

Bilinear and Bilateral Generating Functions for the Gauss' Hypergeometric Polynomials

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Abstract—The object of the present paper is to investigate several general families of bilinear and bilateral generating functions with different argument for the Gauss' hypergeometric polynomials.

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I. INTRODUCTION

IN 1994, S.D. Singh and M.S. Arora [9], gave the semi orthogonal property of the Gauss' hypergeometric polynomials with its application as follows:

$$\int_0^{\infty} x^{-1-b-m}(1+x)^{b-c-m} A_m^{(b,c)}(x) A_n^{(b,c)}(x) dx = 0, \text{ if } m < n$$

$$= \frac{(b)_n n! \Gamma(c) \Gamma(-b) \Gamma(1+b)}{(c)_n \Gamma(1+b+n) \Gamma(c-b)}, \text{ if } m = n \quad (1)$$

where $Re(c) > 0, Re(b) < -m, Re(b) > -n \implies m = n, b \neq -n$.

Later, in 2001, I.K. Khanna and V. Srinivasa Bhagavan [5] derive the generating functions by using the representations of the Lie group $SL(2,C)$ (the complex special linear group).

The present paper is the extension of our earlier paper [6] in which Gauss' hypergeometric polynomials is defined by the relation

$$A_n^{(b,c)}(x) = x^n {}_2F_1 \left[\begin{matrix} -n, b \\ c \end{matrix}; -\frac{1}{x} \right]$$

$$= \sum_{r=0}^{\infty} (-1)^r \frac{(-n)_r (b)_r}{(c)_r r!} x^{n-r}, \quad n = 0, 1, 2, \dots \quad (2)$$

provided that c is not zero nor a negative integer.

In view of the relation [see, E.D. Rainville [3], Th. 20, pp. 60],

$${}_2F_1[a, b; c; z] = (1-z)^{-a} {}_2F_1 \left[a, c-b; c; \frac{z}{z-1} \right] \quad (3)$$

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the relation (2) can be written in an elegant form as

$$A_n^{(b,c)}(x) = (1+x)^n {}_2F_1 \left[\begin{matrix} -n, c-b \\ c \end{matrix}; \frac{1}{1+x} \right] \quad (4)$$

Also, by reversing the order of summation, (2) and (4) can be written as

$$A_n^{(b,c)}(x) = \frac{(b)_n}{(c)_n} {}_2F_1 \left[\begin{matrix} -n, 1-c-n \\ 1-b-n \end{matrix}; -x \right] \quad (5)$$

and

$$A_n^{(b,c)}(x) = (-1)^n \frac{(c-b)_n}{(c)_n} \times {}_2F_1 \left[\begin{matrix} -n, 1-c-n \\ 1+b-c-n \end{matrix}; 1+x \right] \quad (6)$$

Some of the definitions and notations used in the present paper are as follows:

Appell's functions of two variables are given by (see [7]).

$$F_1[a, b, b'; c; x, y] = \sum_{n,k=0}^{\infty} \frac{(a)_{n+k} (b)_n (b')_k}{n! k! (c)_{n+k}} x^n y^k \quad (7)$$

$$F_2[a, b, b'; c, c'; x, y] = \sum_{n,k=0}^{\infty} \frac{(a)_{n+k} (b)_n (b')_k}{n! k! (c)_n (c')_k} x^n y^k \quad (8)$$

$$F_3[a, a', b, b'; c; x, y] = \sum_{n,k=0}^{\infty} \frac{(a)_n (a')_k (b)_n (b')_k}{n! k! (c)_{n+k}} x^n y^k \quad (9)$$

$$F_4[a, b, c, c'; x, y] = \sum_{n,k=0}^{\infty} \frac{(a)_{n+k} (b)_{n+k}}{n! k! (c)_n (c')_k} x^n y^k \quad (10)$$

Saran's functions for three variables are given by (see [8]).

$$F_E[\alpha_1, \alpha_1, \alpha_1, \beta_1, \beta_2, \beta_2; \gamma_1, \gamma_2, \gamma_3; x, y, z]$$

$$= \sum_{m,n,p=0}^{\infty} \frac{(\alpha_1)_{m+n+p} (\beta_1)_m (\beta_2)_{n+p}}{(\gamma_1)_m (\gamma_2)_n (\gamma_3)_p m! n! p!} x^m y^n z^p \quad (11)$$

$$F_G[\alpha_1, \alpha_1, \alpha_1, \beta_1, \beta_2, \beta_3; \gamma_1, \gamma_2, \gamma_2; x, y, z]$$

$$= \sum_{m,n,p=0}^{\infty} \frac{(\alpha_1)_{m+n+p} (\beta_1)_m (\beta_2)_n (\beta_3)_p}{(\gamma_1)_m (\gamma_2)_{n+p} m! n! p!} x^m y^n z^p \quad (12)$$

$$F_S[\alpha_1, \alpha_2, \alpha_2, \beta_1, \beta_2, \beta_3; \gamma_1, \gamma_1, \gamma_1; x, y, z]$$

$$= \sum_{m,n,p=0}^{\infty} \frac{(\alpha_1)_m (\alpha_2)_{n+p} (\beta_1)_m (\beta_2)_n (\beta_3)_p}{(\gamma_1)_{m+n+p} m! n! p!} x^m y^n z^p \quad (13)$$

Lauricella's hypergeometric functions for n variables is defined by (see [4]).

$$F_C^{(n)} [a, b; c_1, \dots, c_n; x_1, \dots, x_n] = \sum_{m_1, \dots, m_n=0}^{\infty} \frac{(a)_{m_1+\dots+m_n} (b)_{m_1+\dots+m_n}}{(c_1)_{m_1} \dots (c_n)_{m_n}} \times \frac{x_1^{m_1}}{m_1!} \dots \frac{x_n^{m_n}}{m_n!} \quad (14)$$

$$F_D^{(n)} [a, b_1, \dots, b_n; c; x_1, \dots, x_n] = \sum_{m_1, \dots, m_n=0}^{\infty} \frac{(a)_{m_1+\dots+m_n} (b_1)_{m_1} \dots (b_n)_{m_n}}{(c)_{m_1+\dots+m_n}} \times \frac{x_1^{m_1}}{m_1!} \dots \frac{x_n^{m_n}}{m_n!} \quad (15)$$

Confluent form of Lauricella's functions for n variables is defined by (see [4]).

$$\psi_2^{(n)} [a, c_1, \dots, c_n; x_1, \dots, x_n] = \sum_{m_1, \dots, m_n=0}^{\infty} \frac{(a)_{m_1+\dots+m_n}}{(c_1)_{m_1} \dots (c_n)_{m_n}} \frac{x_1^{m_1}}{m_1!} \dots \frac{x_n^{m_n}}{m_n!} \quad (16)$$

Similarly, a general triple hypergeometric series $F^{(3)}[x, y, z]$ (see [4], pp. 69) is defined as

$$F^{(3)}[x, y, z] = F^{(3)} \left[\begin{matrix} (a) :: (b); (b'); (b'') \\ (e) :: (g); (g'); (g'') \end{matrix} ; \begin{matrix} (c); (c'); (c'') \\ (h); (h'); (h'') \end{matrix} ; x, y, z \right] = \sum_{m, n, p=0}^{\infty} \Lambda(m, n, p) \frac{x^m y^n z^p}{m! n! p!} \quad (17)$$

where for convenience

$$\Lambda(m, n, p) = \frac{\prod_{j=1}^A (a_j)_{m+n+p} \prod_{j=1}^B (b_j)_{m+n} \prod_{j=1}^{B'} (b'_j)_{n+p}}{\prod_{j=1}^E (e_j)_{m+n+p} \prod_{j=1}^G (g_j)_{m+n} \prod_{j=1}^{G'} (g'_j)_{n+p}} \times \frac{\prod_{j=1}^{B''} (b''_j)_{p+m} \prod_{j=1}^C (c_j)_m \prod_{j=1}^{C'} (c'_j)_n \prod_{j=1}^{C''} (c''_j)_p}{\prod_{j=1}^H (h_j)_m \prod_{j=1}^{H'} (h'_j)_n \prod_{j=1}^{H''} (h''_j)_p \prod_{j=1}^{G''} (g''_j)_{p+m}}$$

II. BILINEAR GENERATING FUNCTIONS

By using the definition (2) and the Gaussian hypergeometric transformation (see, Rainville [3], Th. 21, pp. 60)

$${}_2F_1[a, b; c; z] = (1-z)^{c-a-b} {}_2F_1[c-a, c-b; c; z] \quad (18)$$

We thus obtain the bilinear generating function

$$\sum_{n=0}^{\infty} \frac{(c+b)_n (c+m)_n}{(1+d)_n n!} A_{m+n}^{(-b-n, c)}(x) A_n^{(-d-n, e)}(y) t^n = (1+x)^m \left(\frac{x}{1+x} \right)^{-b-c} F_c^{(3)} [c+m, c+b; c, e, 1+d;$$

$$\left. -\frac{1}{x}, -\frac{(1+x)^2 t}{x}, \frac{(1+x)^2 y t}{x} \right] \quad (19)$$

where $F_c^{(3)}$ denote the Lauricella's function defined by (14, with $n = 3$). An interesting special case of the generating function (19) would occurs when we set, $m = 0$, $d = b$, $e = c$, and appealing the hypergeometric reduction formula (see, B.L. Sharma [1], pp. 716, (2.4)).

$$F_c^{(3)}[\alpha + \beta + 1, \beta + 1; \alpha + 1, \beta + 1, \beta + 1; x, y, z] = (1+x-y-z)^{-\alpha-\beta-1} \times F_4 \left[\frac{\alpha + \beta + 1}{2}, \frac{\alpha + \beta + 2}{2}; \alpha + 1, \beta + 1; X, Y \right] \quad (20)$$

where, $X = \frac{4xz}{(1+x-y-z)^2}$, $Y = \frac{4yz}{(1+x-y-z)^2}$ yields the generating relation

$$\sum_{n=0}^{\infty} \frac{(c+b)_n (c)_n}{(1+b)_n n!} A_n^{(-b-n, c)}(x) A_n^{(-b-n, c)}(y) t^n = \{1 + (1+x)(1+y)t\}^{-b-c} \times F_4 \left[\frac{1}{2}(c+b), \frac{1}{2}(c+b+1); 1+b, c; \xi, \zeta \right] \quad (21)$$

where, $\xi = \frac{4xyt}{(1+(1+x)(1+y)t)^2}$, $\zeta = \frac{4t}{(1+(1+x)(1+y)t)^2}$ and F_4 is the Appell's function defined by (10).

Another bilinear generating function are obtained by using (2), which in conjunction with ([6], (2.25)),

$$\sum_{n=0}^{\infty} \frac{(\lambda)_n (1+c)_n}{(1+b+c)_n n!} A_{m+n}^{(b, -c-n)}(x) t^n = \frac{(1+b+c-m)_m}{(1+c-m)_m} x^m (1-xt)^{-\lambda} \times F_1 \left[b, -m, \lambda; 1+b+c-m; \frac{1+x}{x}, -\frac{(1+x)t}{1-xt} \right] \quad (22)$$

readily gives the relation

$$\sum_{n=0}^{\infty} \frac{(\lambda)_n (1+c)_n}{(1+b+c)_n n!} A_n^{(b, -c-n)}(x) A_n^{(d, e)}(y) t^n = (1-xyt)^{-\lambda} \sum_{n=0}^{\infty} \frac{(\lambda)_n (d)_n (b)_n}{(e)_n (1+b+c)_n n!} (\chi)^n \times F_2 [\lambda + n, d + n, b + n; e + n, 1 + b + c + n; \psi, \omega] \quad (23)$$

where, $\frac{\chi}{-(1+x)} = \frac{\psi}{x} = \frac{\omega}{-(1+x)y} = \frac{t}{1-xyt}$ and F_2 is the Appell's function defined by (8)

The second member of (23) can indeed be written in terms of Srivastava triple hypergeometric series $F^{(3)}[x, y, z]$ defined by (17), and we thus obtain the alternative form of the bilinear generating function (23) as,

$$\sum_{n=0}^{\infty} \frac{(\lambda)_n (1+c)_n}{(1+b+c)_n n!} A_n^{(b, -c-n)}(x) A_n^{(d, e)}(y) t^n = (1-xyt)^{-\lambda} \times F^{(3)} \left[\begin{matrix} \lambda :: d; \dots; b \\ \dots :: e; \dots; 1+b+c \end{matrix} ; \dots; \chi, \psi, \omega \right] \quad (24)$$

Again, when we set $\lambda = 1 + b + c$ in (24), along with ([7], pp. 35, (10))

$$F_2[a, b, b'; a, c'; x, y] = (1-x)^{-b} F_1 \left[b', b, a-b; c'; \frac{y}{1-x}, y \right] \quad (25)$$

Moreover, the power series identity ([4], 1.6(2)).

$$\sum_{m,n=0}^{\infty} f(m+n) \frac{x^m y^n}{m! n!} = \sum_{N=0}^{\infty} f(N) \frac{(x+y)^N}{N!} \quad (26)$$

We obtain generating function in the form

$$\begin{aligned} & \sum_{n=0}^{\infty} \frac{(1+c)_n}{n!} A_n^{(b,-c-n)}(x) A_n^{(d,e)}(y) t^n \\ &= (1+yt)^{-b} (1-xyt)^{-c-1} \\ & \quad \times F_1 \left[d, b, 1+c; e; -\frac{1}{1+yt}, \frac{xt}{1-xyt} \right] \end{aligned} \quad (27)$$

where F_1 is the Appell's function defined by (7).

In view of the definition (2) and (4), which in conjunction with (18), we obtain some more bilinear generating function for $A_n^{(b,c)}(x)$ as given below:

$$\begin{aligned} & \sum_{n=0}^{\infty} \frac{(c+m)_n (1+e)_n}{(\lambda)_n n!} A_{m+n}^{(b,c)}(x) A_n^{(-d-n,-e-n)}(y) t^n \\ &= (1+x)^m \left(\frac{x}{1+x} \right)^{b-c} \\ & \quad \times F_G [c+m, c+m, c+m, c-b, 1+e, d-e; \\ & \quad c, \lambda, \lambda; -\frac{1}{x}, (1+x)(1+y)t, (1+x)t] \end{aligned} \quad (28)$$

Alternatively, equivalently using (2) along with (5), we obtain

$$\begin{aligned} & \sum_{n=0}^{\infty} \frac{(1+e)_n}{n!} A_{m+n}^{(b-n,c-n)}(x) A_n^{(d,-e-n)}(y) t^n \\ &= \frac{(b)_m}{(c)_m} (1+x)^{c+m-1} \\ & \quad \times F_G [1-b, 1-b, 1-b, 1-c-m, 1+e, d; \\ & \quad 1-b-m, 1-c, 1-c; \frac{x}{1+x}, yt, -t] \end{aligned} \quad (29)$$

where in (28) and (29) F_G are the Saran's function defined by (12).

Further, we obtain some more bilinear generating function by using the relation (2) along with (3) in an elegant form as

$$\begin{aligned} & \sum_{n=0}^{\infty} \frac{(\lambda)_n (c+m)_n}{(1+d)_n n!} A_{m+n}^{(b,c)}(x) A_n^{(-d-n,e)}(y) t^n \\ &= (1+x)^m \left(\frac{x}{1+x} \right)^{b-c} \\ & \quad \times F_E [c+m, c+m, c+m, c-b, \lambda, \lambda; \\ & \quad c, e, 1+d; -\frac{1}{x}, -(1+x)t, (1+x)yt] \end{aligned} \quad (30)$$

or, equivalently

$$\begin{aligned} & \sum_{n=0}^{\infty} \frac{(\lambda)_n (1-c)_n}{(1+d)_n n!} A_{m+n}^{(b-n,c-n)}(x) A_n^{(-d-n,e)}(y) t^n \\ &= \frac{(b)_m}{(c)_m} (1+x)^{c+m-1} \\ & \quad \times F_E [1-b, 1-b, 1-b, 1-c-m, \lambda, \lambda; \\ & \quad 1-b-m, e, 1+d; \frac{x}{1+x}, -t, yt] \end{aligned} \quad (31)$$

where in (30) and (31) F_E is the Saran's function defined by (11).

III. BILATERAL GENERATING FUNCTIONS

The polynomials $A_n^{(b,c)}(x)$ admits several bilateral generating functions. Firstly, we introduce three bilateral generating function by using the relation (2), each of which involved the Gaussian hypergeometric ${}_2F_1$ function in terms of the Lauricella's triple hypergeometric series F_4, F_8 and F_7 (which, in the notation used by Saran's [8], are F_E, F_G, F_S respectively) are as follows:

$$\begin{aligned} & \sum_{n=0}^{\infty} \frac{(\lambda)_n (\mu)_n}{(1+b)_n n!} A_n^{(-b-n,c)}(x) {}_2F_1 [\lambda+n, \beta; \gamma; y] t^n \\ &= F_E [\lambda, \lambda, \lambda, \beta, \mu, \mu; \gamma, 1+b, c; y, xt, -t] \end{aligned} \quad (32)$$

$$\begin{aligned} & \sum_{n=0}^{\infty} \frac{(\lambda)_n (1+c)_n}{(\mu)_n n!} A_n^{(b,-c-n)}(x) {}_2F_1 [\lambda+n, \beta; \gamma; y] t^n \\ &= F_E [\lambda, \lambda, \lambda, \beta, 1+c, b; \gamma, \mu, \mu; y, xt, -t] \end{aligned} \quad (33)$$

and

$$\begin{aligned} & \sum_{n=0}^{\infty} \frac{(\lambda)_n (1+c)_n}{(\mu)_n n!} A_n^{(b,-c-n)}(x) {}_2F_1 [\beta, \gamma; \mu+n; y] t^n \\ &= F_S [\beta, \lambda, \lambda, \gamma, 1+c, b; \mu, \mu, \mu; y, xt, -t] \end{aligned} \quad (34)$$

Now, by using the definition (2) along with Laguerre polynomials (see [3], pp. 200, (1)), yields the generating function in the form

$$\begin{aligned} & \sum_{n=0}^{\infty} \binom{m+n}{n} \frac{L_{m+n}^{(\alpha)}(x) A_n^{(-b-n,c)}(y) t^n}{(1+b)_n} \\ &= \binom{\alpha+m}{m} e^x \\ & \quad \times \psi_2^{(3)} [\alpha+m+1; \alpha+1, c, 1+b; -x, -t, yt] \end{aligned} \quad (35)$$

$$\begin{aligned} & \sum_{n=0}^{\infty} \binom{m+n}{n} L_{m+n}^{(\alpha)}(x) A_n^{(b,c)}(y) t^n \\ &= \binom{\alpha+m}{m} e^x (1-yt)^{-\alpha-m-1} \\ & \quad \times \psi_1 \left[\alpha+m+1, b; c, 1+\alpha; \frac{t}{1-yt}, -\frac{x}{1-yt} \right] \end{aligned} \quad (36)$$

Alternatively, equivalently using (5), we obtain

$$\sum_{n=0}^{\infty} \binom{m+n}{n} \frac{(1-c)_n}{(1-b)_n} L_{m+n}^{(\alpha)}(x) A_n^{(b-n, c-n)}(y) t^n$$

$$= \binom{\alpha+m}{m} e^x (1-t)^{-\alpha-m-1}$$

$$\times \psi_1 \left[\alpha+m+1, 1-c; 1-b, 1+\alpha; \frac{yt}{1-t}, \frac{-x}{1-t} \right] \quad (37)$$

where, in (35) $\psi_2^{(3)}$ is the confluent form of Lauricella's function defined by (16), with $n = 3$ and in (36) and (37) ψ_1 is the confluent hypergeometric function of two variables (see [4], pp. 59, (41)).

The generalized heat polynomials $P_{n,\nu}(x, u)$ defined by (Haimo [2], p.736, (2.1)).

$$P_{n,\nu}(x, u) = \sum_{k=0}^n 2^{2k} \binom{n}{k} \frac{\Gamma(\nu+n+\frac{1}{2})}{\Gamma(\nu+n-k+\frac{1}{2})} x^{2n-2k} u^k \quad (38)$$

By reversing the order of summation, (38) can be written as

$$P_{n,\nu}(x, u) = (4u)^n \left(\nu + \frac{1}{2}\right)_n \sum_{k=0}^n \frac{(-n)_k}{\left(\nu + \frac{1}{2}\right)_k} \frac{1}{k!} \left(\frac{-x^2}{4u}\right)^k$$

$$= (4u)^n n! L_n^{(\nu-\frac{1}{2})} \left(\frac{-x^2}{4u}\right) \quad (39)$$

Further, involving the relation (39) with (2) and (5), another form of generating function equivalent to (35), (36) and (37) are obtained,

$$\sum_{n=0}^{\infty} \frac{P_{m+n,\nu}(x, u) A_n^{(-b-n, c)}(y)}{(1+b)_n n!} t^n$$

$$= (4u)^m \left(\nu + \frac{1}{2}\right)_m \exp\left(\frac{-x^2}{4u}\right)$$

$$\times \psi_2^{(3)} \left[\nu+m+\frac{1}{2}; \nu+\frac{1}{2}, c, 1+b; \frac{x^2}{4u}, -4ut, 4uyt \right] \quad (40)$$

$$\sum_{n=0}^{\infty} P_{m+n,\nu}(x, u) A_n^{(b,c)}(y) \frac{t^n}{n!}$$

$$= (4u)^m \left(\nu + \frac{1}{2}\right)_m \exp\left(\frac{-x^2}{4u}\right) (1-4uyt)^{-\nu-m-\frac{1}{2}}$$

$$\times \psi_1 \left[\nu+m+\frac{1}{2}, b; c, \nu+\frac{1}{2}; \frac{4ut}{1-4uyt}, \frac{x^2}{4u(1-4uyt)} \right] \quad (41)$$

$$\sum_{n=0}^{\infty} \frac{(1-c)_n}{(1-b)_n} P_{m+n,\nu}(x, u) A_n^{(b-n, c-n)}(y) \frac{t^n}{n!}$$

$$= (4u)^m \left(\nu + \frac{1}{2}\right)_m \exp\left(\frac{-x^2}{4u}\right) (1-4ut)^{-\nu-m-\frac{1}{2}}$$

$$\times \psi_1 \left[\nu+m+\frac{1}{2}, 1-c; 1-b, \nu+\frac{1}{2}; \frac{4uyt}{1-4ut}, \frac{x^2}{4u(1-4ut)} \right] \quad (42)$$

Again using the definition (2), along with Jacobi polynomials (see [3], (1), pp. 254), which in conjunction with (3) yields the generating relations

$$\sum_{n=0}^{\infty} \frac{(1+\alpha+\beta)_n}{(1+b)_n} A_n^{(-b-n, c)}(x) P_n^{(\alpha, \beta)}(y) t^n$$

$$= \left(\frac{1+y}{2}\right)^{-\alpha-\beta-1} F_c^{(3)} [1+\alpha+\beta, 1+\alpha;$$

$$1+b, c, 1+\alpha; -\frac{2t}{1+y}, \frac{y-1}{1+y}, \frac{2xt}{1+y}] \quad (43)$$

$$\sum_{n=0}^{\infty} \frac{(\lambda)_n}{(1+b)_n} A_n^{(-b-n, c)}(x) P_n^{(\alpha, \beta-n)}(y) t^n$$

$$= \left(\frac{1+y}{2}\right)^{-\alpha-\beta-1} F_E [1+\alpha, 1+\alpha, 1+\alpha,$$

$$1+\alpha+\beta, \lambda, \lambda; 1+\alpha, 1+b, c; \frac{y-1}{y+1}, xt, -t] \quad (44)$$

$$\sum_{n=0}^{\infty} \frac{(1+c)_n}{(\lambda)_n} A_n^{(b, -c-n)}(x) P_n^{(\alpha, \beta-n)}(y) t^n$$

$$= \left(\frac{1+y}{2}\right)^{-\alpha-\beta-1} F_G [1+\alpha, 1+\alpha, 1+\alpha,$$

$$1+\alpha+\beta, 1+c, b1+\alpha, \lambda, \lambda; \frac{y-1}{y+1}, xt, -t] \quad (45)$$

Next, some more generating functions are expressed by using (2), which in conjunction with Lauricella's triple hypergeometric function $F_C^{(S)}$ and $F_D^{(S)}$.

$$\sum_{n=0}^{\infty} \frac{(\lambda)_n (\mu)_n}{(1+b)_n n!} A_n^{(-b-n, c)}(x)$$

$$\times F_C^{(S)} [\lambda+n, \mu+n; \rho_1, \dots, \rho_s; z_1, \dots, z_s] t^n$$

$$= F_C^{(S+2)} [\lambda, \mu; \rho_1, \dots, \rho_s, c, 1+b; z_1, \dots, z_s, -t, xt] \quad (46)$$

and

$$\sum_{n=0}^{\infty} \frac{(\lambda)_n (1+c)_n}{(\mu)_n n!} A_n^{(b, -c-n)}(x)$$

$$\times F_D^{(S)} [\lambda+n, \nu_1, \dots, \nu_s; \mu+n; z_1, \dots, z_s] t^n$$

$$= F_D^{(S+2)} [\lambda, \nu_1, \dots, \nu_s, b, 1+c; \mu; z_1, \dots, z_s, -t, xt] \quad (47)$$

where in (46) and (47) $F_C^{(S+2)}$ and $F_D^{(S+2)}$ denote the Lauricella's triple hypergeometric function defined by (14) and (15) with $n = s + 2$.

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