

# 3D Frictionless Contact Case between the Structure of E-Bike and the Ground

Lele Zhang, Hui Leng Choo, Alexander Konyukhov, Shuguang Li

**Abstract**—China is currently the world's largest producer and distributor of electric bicycle (e-bike). The increasing number of e-bikes on the road is accompanied by rising injuries and even deaths of e-bike drivers. Therefore, there is a growing need to improve the safety structure of e-bikes. This 3D frictionless contact analysis is a preliminary, but necessary work for further structural design improvement of an e-bike. The contact analysis between e-bike and the ground was carried out as follows: firstly, the Penalty method was illustrated and derived from the simplest spring-mass system. This is one of the most common methods to satisfy the frictionless contact case; secondly, ANSYS static analysis was carried out to verify finite element (FE) models with contact pair (without friction) between e-bike and the ground; finally, ANSYS transient analysis was used to obtain the data of the penetration  $p(u)$  of e-bike with respect to the ground. Results obtained from the simulation are as estimated by comparing with that from theoretical method. In the future, protective shell will be designed following the stability criteria and added to the frame of e-bike. Simulation of side falling of the improved safety structure of e-bike will be confirmed with experimental data.

**Keywords**—Frictionless contact, penalty method, e-bike, finite element.

## I. INTRODUCTION

IN China, due to the low cost, simple technology and efficient personal mobility, e-bike sales have grown far faster than the sales of any other mode of transportation in China [1]. Comparing with bicycle, it allows people to commute for longer distances and carry more cargo within a relatively short period of time. Meanwhile, triggered by Chinese local governments' efforts to restrict motorcycles in city centers, e-bikes are the most optimistic projection for substituting motorcycles [2]. The cost of an e-bike is typically around RMB 2,000, less than one tenth of that of a car [3]. It is affordable to most Chinese people. Fig. 1 shows the growth in e-bike sales from 2005 to 2010 [4]. The sales of e-bikes increased by about three times from 2005, reaching 27 million units in 2010.

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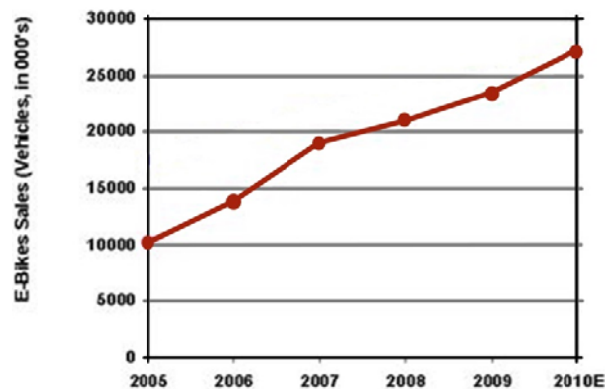


Fig. 1 Historical e-bike sales growth, China: 2005-2010 [4]

However, with the widespread use of e-bikes, the rate of accidents increased as well. According to Chinese government data, the deaths involving e-bikes accidents increased by almost 170 times from 34 in 2001 to 5752 in 2013 across the nation, as shown in Fig. 2.

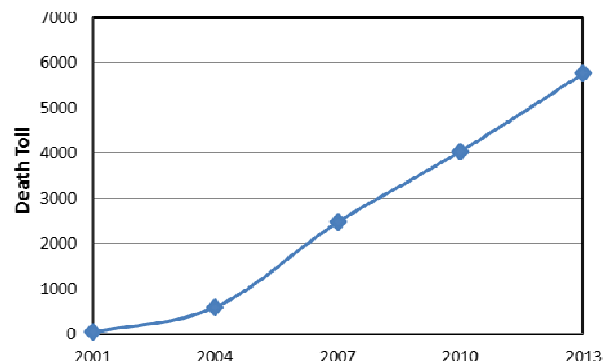


Fig. 2 Deaths associate with e-bike accident from 2001 to 2013 [5]

In order to improve the road safety associated with e-bike accidents, one of the aims of the current work is to improve the structure of e-bike. An FE model of e-bike is established and used as geometry model for finite element analysis. This contact process simulation is necessary to obtain critical data which will be used in further design of the protective structure.

## II. CONTACT THEORY

### A. Frictionless Contact Description

Contact mechanics is the study of the deformation caused by solid bodies that touch each other at one or more points [6]. Contact mechanics can be split up into two components which are the pressures or adhesion acting perpendicularly between

the surfaces of the contacting bodies and the frictional stresses acting tangentially between the surfaces. This paper focuses mainly on the aspect in normal direction. That contact ignored the interaction in tangent direction is exact the frictionless contact mechanics.

**B. Simplest Spring Mass System – Penalty Method**

The simplest system in the contact mechanics is a suspended system consisting of a mass point, attached to a spring of which stiffness is  $c$ , as shown in Fig. 3 [7].

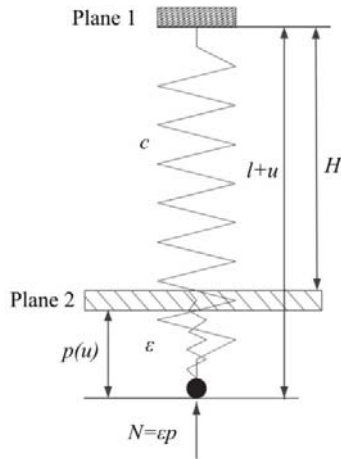


Fig. 3 Mechanical interpretation of the penalty method

$H$  is the height between Plane 1 and Plane 2,  $l$  is the unstretched length of the spring and  $\epsilon$  is the penalty parameter which, in this case, is the stiffness of the additional spring attached to the rigid Plane 2. The deformation of the spring is  $u$ , and  $p(u)$  is the function of contact penetration in this system [7].

$$p(u) = l + u - H \quad (1)$$

The normal contact force  $N$  with the penalty method can be obtained as:

$$N = \epsilon p(u) \quad (2)$$

**C. Weak Formulation – Regularization with Penalty Method**

Considering contact in 3D case in coordinate system (XYZ), in Fig. 4 (a) [8]. The definition of “master” is the body acting as the observer and “slave” is the observed body that consists of arbitrary contact point “s”. The following coordinate system is expressed as:

$$\mathbf{r}(\xi^1, \xi^2, \xi^3) = \boldsymbol{\rho}(\xi^1, \xi^2) + \xi^3 \mathbf{n} \quad (3)$$

where  $\xi^3$  is initial normal gap in the reference configuration (the distance between point  $S$  and  $C$ ),  $\xi^1, \xi^2$  are local convective coordinates defining on the surface coordinate lines (see Fig. 4 (b)).

The virtual work  $\delta W$  can be written in the following form [7]:

$$\delta W = \int_s [ \underbrace{N \delta \xi^3}_{\text{normal part}} + T^1 \underbrace{(\rho_{\xi^1 \xi^2} \cdot \rho_{\xi^1 \xi^2})}_{\text{frictional part}} ] ds \quad (4)$$

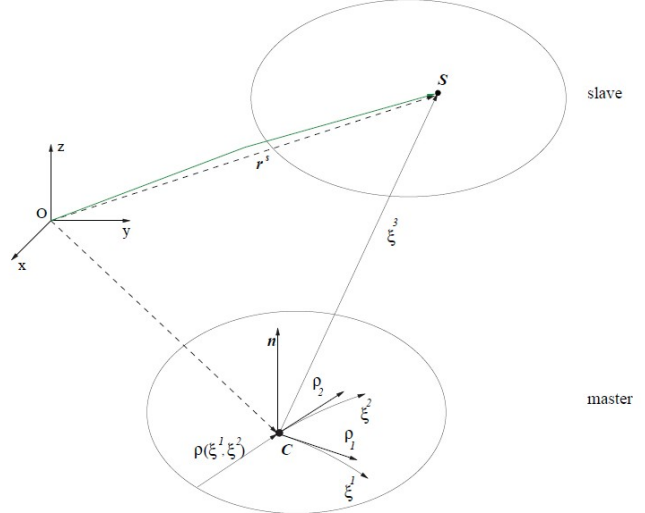


Fig. 4 (a) Contact kinematics in 3D case

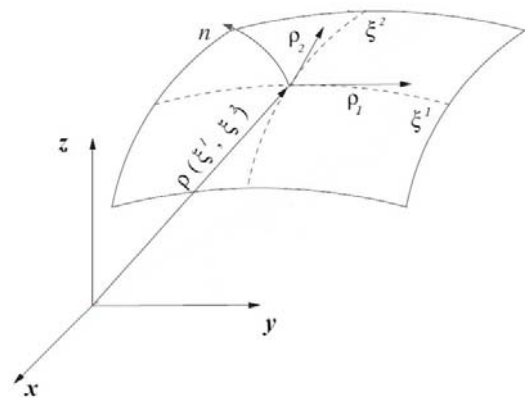


Fig. 4 (b) Definition of surface coordinate system

In the case of the penalty method, the contact term of normal part is a function with the penalized tractions  $N$  (normal contact force) [9]. For the frictionless case, tangential term (frictional part) is ignored ( $T^l=0$ ), and the integral is reduced to

$$\delta W = \int_s N \delta \xi^3 ds \quad (5)$$

when the frictionless case in (5) leads with penalty regularization, normal force  $N$  is treated by (6),

$$N = \epsilon H \xi^3 \quad (6)$$

where  $\epsilon$  is the physical normal penalty parameter (refer to section B) and  $H$  is the Heaviside function expressing that when the value of normal gap  $\xi^3$  is non-positive, normal force is existing. In other words,  $\xi^3$  is the exact penetration  $p(u)$  in this case.

The frictionless contact integral in (5) is transformed with penalty regularization in (6) as

$$\delta W = \int_s \epsilon H \xi^3 \delta \xi^3 ds \quad (7)$$

The frictionless case with the Penalty method was used for all following ANSYS simulations.

### III. FE MODEL

The isometric view of the FE model of e-bike and the ground is shown in Fig. 5. The finite element model was constructed as follows: the wheels and the grounds were discretized by shell elements, the frame was discretized by various beam finite elements with corresponding cross-section, the full weight was modelled by the mass element, which has been positioned at the center of mass determined by experiments [10] and corresponding connecting finite elements was used to connect wheels and various frame parts together. In this FE model, the center point of the front wheel was fixed relatively to the frame. The rear wheel was constrained in all directions except for translation in Y-direction. 3D surface to surface contact method was chosen for the contact pair between wheels and the ground. This method, in ANSYS, is associated with the CONTA174 contact element and the 3-D target element (TARGE170). These two types of elements can be perfectly located on shell elements [11]. Therefore, wheels and the ground were all meshed as shell elements.

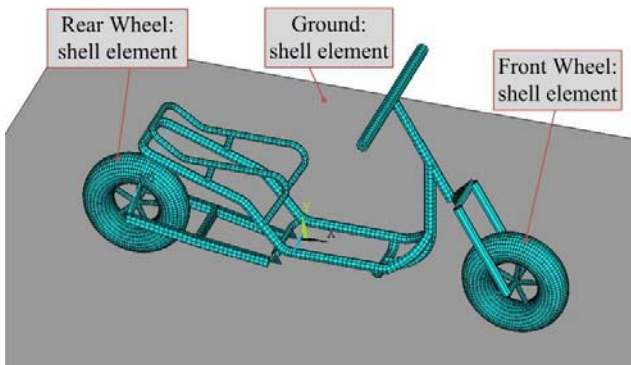


Fig. 4 FE model of e-bike and the ground

### IV. STATIC ANALYSIS OF CONTACT CASE BETWEEN E-BIKE AND THE GROUND – VERIFICATION

Quick static analysis will help to check the mesh quality of FE model and the validity of the boundary conditions and contact pairs.

#### A. Results of Static Analysis

The results obtained from static analysis are shown in Fig. 6. From the results, it can be seen that the rear wheel did not fall through the ground. This demonstrates that the load/constraints and the contact pair were correct. This contact model was used in the next transient analysis.

### V. TRANSIENT ANALYSIS OF E-BIKE

From Fig. 6 (a), it can be seen that Node 3392 is the closest node to the ground on the rear wheel, of which results were

chosen to represent the following ANSYS analysis results. The original position of Node 3392 was set as “0” and dropping in negative Y direction.

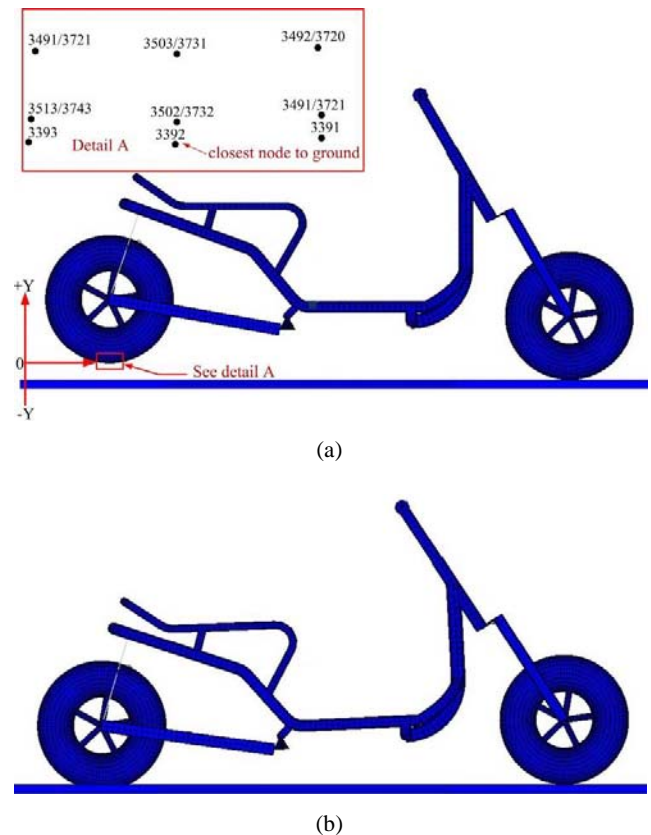


Fig. 5 (a) Drop process and (b) contact process of e-bike

#### A. Optimization of $\epsilon$ for Contact within the Penalty Method

In ANSYS,  $FKN$  represents the normal penalty parameter  $\epsilon$  (refer to II(B)). It is the key parameter using penalty algorithm for ANSYS simulation of frictionless contact. It is proportional to Young's modulus  $E$  of the underlying shell finite element. The ratio,  $c$ , of  $FKN$  to  $E$  is a constant,  $FKN=c * E$ , where  $c$  is generally changing from 0.1 to 1. From (2), specifying a value of  $FKN$  higher than  $1.0 * E$  leads to a smaller penetration.

In order to obtain the distinct differences of results using the penalty method, this parameter was ranged from 0.001 to 10.

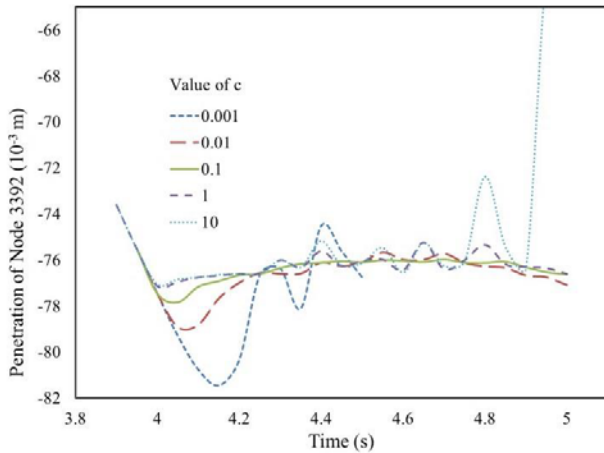


Fig. 6 Penetration  $p(u)$  of Node 3392 with penalty parameters from 0.001 to 10

In Fig. 7,  $p(u)$  of Node 3392 is shown by 5 different types of lines with respect to various value of  $FKN$ . Comparing the 5  $FKN$  values, the curve when  $FKN=0.1 * E$  shows that the process of contact is more stable. This parameter has the best convergence property together with a reasonable penetration value. Thus, “0.1” was determined and used in this contact simulation.

### B. Results of Transient Analysis

The transient results are shown in Fig. 8. Because the mass of the e-bike is always constant, its magnitude was set as “1 kg” in this simulation. The real result can be obtained by scaling with the real mass using the “dimensionality method” in mechanics. Then Kinetic Energy ( $E_k$ ) during all process is,

$$E_k = \frac{1}{2}mv^2 = \frac{1}{2} * 1 * v^2 = 0.5v^2 \quad (8)$$

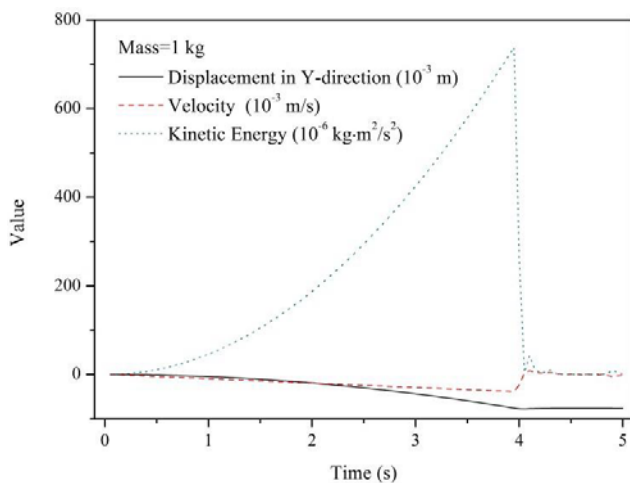


Fig. 7 Results of Node 3392

It can be seen that energy increases parabolically with distance of falling during the drop process, and decreases sharply when in contact with the ground. Velocity was found to increase linearly with falling distance and also decrease abruptly after e-bike interacts with the ground.

TABLE I  
 COMPARISON BETWEEN RESULTS OF TRANSIENT ANALYSIS AND THEORETICAL ANALYSIS

Physical quantity	Theoretical results [ $R_t$ ] [10]	ANSYS results [ $R_a$ ]	Ratio [ $R_a/R_t$ ]
initial contact time ( $t_0$ ) (s)	3.87	3.95	1.02
displacement at $t_0$ ( $10^{-3}$ m)	-76.06	-75.53	1.03
velocity at $t_0$ ( $10^{-3}$ m/s)	-37.99	-38.45	1.01
time of maximum displacement ( $t_m$ ) ( $10^{-3}$ m)	4.14	4.05	0.98
maximum displacement ( $10^{-3}$ m)	-78.56	-77.83	0.99
Maximum penetration ( $10^{-3}$ m)	2.50	2.30	0.92
velocity at $t_m$ [ $10^{-3}$ m/s]	0	3.07	-

The comparison of results between theoretical and numerical analysis is shown in Table I. E-bike starts to drop under the gravity acceleration when  $t=0$ . The time of the closest point of rear wheel contact with the ground is called the initial contact time ( $t_0$ ). Immediately after contact, e-bike continues to penetrate until the velocity is equal to zero and after that it will rebound. In terms of the law of conservation of mechanical energy, when  $v=0$ , the penitential energy reaches its extremum. This is the maximum penetration displacement [12].

The ratios of all quantities (except the velocity) are close to “1”. This demonstrates that this simulation is reliable. In the last row of Table I, magnitude of velocity corresponds to the value of maximum displacement. Data obtained from numerical method is only an approximate value. In this simulation, data was collected by every 0.05s, which resulted in small value of velocity “3.07 x10-3 m/s” in the maximum penetration.

### VI. FUTURE WORK

This contact model will be used in further rebounding and side falling analysis of the e-bike. The rebounding motion in the simulation will be verified with experimental data. Side contact and rebounding of e-bike with and without a designed protective shell will also be carried out in the future.

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