

Comparative Study of Intuitionistic and Generalized Neutrosophic Soft Sets

Debabrata Mandal

Abstract—The aim of this paper is to define several operations such as Intersection, Union, OR, AND operations of intuitionistic (resp. generalized) neutrosophic soft sets in the sense of Maji and compare these with intuitionistic (resp. generalized) neutrosophic soft sets in the sense of Said et al via examples. At the end of the paper, a new concept - extension is introduced, which can be used to refine our choices in case of decision making.

Keywords—AND, OR, Union, Intersection, Extension, Decision making.

I. INTRODUCTION

MOST of the problems in economics, engineering and environment have various uncertainties. We cannot successfully use the classical methods because of various uncertainties typical for these problems. To solve this problem, the concept of fuzzy sets was introduced by Zadeh [9] in 1965 where each element have a degree of membership and has been extensively applied to many scientific fields. As a generalization of fuzzy sets, the intuitionistic fuzzy set was introduced by Atanassov [1] in 1986, where besides the degree of membership of each element there was considered a degree of non-membership with (membership value + non-membership value) ≤ 1 .

There are also several well-known theories, such as, rough sets, vague sets, interval-valued sets etc. which can be considered as mathematical tools for dealing with uncertainties. But all these theories have their inherent difficulties.

To overcome these difficulties, Molodtsov [6] introduced the soft sets which can be seen as a new mathematical tool for dealing with uncertainties. In the soft set theory, the problem of setting the membership function does not arise which makes the theory easily applies to many different fields. Maji et al [5] pointed out several directions for the applications of soft sets. They also studied several operations on the theory of soft sets. Feng et al [3] generalized the soft set theory to the theory of fuzzy soft set and used it in case of decision making. At present, works on soft set theory are progressing rapidly.

In 2005, Smarandache introduced Neutrosophic set [8] which can distinguish between absolute membership and relative membership and use it in non-standard analysis such as result of sport games (winning/defeating/ tie), votes, from yes/no/NA, from decision making and control theory etc. Here he combined the non-standard analysis with a tri-component logic/set/probability theory and philosophy. Motivated by this idea and combining it with the theory of soft sets, in 2013, Maji [4] introduced and studied 'Neutrosophic soft sets'. As

Debabrata Mandal is with the Department of Mathematics, Raja Peary Mohan College, Uttarpara, Hooghly-712258, India. (e-mail: dmandaljumath@gmail.com).

a continuation of it, Said et al [7] studied 'Intuitionistic neutrosophic soft sets'. But here 'intuitionistic neutrosophic set' is in the sense of Bhowmik et al [2].

The aim of this paper is to define 'intuitionistic neutrosophic soft set' in the sense of Maji [4] and compare the obtained results with Said et al [7]. Here we also introduce a new concept which will be useful to refine our choices in case of decision making.

II. PRELIMINARIES

We now recall following definitions from [4] for subsequent use.

Definition 1 A neutrosophic set A on the universe of discourse X is defined as $A = \{ \langle x, T_A(x), I_A(x), F_A(x) \rangle, x \in X \}$, where $T, I, F : X \rightarrow]-0, 1^+[$ and $-0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3^+$. From philosophical point of view, the neutrosophic set takes the value from real standard or non-standard subsets of $] -0, 1^+[$. But in real life application in scientific and engineering problems it is difficult to use neutrosophic set with value from real standard or non-standard subset of $] -0, 1^+[$. Hence we consider the neutrosophic set which takes the value from the subset of $[0, 1]$.

Definition 2 Let U be an initial universe set and E be a set of parameters. Let $\mathcal{P}(U)$ denotes the power set of U . Consider a nonempty set $A, A \subseteq E$. A pair (F, A) is called a soft set over U , where F is a mapping given by $F : A \rightarrow \mathcal{P}(U)$.

Definition 3 Let U be an initial universe set and E be a set of parameters. Consider $A \subseteq E$. Let $P(U)$ denotes the set of all neutrosophic sets of U . The collection (F, A) is termed to be the soft neutrosophic set over U , where F is a mapping given by $F : A \rightarrow P(U)$.

Definition 4 Let (H, A) and (G, B) be two neutrosophic soft sets over the common universe U . Then the union of (H, A) and (G, B) is denoted by $(H, A) \cup (G, B)$ and is defined by $(H, A) \cup (G, B) = (K, C)$, where $C = A \cup B$ and the truth-membership, indeterminacy-membership and falsity-membership of (K, C) are as follows:

$$\begin{aligned} T_{K(e)}(m) &= T_{H(e)}(m), \text{ if } e \in A \setminus B \\ &= T_{G(e)}(m), \text{ if } e \in B \setminus A \\ &= \max\{T_{H(e)}(m), T_{G(e)}(m)\}, \text{ if } e \in A \cap B \\ I_{K(e)}(m) &= I_{H(e)}(m), \text{ if } e \in A \setminus B \\ &= I_{G(e)}(m), \text{ if } e \in B \setminus A \\ &= \frac{I_{H(e)}(m) + I_{G(e)}(m)}{2}, \text{ if } e \in A \cap B \end{aligned}$$

$$\begin{aligned} F_{K(e)}(m) &= F_{H(e)}(m), \text{ if } e \in A \setminus B \\ &= F_{G(e)}(m), \text{ if } e \in B \setminus A \\ &= \min\{F_{H(e)}(m), F_{G(e)}(m)\}, \text{ if } e \in A \cap B \end{aligned}$$

Definition 5 Let (H, A) and (G, B) be two neutrosophic soft sets over the common universe U . Then the intersection of (H, A) and (G, B) is denoted by " $(H, A) \cap (G, B)$ " and is defined by $(H, A) \cap (G, B) = (K, C)$, where $C = A \cap B$ and the truth-membership, indeterminacy-membership and falsity-membership of (K, C) are as follows:

$$\begin{aligned} T_{K(e)}(m) &= \min\{T_{H(e)}(m), T_{G(e)}(m)\}, \\ I_{K(e)}(m) &= \frac{I_{H(e)}(m) + I_{G(e)}(m)}{2}, \\ F_{K(e)}(m) &= \max\{F_{H(e)}(m), F_{G(e)}(m)\}. \end{aligned}$$

for all $e \in C$ and $m \in U$.

Definition 6 Let (H, A) and (G, B) be two neutrosophic soft sets over the common universe U . Then the 'AND' of (H, A) and (G, B) is denoted by $(H, A) \wedge (G, B)$ and is defined by $(H, A) \wedge (G, B) = (K, A \times B)$, where the truth-membership, indeterminacy-membership and falsity-membership of $(K, A \times B)$ are as follows:

$$\begin{aligned} T_{K(\alpha, \beta)}(m) &= \min\{T_{H(\alpha)}(m), T_{G(\beta)}(m)\}, \\ I_{K(\alpha, \beta)}(m) &= \frac{I_{H(\alpha)}(m) + I_{G(\beta)}(m)}{2}, \\ F_{K(\alpha, \beta)}(m) &= \max\{F_{H(\alpha)}(m), F_{G(\beta)}(m)\}. \end{aligned}$$

$\forall \alpha \in A, \beta \in B$ and $m \in U$.

Definition 7 Let (H, A) and (G, B) be two NSSs over the common universe U . Then the 'OR' of (H, A) and (G, B) is denoted by $(H, A) \vee (G, B)$ and is defined by $(H, A) \vee (G, B) = (K, A \times B)$, where the truth-membership, indeterminacy-membership and falsity-membership of $(K, A \times B)$ are as follows:

$$\begin{aligned} T_{K(\alpha, \beta)}(m) &= \max\{T_{H(\alpha)}(m), T_{G(\beta)}(m)\}, \\ I_{K(\alpha, \beta)}(m) &= \frac{I_{H(\alpha)}(m) + I_{G(\beta)}(m)}{2}, \\ F_{K(\alpha, \beta)}(m) &= \min\{F_{H(\alpha)}(m), F_{G(\beta)}(m)\}. \end{aligned}$$

$\forall \alpha \in A, \beta \in B$ and $m \in U$.

Definition 8 The complement of neutrosophic soft set (F, A) is denoted by $(F, A)^c$ and defined as $(F, A)^c = (F^c, \lceil A)$, where $F^c : \lceil A \rightarrow P(U)$ is a mapping given by $F^c(\alpha) =$ neutrosophic soft complement with $T_{F^c(x)} = F_{F(x)}$, $I_{F^c(x)} = I_{F(x)}$ and $F_{F^c(x)} = T_{F(x)}$.

Now we recall some definitions from [7].

Definition 9 An element x of U is called significant w.r.t. neutrosophic set A of U if the degree of truth-membership or indeterminacy-membership or falsity-membership i.e., $T_A(x) \leq 0.5$ or $I_A(x) \leq 0.5$ or $F_A(x) \leq 0.5$.

Otherwise we call it insignificant. Also, for neutrosophic set the truth-membership or indeterminacy-membership or falsity-membership all cannot be significant. We define intuitionistic neutrosophic set by $A = \{ \langle x : T_A(x), I_A(x), F_A(x) \rangle, x \in U \}$ where

$U\}$ where

$$\begin{aligned} \min\{T_A(x), F_A(x)\} &\leq 0.5 \\ \min\{T_A(x), I_A(x)\} &\leq 0.5 \\ \min\{F_A(x), I_A(x)\} &\leq 0.5 \\ \text{with the condition } 0 &\leq T_A(x) + I_A(x) + F_A(x) \leq 2. \end{aligned}$$

The neutrosophic set by $A = \{ \langle x : T_A(x), I_A(x), F_A(x) \rangle, x \in U \}$ is called generalized neutrosophic set if $\min\{T_A(x), I_A(x), F_A(x)\} \leq 0.5$.

Definition 10 Let U be the initial universe set and $A \subset E$ be a set of parameters. Let $N(U)$ denote the set of all intuitionistic neutrosophic sets of U . The collection (F, A) is termed to be the intuitionistic neutrosophic soft set over U , where F is a mapping given by $F : A \rightarrow N(U)$.

Definition 11 Let (H, A) and (G, B) be two intuitionistic neutrosophic soft sets (in short. INSS) over the common universe U . Then the union of (H, A) and (G, B) is denoted by $(H, A) \cup (G, B)$ and is defined by $(H, A) \cup (G, B) = (K, C)$, where $C = A \cup B$ and the truth-membership, indeterminacy-membership and falsity-membership of (K, C) are as follows:

$$\begin{aligned} T_{K(e)}(m) &= T_{H(e)}(m), \text{ if } e \in A \setminus B \\ &= T_{G(e)}(m), \text{ if } e \in B \setminus A \\ &= \max\{T_{H(e)}(m), T_{G(e)}(m)\}, \text{ if } e \in A \cap B \end{aligned}$$

$$\begin{aligned} I_{K(e)}(m) &= I_{H(e)}(m), \text{ if } e \in A \setminus B \\ &= I_{G(e)}(m), \text{ if } e \in B \setminus A \\ &= \min\{I_{H(e)}(m), I_{G(e)}(m)\}, \text{ if } e \in A \cap B \end{aligned}$$

$$\begin{aligned} F_{K(e)}(m) &= F_{H(e)}(m), \text{ if } e \in A \setminus B \\ &= F_{G(e)}(m), \text{ if } e \in B \setminus A \\ &= \min\{F_{H(e)}(m), F_{G(e)}(m)\}, \text{ if } e \in A \cap B \end{aligned}$$

Definition 12 Let (H, A) , (G, B) be two intuitionistic neutrosophic soft sets over the common universe U . Then the intersection of (H, A) and (G, B) is denoted by " $(H, A) \cap (G, B)$ " and is defined by $(H, A) \cap (G, B) = (K, C)$, where $C = A \cap B$ and the truth-membership, indeterminacy-membership and falsity-membership of (K, C) are as follows:

$$\begin{aligned} T_{K(e)}(m) &= \min\{T_{H(e)}(m), T_{G(e)}(m)\}, \\ I_{K(e)}(m) &= \min\{I_{H(e)}(m), I_{G(e)}(m)\}, \\ F_{K(e)}(m) &= \max\{F_{H(e)}(m), F_{G(e)}(m)\}. \end{aligned}$$

for all $e \in C$ and $m \in U$.

Definition 13 Let (H, A) and (G, B) be two intuitionistic neutrosophic soft sets over the common universe U . Then the 'AND' of (H, A) and (G, B) is denoted by $(H, A) \wedge (G, B)$ and is defined by $(H, A) \wedge (G, B) = (K, A \times B)$, where the truth-membership, indeterminacy-membership and falsity-membership of $(K, A \times B)$ are as follows:

$$\begin{aligned} T_{K(\alpha, \beta)}(m) &= \min\{T_{H(\alpha)}(m), T_{G(\beta)}(m)\}, \\ I_{K(\alpha, \beta)}(m) &= \min\{I_{H(\alpha)}(m), I_{G(\beta)}(m)\}, \\ F_{K(\alpha, \beta)}(m) &= \max\{F_{H(\alpha)}(m), F_{G(\beta)}(m)\}. \end{aligned}$$

$\forall \alpha \in A, \beta \in B$ and $m \in U$.

Definition 14 Let (H, A) and (G, B) be two INSSs over the common universe U . Then the 'OR' of (H, A) and (G, B) is denoted by $(H, A) \vee (G, B)$ and is defined by $(H, A) \vee (G, B) = (K, A \times B)$, where the truth-membership, indeterminacy-membership and falsity-membership of $(K, A \times B)$ are as follows:

$$\begin{aligned} T_{K(\alpha, \beta)}(m) &= \max\{T_{H(\alpha)}(m), T_{G(\beta)}(m)\}, \\ I_{K(\alpha, \beta)}(m) &= \min\{I_{H(\alpha)}(m) + I_{G(\beta)}(m)\}, \\ F_{K(\alpha, \beta)}(m) &= \min\{F_{H(\alpha)}(m), F_{G(\beta)}(m)\}. \end{aligned}$$

$\forall \alpha \in A, \beta \in B$ and $m \in U$.

III. MAIN RESULTS

By the operations Union, Intersection, AND, OR etc. of intuitionistic (resp. generalized) neutrosophic soft sets in the sense of Maji [4] we mean the Union, Intersection, AND, OR operations of intuitionistic (resp. generalized) neutrosophic soft sets which follows the Definition 9 and the Definitions 4, 5, 6, 7 respectively, in accordance to Maji [4].

For comparison of results on intuitionistic (resp. generalized) neutrosophic soft sets in the sense of Said et al [7] with Maji [4], it is sufficient to consider at least one value under one parameter.

Result 1 Intersection of two intuitionistic neutrosophic soft sets is not an intuitionistic neutrosophic soft set in sense of Maji [4].

Example 1 Suppose two intuitionistic neutrosophic soft sets (F, A) and (G, B) assume the following values under a common parameter say, x .

$$\begin{aligned} (F, A) &: (0.3, 0.7, 0.4) \\ (G, B) &: (0.2, 0.5, 0.8) \end{aligned}$$

In the sense of Said et al [7] :
 $(F, A) \cap (G, B) = (0.2, 0.5, 0.8)$, intuitionistic.
 In the sense Maji [4] :
 $(F, A) \cap (G, B) = (0.2, 0.6, 0.8)$, not intuitionistic.

Result 2 Intersection of two generalized neutrosophic soft sets is not a generalized neutrosophic soft set in sense of Maji [4].

Example 2 Suppose two generalized neutrosophic soft sets (F, A) and (G, B) assume the following values under a common parameter say, x .

$$\begin{aligned} (F, A) &: (0.7, 0.7, 0.3) \\ (G, B) &: (0.6, 0.5, 0.8) \end{aligned}$$

In the sense of Said et al [7] :
 $(F, A) \cap (G, B) = (0.6, 0.5, 0.8)$, generalized .
 In the sense Maji [4] :
 $(F, A) \cap (G, B) = (0.6, 0.6, 0.8)$, not generalized .

Result 3 The concept of Union of two intuitionistic (resp. generalized) neutrosophic soft sets (F, A) and (G, B) in the sense of Said et al [7] and Maji [4] will be identical only if

$$A \cap B = \phi.$$

Otherwise, suppose under the parameter $x \in A \cap B$

$$\begin{aligned} (F, A) &: (0.3, 0.8, 0.4) \\ (G, B) &: (0.8, 0.4, 0.2) \end{aligned}$$

In the sense of Said et al [7] :
 $(F, A) \cup (G, B) = (0.8, 0.4, 0.2)$, intuitionistic.
 In the sense Maji [4] :
 $(F, A) \cup (G, B) = (0.8, 0.6, 0.2)$, not intuitionistic.

Similarly, we can made similar conclusion for generalized neutrosophic soft set.

Result 4 AND operation of two intuitionistic (resp. generalized) neutrosophic soft sets is not an intuitionistic (resp. generalized) neutrosophic soft set in sense of Maji [4].

Example 3 Suppose two intuitionistic neutrosophic soft sets (F, A) and (G, B) assume the following values under parameter say, x of A and y of B .

$$\begin{aligned} (F, A) / x &: (0.4, 0.8, 0.2) \\ (G, B) / y &: (0.3, 0.4, 0.7) \end{aligned}$$

In the sense of Said et al [7] :
 $((F, A) \wedge (G, B)) / (x, y) = (0.3, 0.4, 0.7)$, intuitionistic.
 In the sense Maji [4] :
 $((F, A) \wedge (G, B)) / (x, y) = (0.3, 0.6, 0.7)$, not intuitionistic.

Similarly, we can proof it for generalized neutrosophic soft set.

Result 5 OR operation of two intuitionistic (resp. generalized) neutrosophic soft sets is not an intuitionistic (resp. generalized) neutrosophic soft set in sense of Maji [4].

Example 4 Suppose two intuitionistic neutrosophic soft sets (F, A) and (G, B) assume the following values under parameter say, x of A and y of B .

$$\begin{aligned} (F, A) / x &: (0.5, 0.7, 0.2) \\ (G, B) / y &: (0.8, 0.4, 0.2) \end{aligned}$$

In the sense of Said et al [7] :
 $((F, A) \vee (G, B)) / (x, y) = (0.8, 0.4, 0.2)$, intuitionistic.
 In the sense Maji [4] :
 $((F, A) \vee (G, B)) / (x, y) = (0.8, 0.55, 0.2)$, not intuitionistic.

Now suppose two generalized neutrosophic soft sets (F, A) and (G, B) assume the following values under parameter say, x of A and y of B .

$$\begin{aligned} (F, A) / x &: (0.3, 0.7, 0.8) \\ (G, B) / y &: (0.7, 0.5, 0.6) \end{aligned}$$

In the sense of Said et al [7] :
 $((F, A) \vee (G, B)) / (x, y) = (0.7, 0.5, 0.6)$, generalized.
 In the sense Maji [4] :
 $((F, A) \vee (G, B)) / (x, y) = (0.7, 0.6, 0.6)$, not generalized.

Remark 1 From definition, it can be seen that 'Complement of intuitionistic (resp. generalized) neutrosophic soft set is also a intuitionistic (resp. generalized) neutrosophic soft set'.

Remark 2 We now extend the concept of neutrosophic soft sets to refine our choices under a common external factor say x , by following way:

Suppose (F, A) be a neutrosophic soft set over U . Extend this as $(F, A < x >)$, where $F : A < x > \rightarrow P(U)$ satisfying

$$T_{A < x >}(m) = \min\{T_A(m), T(x)\}$$

$$I_{A < x >}(m) = \frac{I_A(m) + I(x)}{2}$$

$$F_{A < x >}(m) = \max\{F_A(m), F(x)\}.$$

Example 5 Let $U = \{h_1, h_2, h_3, h_4\}$ – be the set of houses and $E = \{\text{beautiful, costly, moderate}\}$ – be the set of parameters. If we consider the price of a house, there will be some other factor on which all of the parameter depends. Suppose the houses are in rural area; then its price will be automatically less if it is in urban areas due to transport, environment and some other issues. We may consider all of the factors under consideration and then make the decision. But if we refine our choices by dividing or applying the common outer factors then it will be very effective and easy for decision making.

For example, suppose to choice a house the common factor be *rural* ($=x$) or *urban* ($=y$) areas and the neutrosophic values are $F(\text{rural}) = (0.8, 0.7, 0.5)$ and $F(\text{urban}) = (0.3, 0.5, 0.8)$. Now consider the following houses and their corresponding neutrosophic values against the parameters in Table I.

TABLE I

HOUSES WITH NEUTROSOPHIC VALUES

House	beautiful	costly	moderate
h_1	(0.7,0.3,0.2)	(0.7,0.5,0.5)	(0.5,0.4,0.5)
h_2	(0.5,0.7,0.3)	(0.8,0.4,0.5)	(0.7,0.4,0.3)
h_3	(0.6,0.3,0.4)	(0.6,0.4,0.4)	(0.4,0.7,0.2)
h_4	(0.7,0.1,0.3)	(0.8,0.1,0.1)	(0.6,0.3,0.4)

After applying $F(\text{rural}) = (0.8, 0.7, 0.5)$ on Table I, we have

TABLE II

EFFECT OF OUTER FACTOR x ON TABLE I

House	beautiful	costly	moderate
h_1	(0.7,0.5,0.5)	(0.7,0.6,0.5)	(0.5,0.55,0.5)
h_2	(0.5,0.7,0.5)	(0.8,0.55,0.5)	(0.7,0.55,0.5)
h_3	(0.6,0.5,0.5)	(0.6,0.55,0.5)	(0.4,0.7,0.5)
h_4	(0.7,0.4,0.5)	(0.8,0.4,0.5)	(0.6,0.5,0.5)

After applying $F(\text{urban}) = (0.3, 0.5, 0.8)$ on Table I, we have

TABLE III

EFFECT OF OUTER FACTOR y ON TABLE I

House	beautiful	costly	moderate
h_1	(0.3,0.4,0.8)	(0.3,0.5,0.8)	(0.3,0.45,0.8)
h_2	(0.3,0.6,0.8)	(0.3,0.45,0.8)	(0.3,0.45,0.8)
h_3	(0.3,0.4,0.8)	(0.3,0.45,0.8)	(0.3,0.6,0.8)
h_4	(0.3,0.3,0.8)	(0.3,0.3,0.8)	(0.3,0.4,0.8)

Note that Table I is intuitionistic (resp. generalized) neutrosophic soft set, $F(\text{rural})$ is generalized neutrosophic set and $F(\text{urban})$ is intuitionistic (resp. generalized) neutrosophic set. But Table II and Table III are both generalized neutrosophic soft sets.

Decision Making: To refine our choices in case of decision making we first extend our choices on neutrosophic soft sets and then apply the same procedure as stated in [4].

ACKNOWLEDGMENT

The author is thankful to the Editors and the Referees for their valuable comments.

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