The Solution of the Direct Problem of Electrical Prospecting with Direct Current under Conditions of Ground Surface Relief

Balgaisha Mukanova, Tolkyn Mirgalikyzy

Abstract—Theory of interpretation of electromagnetic fields studied in the electrical prospecting with direct current is mainly developed for the case of a horizontal surface observation. However in practice we often have to work in difficult terrain surface. Conducting interpretation without the influence of topography can cause non-existent anomalies on sections. This raises the problem of studying the impact of different shapes of ground surface relief on the results of electrical prospecting's research. This research examines the numerical solutions of the direct problem of electrical prospecting for two-dimensional and three-dimensional media, taking into account the terrain. The problem is solved using the method of integral equations. The density of secondary currents on the relief surface is obtained.

Keywords—Ground surface relief, method of integral equations, numerical method.

I. INTRODUCTION

DUE to the relief redistribution of current density occurs; because of this effect such false anomalies arise [1]. The other works which are carried out by several authors mainly reduced to physical or mathematical modeling of simple models. Nowadays some basic methods of solving direct tasks of electric prospecting are developed: transmission surface method [2], method of integral equations [3]-[5], finite difference method [6], [7], finite element method [8], boundary element method [9].

To this date the developed methods aimed to some extent for the effect of relief did not led to the creation of a unified theory and methodology and have no application data interpretation of electrical direct current to the relief surface. Using the apparatus of the conformal transformation, [10] solves the problem of distortion of separate forms of electric field relief. In the method of finite differences two basic algorithms accounting surface topography observations are developed [1], [11]-[14]. In the finite element method there are several ways of accounting terrain for solving direct problems [15]-[18].

In this work the integral equations method is chosen to solve the direct problem of electrical direct current with the

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relief of the earth surface which has a simple physical meaning and well-established during the two-dimensional simulation [3]-[5], [19]-[22].

II. THE PROBLEM WITH THE EFFECT OF GROUND SURFACE RELIEF

Studies of the influence of the relief on resistivity sounding data were considered in [10]-[18], [23]. Fig. 1 shows the general effects of the relief for remote current source. In this case, the current flow lines diverge on the convex part of the relief and converge on the concave part, and as a result, the directions of electric field lines changes. Because of this effect, the convex shape of the relief reduces, and concave shape increases the value of the apparent resistivity.
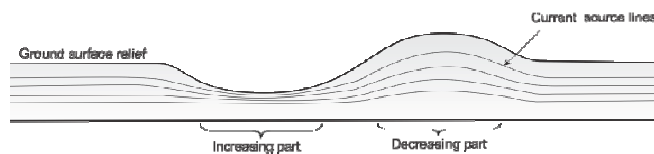

Fig. 1 The general effects of the relief for remote current source

Existing inversion programs are oriented on automatic solving of the inverse problem within the frameworks of 2D-inversion of the electric field, without attendance of any man, which should enter additional, prior information. Due to an influence of the principle of equivalence, a formal improvement of proximity of experimental and theoretically calculated fields leads to sharp increase of objects property contrast in the chosen section. This relatively changes the geometry of the collected section. In most of 2D-inverse programs the solution of inverse problem is performed with minimal contrast of electric properties of the medium. As a result, the solution appears to be smooth and with blurred boundaries, which usually do not exist in real geological situations. In this case 2D inversion is not an ideal way to move from electric field to real or true geologic section. Most authors call such transformation "controlled transformation". In bibliography by most of authors the method of electric tomography is called "Electrical Imaging" or "Resistivity Imaging" [24], which terminologically points to distant likeness between real sections and sections, achieved by results of inversion. Application of distortion effects caused by heterogeneity of the section and the relief produces a complicated picture in resulting geoelectrical pseudosection. Check drilling which is made by the results of geo-physical

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works is very expensive and missing of down holes off ore objects, which was forecasted by geophysic scientists, often has very negative consequences. Without special mathematic modeling on systematization of section distortions, caused by influence of relief during 2D or 3D inversion, it's impossible to achieve final geoelectric sections, which are adequate to true ones.

III. METHODS AND MATHEMATICAL MODEL

In this research, the main instrument of modeling fields is the method of integral equations. The idea of this method is to provide the electric field as a sum of the primary field (generated by current electrodes) and the field of secondary charges (occur when the electric current flows in points of homogeneity violation of the medium and on the surface of the medium): $E = E_0 + E_{sec}$, where E_0 is the vector of the primary electric field, E_{sec} is the vector of total electric field of the secondary charges. Contact boundaries and heterogeneous inclusions of the geoelectric section act as secondary creators of electric field. Field computation problem is reduced to the system of integral equations on density of secondary sources, inducted on contact surfaces of conductive mediums and on surface relief of the medium. The mathematical description of this event leads to the Fredholm equations of type II.

We obtained recently integral equations for solving the direct problem of sounding on local inclusion and buried relief in [19]-[21]. It was shown in [22], that the method of integral equations gives much more accurate field computation and especially their first derivatives in comparison with method of finite volumes. In the works, mentioned above, there were shown such advantages of method of integral equations, as decreasing the dimension of the problem from 3D to 2D and an opportunity of unified (one-for-all) approach for complicated configuration of contacting medium boundaries.

In [3]-[5], [19]-[22], related with application of the method of integral equations to modeling of problem of electric sounding, they did not consider cases of relief form of daylight surface of the Earth. Or the cases were not carried up to systematical numeral modeling.

Presence of relief essentially complicates a problem formulation for the method of integral equations in comparison with a case when the surface is plain. This is due to the fact that for plain surface of discontinuity "earth-air" we can use method of reflections [4], which allows us to get a relatively simple type of integral equations or a system of integral equations. In case of surface relief of discontinuity of the sounding medium and complicated internal boundary, a resulting mathematical model should be carried out as a system of integral equations.

We consider the mathematical model of a direct current sounding method for medium with constant resistivity with any relief on the surface and with three-array of Schlumberger.

It is known from Maxwell's equations that the stationary field electrostatic potential at points in the medium satisfies the differential equation of Laplace:

$$\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} + \frac{\partial^2 \varphi}{\partial z^2} = 0 \tag{1}$$

The condition of decreasing the potential at infinity $\varphi(\infty) = 0$, and the condition on boundary of earth-air must be satisfied:

$$\left.\frac{\partial \varphi}{\partial z}\right|_{z=0} = 0 \tag{2}$$

IV. APPLICATION OF THE METHOD OF INTEGRAL EQUATIONS AND NUMERICAL RESULTS

To calculate the field on the surface relief we assigned some distribution of current density on the surface, which compensate for it so as to satisfy the physical boundary condition:

$$\left.\sigma\frac{\partial \varphi}{\partial n}\right|_{\Gamma} = \left.-\sigma\frac{\partial U_0}{\partial n}\right|_{\Gamma} \tag{3}$$

Here, σ is the layer's electrical conductivity, and U_0 is the potential of a source electrode. For distribution of current densities on the surface relief we used a grid with a logarithmic expanding scale by radial, adapted to the position of the source electrode.

Integral equations can be written for the intensities of secondary sources, and for the values of the potentials. We have built the integral equation for distribution of secondary sources, taking into account the given boundary condition and Green's formula. The following is the integral equation for distribution of secondary sources at points of M on the medium surface, formed under the influence of a single point current source, located at some point on the surface. Under the integrand calculates the mutual influence of all other sources on the source of M. From the integral equation it is required to calculate the current density of secondary source, $I_S(P)$-?

$$I_S(M) = -2\sigma\left(\iint_\Gamma \frac{\partial G(P,M)}{\partial n} I_S(P)dS_P + E_0(M)\right), \tag{4}$$

$$\frac{\partial G(P,M)}{\partial n} = \frac{1}{\sigma \cdot 4\pi} \cdot \frac{\vec{r}_{PM}}{r_{PM}^3} \cdot \vec{1}, \tag{5}$$

where, $I_S(M)$ is the current density of secondary source at point of M on the surface Γ; $E_0(M)$ is the electric field from a primary source at this point M; $G(P,M)$ is Green's function; $I_S(P)$ is the current density of secondary source at point of P, dS_P is the surface area where the source P is. Equation (4) expresses the physical fact, that at the points of M of the surface the total current is zero, except the current source electrode.
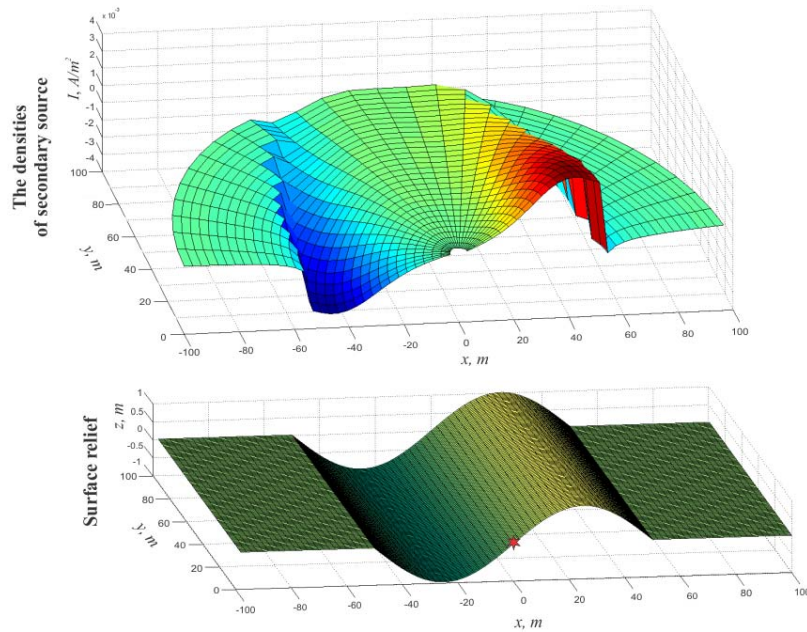
After calculating the distribution of secondary sources, the values of the potentials on the surface are calculated:

$$\frac{\partial \varphi}{\partial r}(M) = U_0(M) + \iint_\Gamma \frac{I_S(P)}{\sigma \cdot 4\pi} \cdot \frac{1}{|r_{PM}|}dS_P, \tag{6}$$

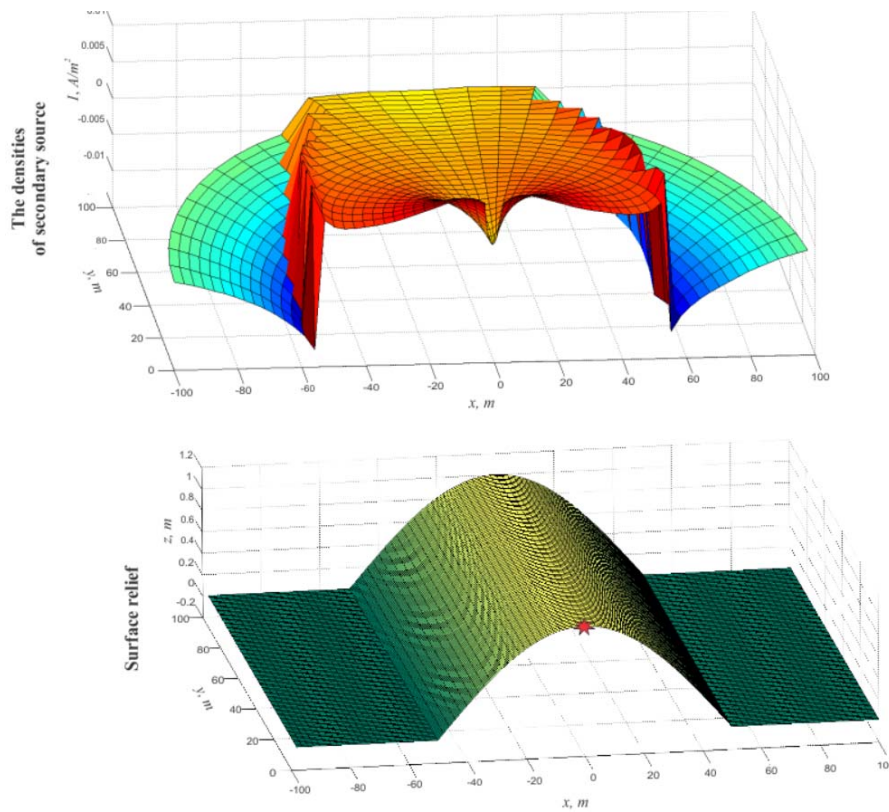$$U_0(M) = \frac{I_S(M)}{\sigma \cdot 2\pi} \cdot \frac{1}{|r_{PM}|}. \tag{7}$$

$U_0(M)$ is the potential of a source electrode.

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Fig. 2 shows the current density of the surface, obtained by solving the integral equation.

(a)

(b)

Fig. 2 Current densities of secondary sources for different shapes of surface relief: A red asterisk indicates the position of the current source electrode; (a) Source electrode is located on a flat surface relief (b) source electrode is located on a convex surface relief

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V. CONCLUSION

The numerical method and computer program solves the direct problem of electrical prospecting with direct current under conditions of ground surface relief. The method is based on the construction of the adapted grid and iterative algorithm. The obtained results allow estimating the effect of topography on the measurements and will improve the quality of work and the development of computer technologies used in geophysics.

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