Analysis of Plates with Varying Rigidities Using Finite Element Method

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***Abstract*—**This paper presents Finite Element Method (FEM) for analyzing the internal responses generated in thin rectangular plates with various edge conditions and rigidity conditions. Comparison has been made between the FEM (ANSYS software) results for displacement, stresses and moments generated with and without the consideration of hole in plate and different aspect ratios. In the end comparison for responses in plain and composite square plates has been studied.

***Keywords*—**ANSYS, Finite Element Method, Plates, Static Analysis.

I. Introduction

In structural mechanics, the basic problem is the determination of the deformational response of various structural components, of different rigidities, subjected to loads. The responses involve determination of stresses, strains and displacements at every point of the structure or it may also involve the determination of the loads when the structure becomes unstable, as in the case of static instability phenomenon.

Objective of this paper is to find the response of plates with and without holes due to static loading. In the paper, internal responses generated in thin rectangular plates of varying dimensions, end conditions and rigidity have been studied through Finite Element Method (FEM) of analysis. Methodology adopted has been explained in Section II and description of the model has been given in Section III. Section IV summarises the results and related discussions on the study. Finally conclusion has been presented in Section V.

The displacement based finite element method, endowed with smartness in solving complex structural problems, manifested itself as a versatile yet most accurate tool of analysis. With the result, a large amount of research to date is available regarding the static and dynamic behaviour of the plate as is evident from [1]-[3]. References [4] and [5] have studies on behaviour of composite and stiffened plates. In recent years, more and more work is been done using finite element method based software packages. Bending analysis of a moderately thick orthotropic sector plate subjected to various loading and boundary conditions has been studied in [6]. Reference [7] gives several useful results by analysing isotropic and composite folded plates made of a high precision composite plate bending element. A very similar work was done in [8] and bending analysis of an isotropic plate of different thickness, loading and boundary conditions was done.

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II. Methodology

Usual FEM equilibrium equations have been used in ANSYS software for analysing of the plate problems. Steel plates of different aspect ratios and rigidity conditions are modelled and analysed for comparison.

A. Description of Element Used in ANSYS Software

Validation check for selecting the proper element has been performed for SHELL 63, SHELL 93, SHELL 99 and SHELL 91; SHELL 63 has been selected for giving the most accurate results for deformation when compared to the results from Classical Methods. SHELL63 has both bending and membrane capabilities. Both normal and in-plane loads are permitted. The element has six degrees of freedom at each node: translations in the nodal x, y, and z directions and rotations about the nodal x, y, and z-axes. Stress stiffening and large deflection capabilities are included.

III. Model Description

FEM (ANSYS software) has been used for plates (Fig. 1) of varying boundary conditions and aspect ratios. The dimensional parameters for all models considered here are summarised in Table I.

TABLE I
Summary of Models Analyzed

Edge Conditions	Rigidity Condition	Aspect Ratios considered (b=500mm)
All edges clamped	(I) Without hole (II) With square hole at centre (III) With square hole at corner	1, 1.5, 2
All edges simply supported	(I) Without hole (II) With rectangular hole at centre	1.0, 1.5, 2.0, 3
Two opposite edges clamped and two simply supported	(I) Without hole (II) With square hole at centre	1, 1.5, 2
Two opposite edges free and two simply supported	(I) Without hole (II) With circular hole at centre	1, 1.5, 2

All 120 mm thick plates have been modelled as plain plates and analysed for a uniformly distributed load of intensity 5.88 x 10^{-3}N/mm^2. Other material properties are:

- Poisson's Ratio, μ = 0.3
- Young's Modulus of Elasticity, E = 2 x 10^5 N/mm^2.

To examine the versatility and robustness of FEM, a few examples of composite plates with and without holes have been studied. Side of square plate, b = 500mm, thickness = 60mm for both plates and Young's Modulus of Elasticity for plate 1, E_1= 2 x 10^4N/mm^2 and for plate 2, E_2= 2 x 10^6 N/mm^2.

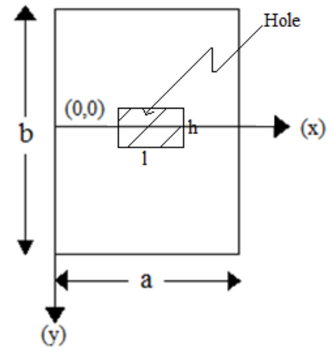


Fig. 1 Arrangement of plate and hole in Cartesian coordinates

IV. Results and Discussion

Deflection and moment responses of various plates have been summarized in Tables II-VI. Contour plots for deformation in z-direction and stresses in x and y directions have been presented in Figs. 2-13. In deformation contours, regions in red represent the regions of maximum deflection and stress. Decreasing values are represented by shades of yellow, green and blue with regions of dark blue representing zero deformation. For stress contours, blue shade defines negative limit while red defines the positive limit of stresses. Cases I, II and III in Tables II-V represent the rigidity conditions as defined in Table I.

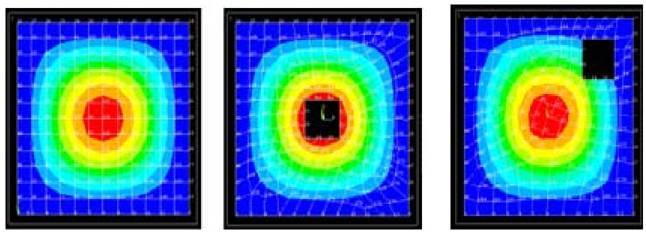
A. All Edges Clamped

Rectangular plates with aspect ratios given in Table II have been analysed as clamped. Both central and corner holes are square with 100mm sides.

TABLE II
Results for Plates with All Edges Clamped

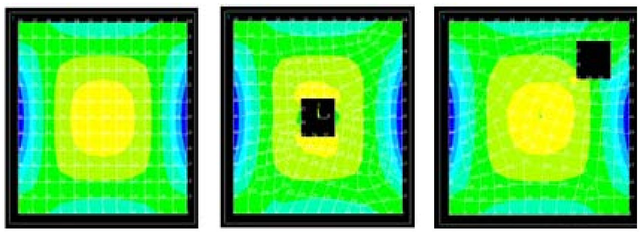
Aspect ratio (a/b)	w_{max}(mm)		
	I	*II*	*III*
1.0	1.48E-4	1.53E-4	1.50E-4
1.5	2.58E-4	3.24E-4	2.58E-4
2.0	2.99E-4	3.46E-4	2.99E-4
	Mx_{max}(N-mm)		
	I	*II*	*III*
1.0	71.85	70.56	74.99
1.5	76.73	80.69	82.73
2.0	73.61	87.92	82.90
	My_{max}(N-mm)		
	I	*II*	*III*
1.0	71.85	70.93	75.20
1.5	108.26	115.59	111.18
2.0	119.43	117.07	115.94

Figs. 2-4 show the stress and deformation contours in the plate as obtained by FEM in ANSYS.



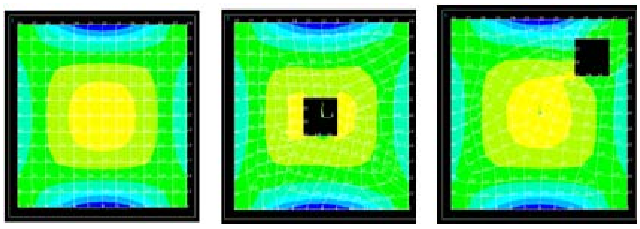
(a) (b) (c)

Fig. 2 Contour of deformation for clamped square plate (a) without hole (b) with square hole at centre and (c) with square hole at a corner



(a) (b) (c)

Fig. 3 Contour of stresses in x direction for clamped square plate (a) without hole (b) with square hole at centre and (c) with square hole at a corner



(a) (b) (c)

Fig. 4 Contour of stresses in y direction for clamped square plate (a) without hole (b) with square hole at centre and (c) with square hole at a corner

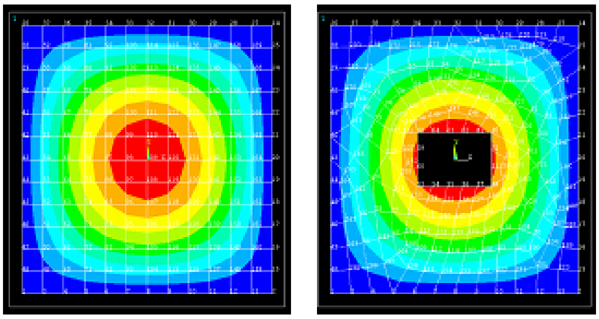
For plate with hole at centre and for one at corner, the deflection increases with increase of aspect ratio. The relation is not completely linear and is different from that of plain plate. Maximum moments generated increase sharply up to a certain value and then gradual increase is seen in all cases. The values of deflection and moments generated increase in case of plate with hole for all aspect ratios.

B. All Edges Simply Supported

Rectangular plates with aspect ratios given in Table III have been analysed as simply supported. Rectangular holes with l = 150mm and h = 100mm have been provided.

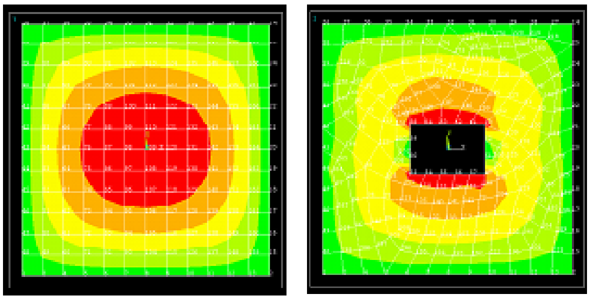
TABLE III
RESULTS FOR PLATES WITH ALL EDGES SIMPLY SUPPORTED

Aspect ratio (a/b)	w_{max}(mm)		Mx_{max}(N-mm)		My_{max}(N-mm)	
	I	*II*	*I*	*II*	*I*	*II*
1.0	4.68E- 4	5.54E-4	71.15	87.10	71.15	96.70
1.5	1.76E- 4	1.40E-4	53.33	103.05	33.06	222.5
2.0	7.32E- 5	1.59E-4	37.49	114.78	17.43	26.12
3	1.75E- 5	1.67E-4	19.41	99.93	7.55	26.49



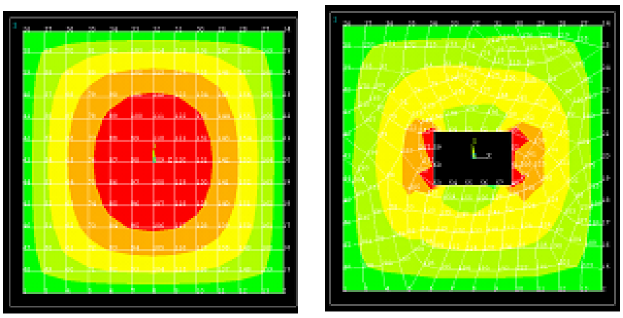
(a) (b)

Fig. 5 Contour of deformation for simply supported square plate (a) without hole and (b) with square hole in centre



(a) (b)

Fig. 6 Contour of stress in x-direction for simply supported square plate (a) without hole and (b) with square hole in centre



(a) (b)

Fig. 7 Contour of stress in y-direction for simply supported square plate (a) without hole and (b) with square hole in centre

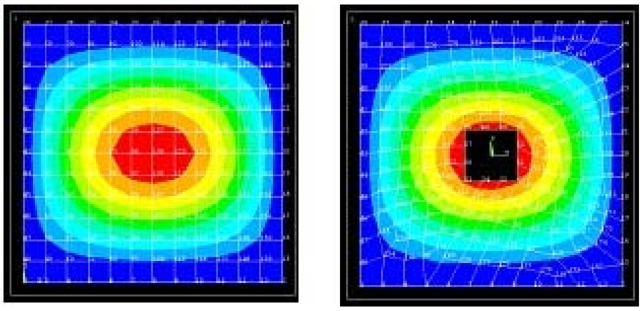
In case of plates without hole, it is seen that, out plane deflection and moments are increasing with decrease of aspect ratio. This may be due to decrease of stiffness of the plate with decrease of aspect ratio. The maximum out plane deflection and stress occurs at centre of the plate. In case of a hole, the deflection decreases with decrease of aspect ratio up to aspect ratio of 1.5 and thereafter increases abruptly. This behavior is different from that in plain plate.

C.Two Opposite Edges Clamped and Two Simply Supported

Rectangular plates with aspect ratios given in Table IV have been analysed with two edges clamped and two edges simply supported. Figs. 8 to 10 show the stress and deformation contours in the plate as obtained by FEM analysis in ANSYS. Here square holes of 100mm sides have been provided.

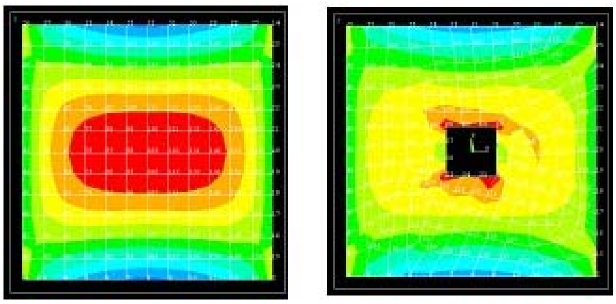
TABLE IV
RESULTS FOR PLATES WITH TWO EDGES CLAMPED AND TWO SIMPLY SUPPORTED

Aspect ratio (a/b)	w_{max}(mm)		Mx_{max}(N-mm)		My_{max}(N-mm)	
	I	*II*	*I*	*II*	*I*	*II*
1.0	2.23E- 4	2.41E-4	36.81	44.90	49.85	97.94
1.5	2.89E- 4	3.13E-4	26.96	42.21	60.61	113.91
2.0	3.07E- 4	3.34E-4	21.28	44.28	62.60	117.78



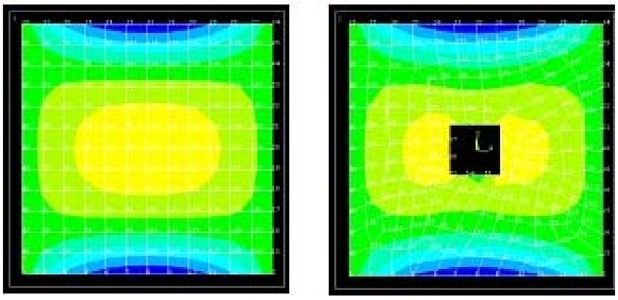
(a) (b)

Fig. 8 Contour of deformation for two edges clamped and two edges simply supported square plate (a) without hole and (b) with square hole in centre



(a) (b)

Fig. 9 Contour of stresses in x-direction for two edges clamped and two edges simply supported square plate (a) without hole and (b) with square hole in centre



(a) (b)

Fig. 10 Contour of stresses in y-direction for two edges clamped and two edges simply supported square plate (a) without hole and (b) with square hole in centre

In case of plates without hole, it is seen that, out plane deflection and moment in y direction are increasing with increase of aspect ratio. The maximum out plane deflection and stress occurs at centre of the plate. In case of a hole, the deflection and maximum moment in y-direction drastically increases with increase of aspect ratio whereas the maximum moment in x-direction first decreases with increase of aspect ratio up to 1.5 and thereafter increases with increase of aspect ratio. This behaviour is different from that in plain plate.

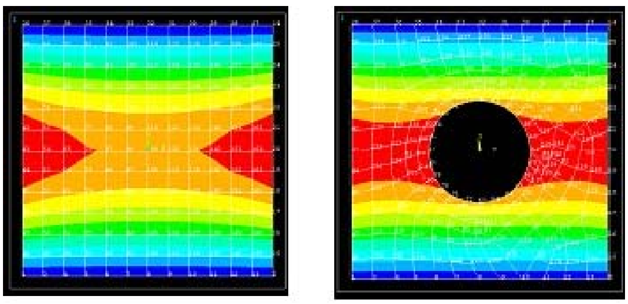
D. Two Opposite Edges Free and Two Simply Supported

Rectangular plates with aspect ratios given in Table V have been analysed with two opposite edges free and two edges simply supported. Circular holes of 100mm radius have been cut from the plates.

TABLE V
RESULTS FOR PLATES WITH TWO OPPOSITE EDGES FREE AND TWO SIMPLY SUPPORTED

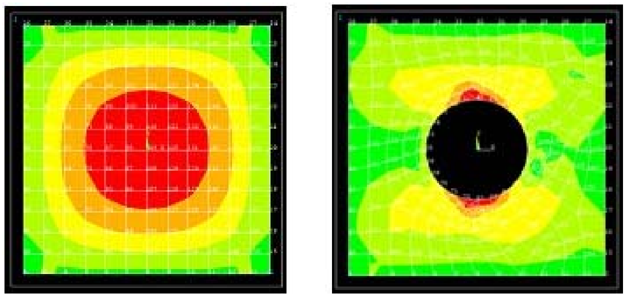
Aspect ratio (a/b)	w_{max}(mm)		Mx_{max}(N-mm)		My_{max}(N-mm)	
	I	*II*	*I*	*II*	*I*	*II*
1.0	1.73E-3	1.95E-3	39.76	53.06	193.16	307.15
1.5	1.75E-3	1.86E-3	49.84	69.40	195.79	300.58
2.0	1.75E-3	1.85E-3	53.64	77.78	196.80	297.72

Figs. 11-13 show the stress and deformation contours in the plate as obtained by FEM in ANSYS.



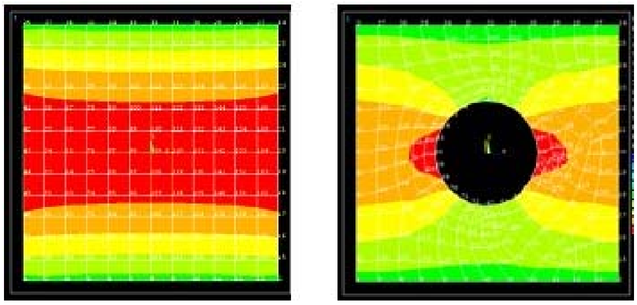
(a) (b)

Fig. 11 Contour of deformation for two opposite edges free and two edges simply supported square plate (a) without hole and (b) with square hole in centre



(a) (b)

Fig. 12 Contour of stresses in x-direction for two opposite edges free and two edges simply supported square plate (a) without hole and (b) with square hole in centre



(a) (b)

Fig. 13 Contour of stresses in y-direction for two opposite edges free and two edges simply supported square plate (a) without hole and (b) with square hole in centre

In case of plates without hole, it is seen that, maximum moments (in x and y direction) are increasing with increase of aspect ratio. The maximum stress occurs at centre of the plate. In case of a hole, the deflection drastically decreases with increase of aspect ratio. The maximum moment in x-direction increases with increase of aspect ratio whereas the maximum moment in y-direction first decreases with increase of aspect ratio up to 2.0 and thereafter increases with increase of aspect ratio. This behavior is different from that in plain plate.

E. Variation in Stresses

For plates with aspect ratio = 1, percentage increase in absolute values of maximum stress have been summarised in Table VI.

TABLE VI
CHANGES IN MAXIMUM STRESS VALUE FOR PLATES WITH AND WITHOUT HOLES

Edge Conditions	Rigidity Condition	σ_{max} (N/mm^2)	% change w.r.t. (I)
All edges clamped	(I) Without hole	-0.029939	-
	(II) With square hole at centre	-0.02955	0.5 %
	(III) With square hole at corner	-0.03133	4.65 %
All edges simply supported	(I) Without hole	0.029648	-
	(II) With rectangular central hole	0.040293	35.9 %
Two opposite edges clamped and two simply supported	(I) Without hole	-0.015339	-
	(II) With square hole at centre	-0.018708	21.96 %
Two opposite edges free and two simply supported	(I) Without hole	0.080484	-
	(II) With circular hole at centre	0.127984	59.01 %

There is a definite increase in values of maximum stress for all end conditions because of the decrease in rigidity. Also, in case of clamped edges, the negative value of stress at edges is prominent even with hole. While in case of simply supported or free edges, stresses in periphery of holes are dominantly greater than those at edges.

F. Comparison between Plain Plate and Composite Plate

The out plane maximum deflection of composite plates is less than the respective plain plate. The maximum moments are more in composite plates as compared to plain plate. These are presented in Table VII. These may be due to increase in stiffness of composite.

TABLE VII
RESULTS FOR PLAIN AND COMPOSITE SIMPLY SUPPORTED PLATES

Type of plate		w_{max} (mm)	Mx_{max} (Nmm)	My_{max} (Nmm)
With hole	Plain	5.54E-4	87.10	96.70
	Composite plate	4.93E-5	340.01	358.61
Without hole	Plain	4.68E- 4	71.15	71.15
	Composite plate	3.98E-5	276.82	276.82

V. CONCLUSION

On the basis of above results and discussion, following conclusions can be made:

1) From all the conditions evaluated above, it can be clearly concluded that stress concentration in periphery of the hole is more than that at any other point on the plate.
2) Maximum deflection decreases with shifting of hole from centre towards the edges for all aspect ratios of clamped plate.
3) For composite plates, the maximum deflection is less whereas maximum moments are more as compared to the respective plain plate.

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