Numerical Evaluation of Nusselt Number on the Hot Wall in Square Enclosure Filled with Nanofluid

A. Ghafouri, A. Falavand Jozaei, M. Salari

***Abstract*—**In this paper, effects of using Alumina-water nanofluid on the rate of heat transfer have been investigated numerically. Physical model is a square enclosure with insulated top and bottom horizontal walls, while the vertical walls are kept at different constant temperatures. Two appropriate models are used to evaluate the viscosity and thermal conductivity of nanofluid. The governing stream-vorticity equations are solved using a second order central finite difference scheme, coupled to the conservation of mass and energy. The study has been carried out for the Richardson number 0.1 to 10 and the solid volume fraction 0 to 0.04. Results are presented by isotherms lines, average Nusselt number and normalized Nusselt number in different range of φ and Ri for forced, combined and natural convection dominated regime. It is found that higher heat transfer rate is predicted when the effects of nanoparticle is taken into account.

***Keywords*—**Nanofluid, Heat Transfer Enhancement, Square Enclosure, Nusselt number.

I. Introduction

THE main restriction of common fluids used for heat transfer application such as water, ethylene glycol, mineral oil or propylene glycol is their low thermal conductivity. Nanofluid, which is a mixture of nano-size particles suspended in base fluid, has a superior thermal conductivity compared to the base fluid [1]. The nanofluid can be used to various engineering applications such as solar collectors, heat exchangers, materials processing, cooling of electronic devices, crystal growth, metal coating and casting, and nuclear system cooling [2]-[4].

There have been many investigations in the past decade on the natural and combined convective heat transfer and fluid flow in lid-driven square enclosure. Oztop and Abu-Nada [5] considered the buoyancy-driven heat transfer in a two-dimensional chamber with different aspect ratio and filled with different types of nanoparticles for the Rayleigh numbers of 10^3 to 5×10^5 using the finite volume method. They concluded that the existence of the nanoparticles resulted in an increase in the rate of heat transfer and average Nusselt number for the whole range of Rayleigh number. Sheikhzadeh et al. [6] performed a numerical study of the heat transfer performance of nanofluids inside two-dimensional rectangular enclosures. Their results indicated that increasing the volume fraction of nanoparticles produced a significant enhancement of the average rate of heat transfer. Also their results showed that the variation of average Nusselt number with increasing the volume fraction of nanoparticles is linear in two studied cases. Oztop et al. [7] investigated the laminar natural convection flow through a square inclined enclosure filled with a CuO nanofluid. They found that heat transfer in the cavity increases by adding nanoparticles. Also their results showed that the rate of increase is greater for the enclosures with low Rayleigh number. Ghafouri and salari [8] numerically studied Laminar Mixed convection in the lid-driven square enclosure filled with CuO nanoparticle using eight different nanofluid viscosity models. They found that the rate of heat transfer is accentuated moderately by falling the Richardson number and rising the solid volume fraction.

Something that is common in most of the cited numerical study on nanofluid combined convection heat transfer problems is the use of the Maxwell-Garnett thermal conductivity model [9] for a nanofluid and hence, predict enhancement of heat transfer because of the presence of nanoparticles and thermal conductivity coefficients of base fluid and nanoparticles. This model does not consider main mechanisms for heat transfer in nanofluids such as Brownian motion and does not consider nanoparticle size or temperature dependence. However, it is evident that the use of the accurate fundamental model for evaluating the thermal conductivity of nanofluids can be very influential. A number of researches have performed in comparative study of variable thermal conductivity models in various configurations. Abu-Nada [10] investigated the effects of thermal conductivity of Al_2O_3-water nanofluid on heat transfer enhancement in natural convection. Two different thermal conductivity models namely [11] and [9] evaluated by comparing their predicted results on Nusselt number. He indicated that at $Ra\geq10^4$, the difference in Nusselt number between the Maxwell-Garnett [9] and Chon et al. model [11] prediction was small. However, there was a deviation in prediction at $Ra=10^3$ and this deviation becomes more significant at high volume fraction of nanoparticles. Similar comparative investigations with pair of models have been reported by [12]-[17].

The main motivation of this study is to investigate numerically the rate of heat transfer on the hot wall of square enclosure utilized with water-Alumina nanofluid and its effect on natural, combined and forced convections heat transfer. In order to achieve this objective, the model of Nguyen et al. is employed for viscosity of nanofluid while Chon et al. [11] model is used to evaluate the thermal conductivity of nanofluid. The results and enhancement in heat transfer rate will be obtained for a wide range of Richardson number and

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volume fraction of the solid particles on isotherms lines, average Nusselt number and normalized Nusselt number.

II. Problem Description

Fig. 1 depicts schematics of the square enclosure considered in this paper. The nanofluid is assumed as incompressible and the flow is assumed to be laminar. It is supposed that the base fluid (i.e. water) and the nanoparticles are in thermal equilibrium and no slip occurs between them. The shape and the size of solid particles are furthermore assumed to be uniform and their diameter to be equal to 38.4 nm. The top and bottom horizontal walls are thermally insulated while the vertical walls are kept at constant but different temperatures. In order to induce the buoyancy effect, the left vertical wall is kept at a relatively high temperature (T_H) and right vertical wall is maintained at a relatively low temperature (T_C). The top wall is moving rightwards while the bottom wall is moving leftwards with a uniform velocity (U_m). The thermo-physical properties of the pure water and the nanoparticle at temperature of 25°C are given in Table I. The thermo-physical properties of the nanofluid are assumed to be constant except for the density variation, which is approximated by the Boussinesq model.

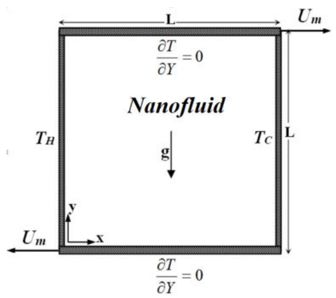

Fig. 1 Schematic of problem

TABLE I
Physical Properties of Base Fluid and Nanoparticles

Physical properties	Fluid phase (Water)	Al_2O_3
C_p (J/kgK)	4179	765
ρ (kg/m^3)	997.1	3970
k (W/mK)	0.613	25
$\beta\times10^{-5}$ (1/K)	21	0.85

III. Governing Equation

The non-dimensional governing equations in the Cartesian coordinate system for the stream function (Ψ), vorticity function (Ω) and thermal transport, respectively, can be written as:

$$\frac{\partial^2\Psi}{\partial X^2}+\frac{\partial^2\Psi}{\partial Y^2}=-\Omega \tag{1}$$

$$U\frac{\partial\Omega}{\partial X}+V\frac{\partial\Omega}{\partial Y}=\frac{\mu_{nf}/\mu_f}{(1-\varphi)+\varphi\frac{\rho_s}{\rho_f}}\frac{1}{\mathrm{Re}}\left(\frac{\partial^2\Omega}{\partial X^2}+\frac{\partial^2\Omega}{\partial Y^2}\right)+\left((1-\varphi)+\varphi\frac{\beta_s}{\beta_f}\right)Ri\frac{\partial\theta}{\partial X} \tag{2}$$

$$U\frac{\partial\theta}{\partial X}+V\frac{\partial\theta}{\partial Y}=\frac{k_{nf}/k_f}{(1-\varphi)+\varphi\frac{(\rho c_p)_s}{(\rho c_p)_f}}\frac{1}{\mathrm{Re}.\mathrm{Pr}}\left(\frac{\partial^2\theta}{\partial X^2}+\frac{\partial^2\theta}{\partial Y^2}\right) \tag{3}$$

where the Reynolds number, Re, the Prandtl number, Pr, and the Richardson number, Ri, are defined as:

$$\mathrm{Re}=\frac{U_m L}{\nu_f},\qquad \mathrm{Pr}=\frac{\nu_f}{\alpha_f},\qquad Ri=\frac{Ra}{\mathrm{Pr}\,\mathrm{Re}^2} \tag{4}$$

The additional dimensionless variables in the above equations are defined as:

$$U=\frac{\partial\Psi}{\partial Y},\quad V=-\frac{\partial\Psi}{\partial X},\quad \Psi=\frac{\psi}{U_m L},\quad \Omega=\frac{\omega L}{U_m} \tag{5}$$

The effective dynamic viscosity of Al_2O_3-water nanofluid is estimated by [18]:

$$\mu_{Al_2O_3}=\exp(3.003-0.04203T-0.5445\varphi+0.0002553T^2+0.0524\varphi^2-1.622\varphi^{-1}) \tag{6}$$

This viscosity in the above equation is expressed in centipoise and the temperature in °C. The viscosity of the base fluid (water) considered to vary with temperature as following:

$$\mu_f=2.414\times10^{-5}\times10^{247.8/(T-140)} \tag{7}$$

The effective thermal conductivity of the nanofluid is approximated by the Chon et al. [11] model:

$$\frac{k_{nf}}{k_f}=1+64.7\varphi^{0.7640}\left(\frac{d_f}{d_s}\right)^{0.3690}\left(\frac{k_s}{k_f}\right)^{0.7476}\mathrm{Pr}_T^{\,0.9955}\,\mathrm{Re}_T^{\,1.2321} \tag{8}$$

where Pr_T, Re_T and l_f are defined as:

$$\mathrm{Pr}_T=\frac{\mu_f}{\rho_f\alpha_f},\quad \mathrm{Re}_T=\frac{\rho_f k_B T}{3\pi\mu_f^2 l_f},\quad l_f=\frac{1}{\sqrt{2m\pi\, d_f^2}} \tag{9}$$

where k_B is the Boltzmann constant (k_B = 1.3807×10^{-23} J/K). Also, l_f is the mean path of base fluid particles given as 0.17 nm [19]. This model considers the effect of nanoparticle size and temperature on nanofluids thermal conductivity with a wide range of temperature between 21 and 70°C. Accuracy of this model was confirmed by the experiments of [20].

IV. Numerical Method

The set of nonlinear coupled governing mass, momentum and energy equations are developed in term of stream function- vorticity formulation. The classical theory of single phase fluids can be applied, where physical properties of nanofluid are taken as a function of properties of both constituents and their concentrations. In order to numerically solve, the governing equation for two dimensional velocity and temperature field, (1)–(3) with the corresponding boundary conditions are approximated by second-order central difference scheme. Successive over relaxation (SOR) scheme is used to solve stream function equation. The numerical solution is made by solving sequentially the stream function, vorticity and temperature equations and advanced into a next time level by solving systems of each equation that is numerically modeled in FORTRAN. The convergence criterion is defined by the following expression:

$$\varepsilon = \frac{\sum_{j=1}^{j=M}\sum_{i=1}^{i=N}\left|\zeta^{n+1}-\zeta^{n}\right|}{\sum_{j=1}^{j=M}\sum_{i=1}^{i=N}\left|\zeta^{n+1}\right|} \le 10^{-6} \tag{10}$$

where ε represents the tolerance, also M and N are the number of grid points in the X and Y direction, respectively. The symbol ζ denotes any scalar transport quantity namely Ψ, Ω or θ.

The local and average heat transfer rates of the chamber can be presented by means of the local and average Nusselt numbers. The local Nusselt number is calculated along the left heated wall (11) and the average Nusselt number is determined by integrating the local Nusselt number along the heated wall (12):

$$Nu(X) = \frac{k_{nf}}{k_f}\frac{\partial\theta}{\partial X}\Big|_{X=0} \tag{11}$$

$$Nu_{avg} = \int_0^1 Nu(X)dY \tag{12}$$

V. Code Validation

The governing equations have been solved for the natural convection flow in a cavity filled by pure fluid, in order to compare the results with those obtained by [21]-[23], [14]. This comparison revealed good agreements between the results, which are shown in Table II.

TABLE II
COMPARISONS OF THE PRESENT RESULTS FOR THE AVERAGE NUSSELT NUMBER OF THE HOT WALL WITH THE RESULTS OF OTHER INVESTIGATORS

	$Ra=10^4$	$Ra=10^5$	$Ra=10^6$
Present work	2.246	4.521	8.984
Sheikhzadeh et al. [14]	2.242	4.514	8.790
Khanafer et al. [21]	2.245	4.522	8.826
Fusegi et al. [22]	2.302	4.646	9.012
Markatos and Pericleous [23]	2.201	4.430	8.754

VI. Result and Discussion

Combined convection heat transfer is studied numerically for Alumina-water nanofluid in a square enclosure. Calculations were accomplished for Richardson numbers of 0.1,1 and 10, and nanoparticle volume fraction of 0.01, 0.02, 0.03, and 0.04. Appropriate choices of viscosity models and thermal conductivity models have strongly influence on predict heat transfer rate and Nusselt number correctly. Results are presented by average Nusselt number and normalized Nusselt number on the hot wall.

Figs. 2 and 3 represent variation of the average Nusselt number and normalized Nusselt number by nanoparticle volume fraction at *Re*=100. As seen from the figures, average Nusselt number and normalized Nusselt number increase with nanoparticle volume fraction (φ=0 to 4%).

At φ=4% Fig. 2 indicate that for small *Ri* (*Ri*=0.1, purely forced convection regime) the highest average Nusselt number (Nu_{ave}=8.1) is observed at forced convection heat transfer with using the Chon et al. [11] thermal conductivity model by considering the role of Brownian motion, temperature and nanoparticles size.

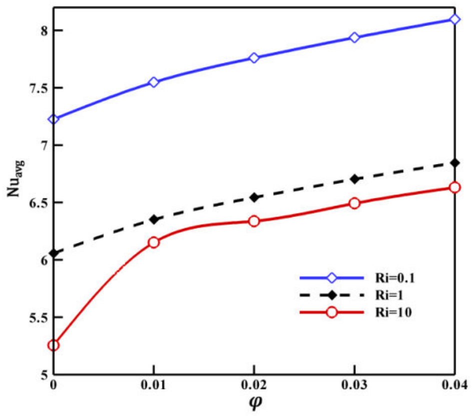

Fig. 2 Effect of different solid volume fractions of nanofluid on the average Nusselt number at different *Ri*

Fig. 3 depicts the Effect of different solid volume fractions of nanofluid on the normalized Nusselt number at different Richardson number. It is indicated that the highest and lowest assess are related to Richardson number of 10 (natural dominated convection) and 0.1 (Forced dominated convection) by 26% and 12%, respectively.

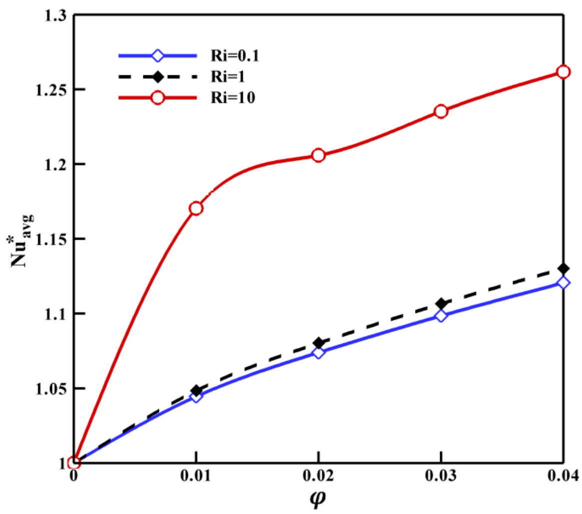

Fig. 3 Effect of different solid volume fractions of nanofluid on the normalized Nusselt number at different *Ri*

Figs. 4 and 5 demonstrate the effect of various Richardson number on the average Nusselt number and normalized Nusselt number at Re=100 and φ=4% compare with base fluid Nusselt number. It can be seen that average Nusselt number decreases with the increase of Ri. Nonetheless by the increasing of Richardson number, normalized Nusselt number (Nu^*_{ave}) will be rising proportional to nanoparticle volume fraction.

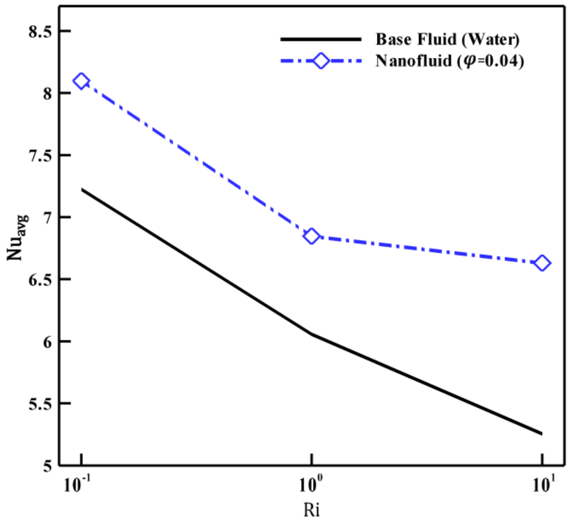

Fig. 4 Effect of different Richardson number on the average Nusselt number for φ=0 %(——) and φ=4 %(-----)

Moreover, at Re=100 and φ=4% the highest enhancement in average Nusselt number is observed at natural convection heat transfer 26% and this value at φ=2% is equal to 21%.

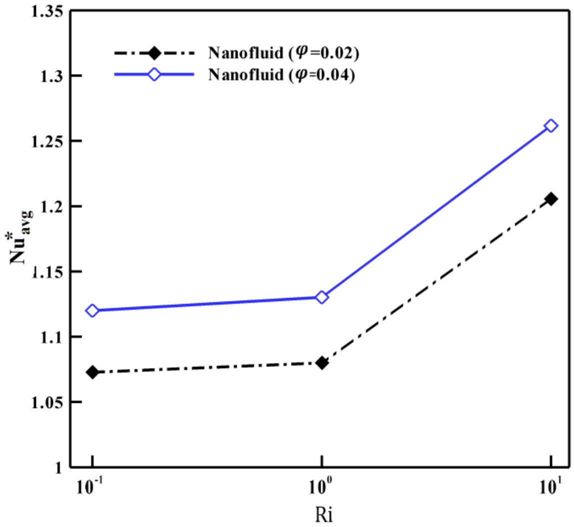

Fig. 5 Effect of different Richardson number on the normalized Nusselt number for φ=2 %(-----) and φ=4 %(——)

VII. Conclusions

Laminar combined convection in two-dimensional square enclosure filled with a water-Alumina nanofluid was numerically studied. The top wall moved to the right while the bottom wall moved to the left at the same constant speed. Two horizontal walls of the enclosure were kept insulated while the right and left walls were maintained isothermally, however the temperature of the left wall was higher than the right wall. The governing equations were given in term of the stream function-vorticity formulation in the non-dimensionalized form and then solved numerically by second-order central difference scheme. The model of Nguyen et al. is employed for viscosity of nanofluid while Chon et al. model is used to evaluate the thermal conductivity of nanofluid. In these models, the effects of nanofluid bulk temperature, nanoparticles size, nanoparticles volume fraction and Brownian motion were incorporated. The main findings are listed as follows:

- The Nusselt number and thus heat transfer from the left wall of enclosure was enhanced with increasing volume fraction of nanoparticle.
- Heat transfer in square enclosure was enhanced by decreasing the value of Richardson number keeping constant the other parameters.
- It is indicated that by the increasing of Ri the average Nusselt number will be declining and normalized Nusselt number will be increasing, respectively.

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