated in all previous excursions and c is a component which defines the rate of deterioration. Reasonable range of c is between 1.0 and 2.0 [18]. Although the parameter *c* affects the cyclic deterioration, [3] suggested a constant value of 1 for *c* and this suggestion is followed in this study.

$$E_t = \gamma F_y u_y \tag{2}$$

γ expresses the hysteretic energy dissipation capacity as a function of twice the elastic strain energy at yielding ($f_y u_y$). The parameter γ can have different values for each deterioration mode. Different indices are used for different modes; γ_s is for *basic strength deterioration*, γ_c is for *post-capping strength deterioration*, γ_u is for *unloading stiffness deterioration* and γ_a is for *accelerated reloading stiffness deterioration*. However the results determined by using the same value of γ for all deterioration modes are sufficient for the effect of cyclic deterioration [3]. Deterioration modes are described briefly below, however detailed information can be seen in [3].

The parameters effect on the cyclic degradation γ, u_c/u_y, α_c. γ is parameter of the rate of the degradation and it is assumed in this study γ = 50, 100 and 150 as severe, moderate and low degradation, respectively [14]. u_c/u_y is denoted as ductility capacity however the term ductility is not used as its traditional meaning. In this study u_c/u_y is the ratio between corresponding displacement of peak and yield strength. u_c/u_y = 2, 4, 6 represent non-ductile, medium ductile and very ductile structures, respectively [2]. α_c that is used to define post-capping stiffness ratio are -6% [14], -14% and -21% [11] which represent small, medium and large slope, respectively. -14% is assumed as the medium slope in this study. 27 combinations of degradation parameters are used in this study to define the all possibilities of degradation and these combinations are given in Table I.

TABLE I
CONSIDERED COMBINATIONS OF DETERIORATION PARAMETERS

Name	γ	α_c	u_c/u_y
γ50_α_c6_u_c/u_y2	50	-6%	2
γ50_α_c6_u_c/u_y4	50	-6%	4
γ50_ α_c6_u_c/u_y6	50	-6%	6
γ50_α_c14_u_c/u_y2	50	-14%	2
γ50_α_c14_u_c/u_y4	50	-14%	4
γ50_α_c14_u_c/u_y6	50	-14%	6
γ50_α_c21_u_c/u_y2	50	-21%	2
γ50_α_c21_u_c/u_y4	50	-21%	4
γ50_α_c21_u_c/u_y6	50	-21%	6
γ100_α_c6_u_c/u_y2	100	-6%	2
γ100_α_c6_u_c/u_y4	100	-6%	4
γ100_α_c6_u_c/u_y6	100	-6%	6
γ100_α_c14_u_c/u_y2	100	-14%	2
γ100_α_c14_u_c/u_y4	100	-14%	4
γ100_α_c14_u_c/u_y6	100	-14%	6
γ100_α_c21_u_c/u_y2	100	-21%	2
γ100_α_c21_u_c/u_y4	100	-21%	4
γ100_α_c21_u_c/u_y6	100	-21%	6
γ150_α_c6_u_c/u_y2	150	-6%	2
γ150_α_c6_u_c/u_y4	150	-6%	4
γ150_α_c6_u_c/u_y6	150	-6%	6
γ150_α_c14_u_c/u_y2	150	-14%	2
γ150_α_c14_u_c/u_y4	150	-14%	4
γ150_α_c14_u_c/u_y6	150	-14%	6
γ150_α_c21_u_c/u_y2	150	-21%	2
γ150_α_c21_u_c/u_y4	150	-21%	4
γ150_α_c21_u_c/u_y6	150	-21%	6

IV. ANALYSIS

The global collapse was considered within the scope of this study. There are two main criteria to define the collapse: when the post-capping branch intersects the horizontal axis (dynamic instability occurs) or when the parameter β_i exceeds 1 (that means hysteretic energy capacity has been exhausted) system collapses. According to results of this study, post-capping branch reaches the horizontal axis before hysteretic energy capacity exhausts. That means collapse occurs only because of the dynamic instability.

Nonlinear time history analyses were performed via MATLAB using Newmark-Beta method based on hysteretic behavior mentioned above. Analyses were performed for SDOF systems with previously mentioned values of R_y, α_s and T. The damping ratio is 5% for all systems. With 27 degradation combinations and 160 ground motion records, 4121280 nonlinear time history analyses were performed to determine the *collapse period (T_{col})* for each considered system. If the structure collapses under the effect of more than 50% of the considered records then it is assumed that its period is median collapse period [14].

V. RESULTS

Collapse periods of the considered systems for all site classes with different combinations of degrading parameters are given in Fig. 3. It is clear from Fig. 3 that collapse period T_{col} increases while strength reduction factor R_y increases. According to the results of this study, considering only elastic-perfectly plastic system (α_s = 0%) yields results conservative enough in the estimation of T_{col}. Site class has significant effect on T_{col} thus site class should be considered individually.

Cyclic degradation is the function of the parameters γ, u_c/u_y, α_c. Significance of the effect of the degradation parameters on T_{col} is not same for each degradation parameters.

u_c/u_y and α_c have significant effect on T_{col} while the effect of γ is not significant as much as u_c/u_y and α_c. Detailed information on the effect of each parameter can be seen from [19].

VI. CONCLUSION

Nonlinear regression analysis was made using least squares method and an equation was proposed for T_{col} as a function of R_y, u_c/u_y and α_c. The proposed equation is given in (3). Some combinations of the considered parameters were excluded from regression analyses so that more realistic results can be obtained. For example, a system with severe degradation and large post-capping slope is assumed not to be ductile.

$$T_{col} = 0.1 + x_1 R_y^{x_2}\left(\left(\frac{u_c}{u_y}\right)^{x_3} + \alpha_c^{x_4}\right) \tag{3}$$

The observed and predicted values of T_{col} are given in Fig. 4 for each site class. It is clear from figure that the results of the proposed equation for T_{col} have a good agreement with the theoretical values of T_{col}.

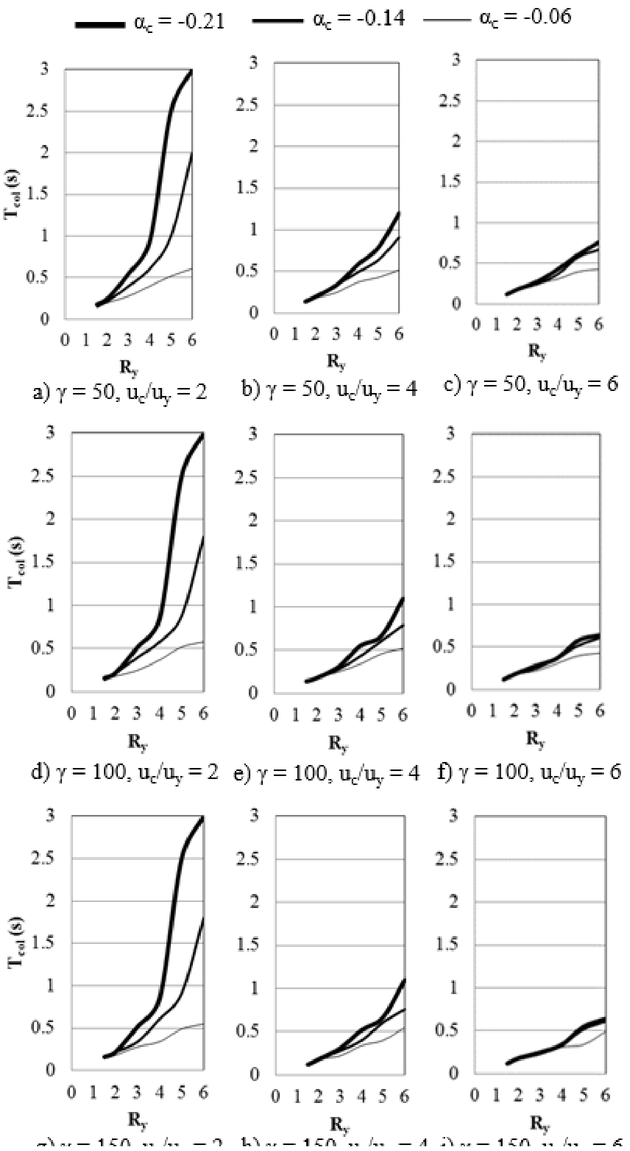

Fig. 3 Collapse period of all degradation combinations for mean of all site classes

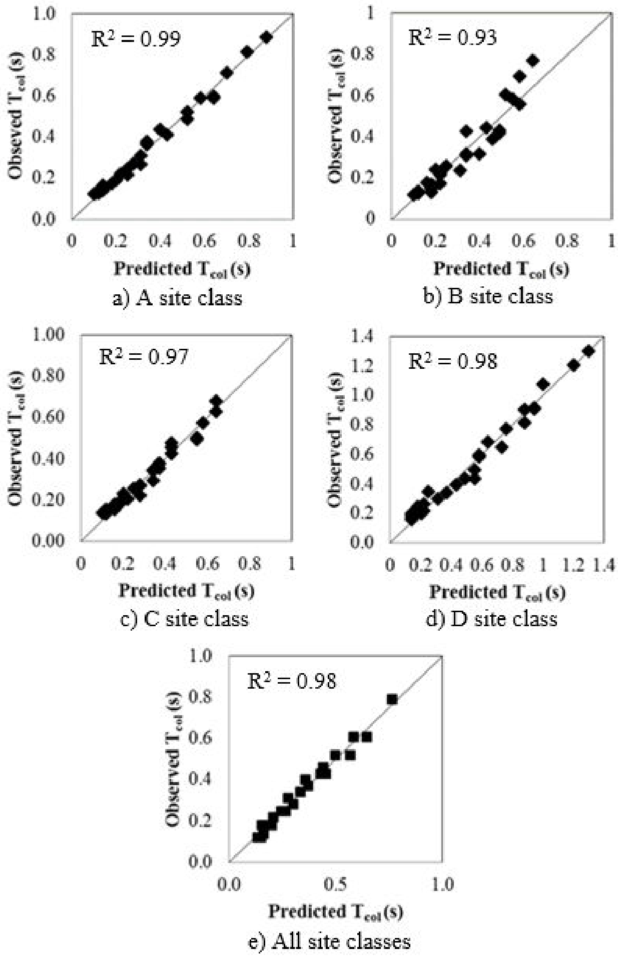

Fig. 4 Observed and predicted T_{col} for different site classes and moderate degradation (γ = 100)

Coefficients of (3) are given in Table II.

TABLE II
COEFFICIENT VALUES OF (3)

Site Class	Degradation Level	x_1	x_2	x_3	x_4	Correlation Coefficient
A	Severe	**0.0760**	**1.9946**	**-2.1919**	**0.6819**	**0.99**
	Moderate	**0.0500**	**2.0508**	**-1.6188**	**0.6950**	**0.99**
	Low	**0.0685**	**1.9516**	**-1.5128**	**0.8597**	**0.99**
B	Severe	**0.1080**	**2.5910**	**-3.3714**	**1.4010**	**0.97**
	Moderate	**0.0736**	**2.0795**	**-1.4281**	**1.2840**	**0.93**
	Low	**0.0520**	**1.9192**	**-1.6044**	**0.65463**	**0.98**
C	Severe	**0.1127**	**1.9430**	**-3.3559**	**0.9167**	**0.96**
	Moderate	**0.0660**	**1.6114**	**-1.7429**	**0.5218**	**0.97**
	Low	**0.0791**	**1.5530**	**-1.1348**	**0.8600**	**0.95**
D	Severe	**0.0902**	**1.7420**	**-0.6007**	**0.6929**	**0.97**
	Moderate	**0.1093**	**1.7232**	**-1.5604**	**0.5378**	**0.98**
	Low	**0.1735**	**1.6249**	**-1.9407**	**0.6922**	**0.96**
All	Severe	**0.1123**	**2.0338**	**-2.7238**	**0.9330**	**0.98**
	Moderate	**0.0873**	**1.7190**	**-1.4146**	**0.7963**	**0.98**
	Low	**0.0600**	**1.8424**	**-1.3995**	**0.6928**	**0.98**

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