World Academy of Science, Engineering and Technology
International Journal of Mathematical and Computational Sciences
Vol:8, No:10, 2014

Bilinear and Bilateral Generating Functions for the Gauss' Hypergeometric Polynomials

Manoj Singh, Mumtaz Ahmad Khan, Abdul Hakim Khan

Abstract—The object of the present paper is to investigate several general families of bilinear and bilateral generating functions with different argument for the Gauss' hypergeometric polynomials.

Mathematics Subject Classification(2010): Primary 42C05, Secondary 33C45.

Keywords—Appell's functions, Gauss hypergeometric functions, Heat polynomials, Kampe' de Fe'riet function, Laguerre polynomials, Lauricella's function, Saran's functions.

I. Introduction

In 1994, S.D. Singh and M.S. Arora [9], gave the semi orthogonal property of the Gauss' hypergeometric polynomials with its application as follows:

$$\int_0^\infty x^{-1-b-m}(1+x)^{b-c-m}A_m^{(b,c)}(x)A_n^{(b,c)}(x)dx$$

$$= 0,\ if\ m < n$$

$$= \frac{(b)_n\ n!\ \Gamma(c)\ \Gamma(-b)\ \Gamma(1+b)}{(c)_n\ \Gamma(1+b+n)\ \Gamma(c-b)},\ if\ m = n \quad (1)$$

where $Re(c) > 0$, $Re(b) < -m$, $Re(b) > -n \implies m = n,\ b \neq -n$.

Later, in 2001, I.K. Khanna and V. Srinivasa Bhagavan [5] derive the generating functions by using the representations of the Lie group SL(2,C)(the complex special linear group).

The present paper is the extension of our earlier paper [6] in which Gauss' hypergeometric polynomials is defined by the relation

$$A_n^{(b,c)}(x) = x^n {}_2F_1\left[\begin{array}{c} -n, b\ ; \\ c\ \ ; \end{array}\ -\frac{1}{x}\right]$$

$$= \sum_{r=0}^{\infty}(-1)^r\frac{(-n)_r(b)_r}{(c)_r\ r!}x^{n-r},\ n = 0, 1, 2, ... \quad (2)$$

provided that c is not zero nor a negative integer.

In view of the relation [see, E.D. Rainville [3], Th. 20, pp. 60],

$${}_2F_1[a,\ b;\ c;\ z] = (1-z)^{-a}{}_2F_1\left[a,\ c-b;\ c;\frac{z}{z-1}\right] \quad (3)$$

M. Singh is with Department of Mathematics, College of Science, Jazan University, Jazan, Saudi Arabia (e-mail: manojsingh221181@gmail.com).

M. A. Khan is with Department of Applied Mathematics, Faculty of Engineering, Aligarh Muslim University, Aligarh - 202002, U.P., India (e-mail: mumtaz_ahmad_khan_2008@yahoo.com).

A. H. Khan is with Department of Applied Mathematics, Faculty of Engineering, Aligarh Muslim University, Aligarh - 202002, U.P., India (e-mail: ahkhanamu@gmail.com).

the relation (2) can be written in an elegant form as

$$A_n^{(b,c)}(x) = (1+x)^n {}_2F_1\left[\begin{array}{c} -n, c-b\ ; \\ c\ ; \end{array}\ \frac{1}{1+x}\right] \quad (4)$$

Also, by reversing the order of summation, (2) and (4) can be written as

$$A_n^{(b,c)}(x) = \frac{(b)_n}{(c)_n}\ {}_2F_1\left[\begin{array}{c} -n, 1-c-n\ ; \\ 1-b-n\ ; \end{array}\ -x\right] \quad (5)$$

and

$$A_n^{(b,c)}(x) = (-1)^n\frac{(c-b)_n}{(c)_n}$$

$$\times {}_2F_1\left[\begin{array}{c} -n, 1-c-n\ ; \\ 1+b-c-n\ ; \end{array}\ 1+x\right] \quad (6)$$

Some of the definitions and notations used in the present paper are as follows:

Appell's functions of two variables are given by (see [7]).

$$F_1[a,b,b';c;x,y] = \sum_{n,k=0}^{\infty}\frac{(a)_{n+k}(b)_n(b')_k}{n!\ k!\ (c)_{n+k}}x^ny^k \quad (7)$$

$$F_2[a,b,b';c,c';x,y] = \sum_{n,k=0}^{\infty}\frac{(a)_{n+k}(b)_n(b')_k}{n!\ k!\ (c)_n(c')_k}x^ny^k \quad (8)$$

$$F_3[a,a',b,b';c;x,y] = \sum_{n,k=0}^{\infty}\frac{(a)_n(a')_k(b)_n(b')_k}{n!\ k!\ (c)_{n+k}}x^ny^k \quad (9)$$

$$F_4[a,b;c,c';x,y] = \sum_{n,k=0}^{\infty}\frac{(a)_{n+k}(b)_{n+k}}{n!\ k!\ (c)_n(c')_k}x^ny^k \quad (10)$$

Saran's functions for three variables are given by (see [8]).

$$F_E[\alpha_1,\ \alpha_1,\ \alpha_1,\ \beta_1,\ \beta_2,\ \beta_2;\ \gamma_1,\ \gamma_2,\ \gamma_3;\ x,y,z]$$

$$= \sum_{m,n,p=0}^{\infty}\frac{(\alpha_1)_{m+n+p}\ (\beta_1)_m\ (\beta_2)_{n+p}}{(\gamma_1)_m\ (\gamma_2)_n\ (\gamma_3)_p\ m!\ n!\ p!}x^m\ y^n\ z^p \quad (11)$$

$$F_G[\alpha_1,\ \alpha_1,\ \alpha_1,\ \beta_1,\ \beta_2,\ \beta_3;\ \gamma_1,\ \gamma_2,\ \gamma_2;\ x,y,z]$$

$$= \sum_{m,n,p=0}^{\infty}\frac{(\alpha_1)_{m+n+p}\ (\beta_1)_m\ (\beta_2)_n\ (\beta_3)_p}{(\gamma_1)_m\ (\gamma_2)_{n+p}\ m!\ n!\ p!}x^m\ y^n\ z^p \quad (12)$$

$$F_S[\alpha_1,\ \alpha_2,\ \alpha_2,\ \beta_1,\ \beta_2,\ \beta_3;\ \gamma_1,\ \gamma_1,\ \gamma_1;\ x,y,z]$$

$$= \sum_{m,n,p=0}^{\infty}\frac{(\alpha_1)_m(\alpha_2)_{n+p}(\beta_1)_m(\beta_2)_n(\beta_3)_p}{(\gamma_1)_{m+n+p}\ m!\ n!\ p!}x^my^nz^p \quad (13)$$

Lauricella's hypergeometric functions for n variables is defined by (see [4]).

$$F_C^{(n)}\left[a,b;\ c_1,...,c_n;\ x_1,...,x_n\right] = \sum_{m_1,...,m_n=0}^{\infty} \frac{(a)_{m_1+...+m_n}\ (b)_{m_1+...+m_n}}{(c_1)_{m_1}...(c_n)_{m_n}} \times \frac{x_1^{m_1}}{m_1!}...\frac{x_n^{m_n}}{m_n!} \tag{14}$$

$$F_D^{(n)}\left[a,b_1,...,b_n;\ c\ ;\ x_1,...,x_n\right] = \sum_{m_1,...,m_n=0}^{\infty} \frac{(a)_{m_1+...+m_n}\ (b_1)_{m_1}...(b_n)_{m_n}}{(c)_{m_1+...+m_n}} \times \frac{x_1^{m_1}}{m_1!}...\frac{x_n^{m_n}}{m_n!} \tag{15}$$

Confluent form of Lauricella's functions for n variables is defined by (see [4]).

$$\psi_2^{(n)}\left[a,c_1,...,c_n;\ x_1,...,x_n\right] = \sum_{m_1,...,m_n=0}^{\infty} \frac{(a)_{m_1+...+m_n}}{(c_1)_{m_1}...(c_n)_{m_n}}\ \frac{x_1^{m_1}}{m_1!}...\frac{x_n^{m_n}}{m_n!} \tag{16}$$

Similarly, a general triple hypergeometric series $F^{(3)}[x,y,z]$ (see [4], pp. 69) is defined as

$$F^{(3)}[x,y,z] = F^{(3)}\left[\begin{array}{l}(a) :: (b);(b');(b'')\ :(c);(c');(c'')\ ;\\ (e) :: (g);(g');(g'') : (h);(h');(h'');\end{array} x,y,z\right] = \sum_{m,n,p=0}^{\infty} \Lambda(m,n,p)\frac{x^m}{m!}\frac{y^n}{n!}\frac{z^p}{p!} \tag{17}$$

where for convenience

$$\Lambda(m,n,p) = \frac{\prod_{j=1}^{A}(a_j)_{m+n+p}\ \prod_{j=1}^{B}(b_j)_{m+n}\ \prod_{j=1}^{B'}(b'_j)_{n+p}}{\prod_{j=1}^{E}(e_j)_{m+n+p}\ \prod_{j=1}^{G}(g_j)_{m+n}\ \prod_{j=1}^{G'}(g'_j)_{n+p}} \times \frac{\prod_{j=1}^{B''}(b''_j)_{p+m}\ \prod_{j=1}^{C}(c_j)_m\ \prod_{j=1}^{C'}(c'_j)_n\ \prod_{j=1}^{C''}(c''_j)_p}{\prod_{j=1}^{H}(h_j)_m\ \prod_{j=1}^{H'}(h'_j)_n\ \prod_{j=1}^{H''}(h''_j)_p\ \prod_{j=1}^{G''}(g''_j)_{p+m}}$$

II. Bilinear Generating Functions

By using the definition (2) and the Gaussian hypergeometric transformation (see, Rainville [3], Th. 21, pp. 60]

$${}_2F_1[a,\ b;\ c;\ z] = (1-z)^{c-a-b}{}_2F_1\left[c-a,\ c-b;\ c;z\right] \tag{18}$$

We thus obtain the bilinear generating function

$$\sum_{n=0}^{\infty} \frac{(c+b)_n(c+m)_n}{(1+d)_n\ n!} A_{m+n}^{(-b-n,c)}(x)A_n^{(-d-n,e)}(y)t^n = (1+x)^m\left(\frac{x}{1+x}\right)^{-b-c} F_c^{(3)}\left[c+m\ ,c+b\ ;c\ ,e\ ,1+d\ ; -\frac{1}{x},-\frac{(1+x)^2t}{x},\frac{(1+x)^2yt}{x}\right] \tag{19}$$

where $F_c^{(3)}$ denote the Lauricella's function defined by (14, with $n=3$). An interesting special case of the generating function (19) would occurs when we set, $m=0$, $d=b$, $e=c$, and appealing the hypergeometric reduction formula (see, B.L. Sharma [1], pp. 716, (2.4)).

$$F_c^{(3)}[\alpha+\beta+1,\beta+1\ ;\ \alpha+1,\beta+1,\beta+1\ ;\ x,y,z] = (1+x-y-z)^{-\alpha-\beta-1} \times F_4\left[\frac{\alpha+\beta+1}{2},\frac{\alpha+\beta+2}{2};\alpha+1,\beta+1;X,Y\right] \tag{20}$$

where, $X=\frac{4x}{(1+x-y-z)^2}$, $Y=\frac{4yz}{(1+x-y-z)^2}$ yields the generating relation

$$\sum_{n=0}^{\infty} \frac{(c+b)_n(c)_n}{(1+b)_n\ n!} A_n^{(-b-n,c)}(x)A_n^{(-b-n,c)}(y)t^n = \{1+(1+x)(1+y)t\}^{-b-c} \times F_4\left[\frac{1}{2}(c+b),\frac{1}{2}(c+b+1);1+b,c;\xi,\zeta\right] \tag{21}$$

where, $\xi=\frac{4xyt}{(1+(1+x)(1+y)t)^2}$, $\zeta=\frac{4t}{(1+(1+x)(1+y)t)^2}$ and F_4 is the Appell's function defined by (10).

Another bilinear generating function are obtained by using (2), which in conjunction with ([6], (2.25)),

$$\sum_{n=0}^{\infty} \frac{(\lambda)_n(1+c)_n}{(1+b+c)_n\ n!} A_{m+n}^{(b,-c-n)}(x)t^n = \frac{(1+b+c-m)_m}{(1+c-m)_m}x^m(1-xt)^{-\lambda} \times F_1\left[b,-m,\ \lambda;\ 1+b+c-m;\frac{1+x}{x},-\frac{(1+x)t}{1-xt}\right] \tag{22}$$

readily gives the relation

$$\sum_{n=0}^{\infty} \frac{(\lambda)_n(1+c)_n}{(1+b+c)_n\ n!} A_n^{(b,-c-n)}(x)A_n^{(d,e)}(y)t^n = (1-xyt)^{-\lambda}\sum_{n=0}^{\infty} \frac{(\lambda)_n(d)_n(b)_n}{(e)_n(1+b+c)_n\ n!}(\chi)^n \times F_2\left[\lambda+n,d+n,b+n;e+n,1+b+c+n;\psi,\omega\right] \tag{23}$$

where, $\frac{\chi}{-(1+x)}=\frac{\psi}{x}=\frac{\omega}{-(1+x)y}=\frac{t}{1-xyt}$ and F_2 is the Appell's function defined by (8)

The second member of (23) can indeed be written in terms of Srivastava triple hypergeometric series $F^{(3)}[x,y,z]$ defined by (17), and we thus obtain the alternative form of the bilinear generating function (23) as,

$$\sum_{n=0}^{\infty} \frac{(\lambda)_n(1+c)_n}{(1+b+c)_n\ n!} A_n^{(b,-c-n)}(x)A_n^{(d,e)}(y)t^n = (1-xyt)^{-\lambda} \times F^{(3)}\left[\begin{array}{l}\lambda\ :: d\ ;...;\qquad b\qquad :...;...;...;\\ ...:: e\ ;...;1+b+c:...;...;...;\end{array} \chi,\psi,\omega\right] \tag{24}$$

Again, when we set $\lambda = 1 + b + c$ in (24), along with ([7], pp. 35, (10))

$$F_2[a, b, b'; a, c'; x, y] = (1-x)^{-b} F_1 \left[b', b, a-b; c'; \frac{y}{1-x}, y \right] \quad (25)$$

Moreover, the power series identity ([4], 1.6(2)).

$$\sum_{m,n=0}^{\infty} f(m+n) \frac{x^m}{m!} \frac{y^n}{n!} = \sum_{N=0}^{\infty} f(N) \frac{(x+y)^N}{n!} \quad (26)$$

We obtain generating function in the form

$$\sum_{n=0}^{\infty} \frac{(1+c)_n}{n!} A_n^{(b,-c-n)}(x) A_n^{(d,e)}(y) t^n = (1+yt)^{-b}(1-xyt)^{-c-1} \times F_1 \left[d, b, 1+c; e; -\frac{1}{1+yt}, \frac{xt}{1-xyt} \right] \quad (27)$$

where F_1 is the Appell's function defined by (7).

In view of the definition (2) and (4), which in conjunction with (18), we obtain some more bilinear generating function for $A_n^{(b,c)}(x)$ as given below:

$$\sum_{n=0}^{\infty} \frac{(c+m)_n (1+e)_n}{(\lambda)_n \; n!} A_{m+n}^{(b,c)}(x) A_n^{(-d-n,-e-n)}(y) t^n = (1+x)^m \left(\frac{x}{1+x} \right)^{b-c} \times F_G \left[c+m, \; c+m, \; c+m, \; c-b, \; 1+e, \; d-e; \; c, \; \lambda, \; \lambda; -\frac{1}{x}, \; (1+x)(1+y)t, (1+x)t \right] \quad (28)$$

Alternatively, equivalently using (2) along with (5), we obtain

$$\sum_{n=0}^{\infty} \frac{(1+e)_n}{n!} A_{m+n}^{(b-n,c-n)}(x) A_n^{(d,-e-n)}(y) t^n = \frac{(b)_m}{(c)_m} (1+x)^{c+m-1} \times F_G \left[1-b, \; 1-b, \; 1-b, \; 1-c-m, \; 1+e, \; d; \; 1-b-m, \; 1-c, \; 1-c; \; \frac{x}{1+x}, \; yt, -t \right] \quad (29)$$

where in (28) and (29) F_G are the Saran's function defined by (12).

Further, we obtain some more bilinear generating function by using the relation (2) along with (3) in an elegant form as

$$\sum_{n=0}^{\infty} \frac{(\lambda)_n (c+m)_n}{(1+d)_n \; n!} A_{m+n}^{(b,c)}(x) A_n^{(-d-n,e)}(y) t^n = (1+x)^m \left(\frac{x}{1+x} \right)^{b-c} \times F_E \left[c+m, \; c+m, \; c+m, \; c-b, \; \lambda, \; \lambda; \; c, \; e, \; 1+d; -\frac{1}{x}, -(1+x)t, \; (1+x)yt \right] \quad (30)$$

or, equivalently

$$\sum_{n=0}^{\infty} \frac{(\lambda)_n (1-c)_n}{(1+d)_n \; n!} A_{m+n}^{(b-n,c-n)}(x) A_n^{(-d-n,e)}(y) t^n = \frac{(b)_m}{(c)_m} (1+x)^{c+m-1} \times F_E \left[1-b, \; 1-b, \; 1-b, \; 1-c-m, \; \lambda, \; \lambda; \; 1-b-m, \; e, \; 1+d; \; \frac{x}{1+x}, -t, \; yt \right] \quad (31)$$

where in (30) and (31) F_E is the Saran's function defined by (11).

III. Bilateral Generating Functions

The polynomials $A_n^{(b,c)}(x)$ admits several bilateral generating functions. Firstly, we introduce three bilateral generating function by using the relation (2), each of which involved the Gaussian hypergeometric ${}_2F_1$ function in terms of the Lauricella's triple hypergeometric series F_4, F_8 and F_7 (which, in the notation used by Saran's [8], are F_E, F_G, F_S respectively) are as follows:

$$\sum_{n=0}^{\infty} \frac{(\lambda)_n (\mu)_n}{(1+b)_n \; n!} A_n^{(-b-n,c)}(x) \; {}_2F_1 \left[\lambda+n, \; \beta; \; \gamma; \; y \right] t^n = F_E \left[\lambda, \; \lambda, \; \lambda, \; \beta, \; \mu, \; \mu; \; \gamma, \; 1+b, \; c; \; y, \; xt, -t \right] \quad (32)$$

$$\sum_{n=0}^{\infty} \frac{(\lambda)_n (1+c)_n}{(\mu)_n \; n!} A_n^{(b,-c-n)}(x) \; {}_2F_1 \left[\lambda+n, \; \beta; \; \gamma; \; y \right] t^n = F_E \left[\lambda, \; \lambda, \; \lambda, \; \beta, \; 1+c, \; b; \; \gamma, \; \mu, \; \mu; \; y, \; xt, -t \right] \quad (33)$$

and

$$\sum_{n=0}^{\infty} \frac{(\lambda)_n (1+c)_n}{(\mu)_n \; n!} A_n^{(b,-c-n)}(x) \; {}_2F_1 \left[\beta, \; \gamma; \; \mu+n; \; y \right] t^n = F_S \left[\beta, \; \lambda, \; \lambda, \; \gamma, \; 1+c, \; b; \; \mu, \; \mu, \; \mu; \; y, \; xt, -t \right] \quad (34)$$

Now, by using the definition (2) along with Laguerre polynomials (see [3], pp. 200, (1)), yields the generating function in the form

$$\sum_{n=0}^{\infty} \binom{m+n}{n} \frac{L_{m+n}^{(\alpha)}(x) A_n^{(-b-n,c)}(y)}{(1+b)_n} t^n = \binom{\alpha+m}{m} e^x \times \psi_2^{(3)} \left[\alpha+m+1; \; \alpha+1, \; c, \; 1+b; -x, -t, \; yt \right] \quad (35)$$

$$\sum_{n=0}^{\infty} \binom{m+n}{n} L_{m+n}^{(\alpha)}(x) \; A_n^{(b,c)}(y) t^n = \binom{\alpha+m}{m} e^x (1-yt)^{-\alpha-m-1} \times \psi_1 \left[\alpha+m+1, \; b; \; c, \; 1+\alpha; \frac{t}{1-yt}, -\frac{x}{1-yt} \right] \quad (36)$$

Alternatively, equivalently using (5), we obtain

$$\sum_{n=0}^{\infty}\begin{pmatrix} m+n \\ n \end{pmatrix}\frac{(1-c)_n}{(1-b)_n}L_{m+n}^{(\alpha)}(x)\ A_n^{(b-n,c-n)}(y)t^n$$

$$=\begin{pmatrix} \alpha+m \\ m \end{pmatrix}e^x(1-t)^{-\alpha-m-1}$$

$$\times\psi_1\left[\alpha+m+1,1-c;1-b,1+\alpha;\frac{yt}{1-t},\frac{-x}{1-t}\right] \quad (37)$$

where, in (35) $\psi_2^{(3)}$ is the confluent form of Lauricella's function defined by (16), with $n=3$ and in (36) and (37) ψ_1 is the confluent hypergeometric function of two variables (see [4], pp. 59, (41)).

The generalized heat polynomials $P_{n,\nu}(x,u)$ defined by (Haimo [2], p.736, (2.1)).

$$P_{n,\nu}(x,u)=\sum_{k=0}^{n}2^{2k}\begin{pmatrix} n \\ k \end{pmatrix}\frac{\Gamma(\nu+n+\frac{1}{2})}{\Gamma(\nu+n-k+\frac{1}{2})}x^{2n-2k}u^k \quad (38)$$

By reversing the order of summation, (38) can be written as

$$P_{n,\nu}(x,u)=(4u)^n\left(\nu+\frac{1}{2}\right)_n\sum_{k=0}^{n}\frac{(-n)_k}{\left(\nu+\frac{1}{2}\right)_k\ k!}\left(\frac{-x^2}{4u}\right)^k$$

$$=(4u)^n\ n!\ L_n^{(\nu-\frac{1}{2})}\left(-\frac{x^2}{4u}\right) \quad (39)$$

Further, involving the relation (39) with (2) and (5), another form of generating function equivalent to (35), (36) and (37) are obtained,

$$\sum_{n=0}^{\infty}\frac{P_{m+n,\nu}(x,u)A_n^{(-b-n,\ c)}(y)}{(1+b)_n\ n!}t^n$$

$$=(4u)^m\left(\nu+\frac{1}{2}\right)_m exp\left(\frac{-x^2}{4u}\right)$$

$$\times\psi_2^{(3)}\left[\nu+m+\frac{1}{2};\nu+\frac{1}{2},c,1+b;\frac{x^2}{4u},-4ut,4uyt\right] \quad (40)$$

$$\sum_{n=0}^{\infty}P_{m+n,\nu}(x,u)A_n^{(b,c)}(y)\frac{t^n}{n!}$$

$$=(4u)^m\left(\nu+\frac{1}{2}\right)_m exp\left(\frac{-x^2}{4u}\right)(1-4uyt)^{-\nu-m-\frac{1}{2}}$$

$$\times\psi_1\left[\nu+m+\frac{1}{2},\ b;\ c,\ \nu+\frac{1}{2};\frac{4ut}{1-4uyt},\frac{x^2}{4u(1-4uyt)}\right] \quad (41)$$

$$\sum_{n=0}^{\infty}\frac{(1-c)_n}{(1-b)_n}P_{m+n,\nu}(x,u)A_n^{(b-n,c-n)}(y)\frac{t^n}{n!}$$

$$=(4u)^m\left(\nu+\frac{1}{2}\right)_m exp\left(\frac{-x^2}{4u}\right)(1-4ut)^{-\nu-m-\frac{1}{2}}$$

$$\times\psi_1\left[\nu+m+\frac{1}{2},\ 1-c;1-b,\ \nu+\frac{1}{2}\right.$$

$$\left.;\frac{4uyt}{1-4ut},\frac{x^2}{4u(1-4ut)}\right] \quad (42)$$

Again using the definition (2), along with Jacobi polynomials (see [3], (1), pp. 254), which in conjunction with (3) yields the generating relations

$$\sum_{n=0}^{\infty}\frac{(1+\alpha+\beta)_n}{(1+b)_n}A_n^{(-b-n,c)}(x)P_n^{(\alpha,\beta)}(y)t^n$$

$$=\left(\frac{1+y}{2}\right)^{-\alpha-\beta-1}F_c^{(3)}\left[1+\alpha+\beta,\ 1+\alpha;\right.$$

$$\left.1+b,\ c,\ 1+\alpha;-\frac{2t}{1+y},\frac{y-1}{1+y},\frac{2xt}{1+y}\right] \quad (43)$$

$$\sum_{n=0}^{\infty}\frac{(\lambda)_n}{(1+b)_n}A_n^{(-b-n,c)}(x)P_n^{(\alpha,\beta-n)}(y)t^n$$

$$=\left(\frac{1+y}{2}\right)^{-\alpha-\beta-1}F_E\left[1+\alpha,\ 1+\alpha,\ 1+\alpha,\right.$$

$$\left.1+\alpha+\beta,\ \lambda,\ \lambda;\ 1+\alpha,\ 1+b,\ c\ ;\frac{y-1}{y+1},xt,-t\right] \quad (44)$$

$$\sum_{n=0}^{\infty}\frac{(1+c)_n}{(\lambda)_n}A_n^{(b,-c-n)}(x)P_n^{(\alpha,\beta-n)}(y)t^n$$

$$=\left(\frac{1+y}{2}\right)^{-\alpha-\beta-1}F_G\left[1+\alpha,\ 1+\alpha,\ 1+\alpha,;\right.$$

$$\left.1+\alpha+\beta,\ 1+c,\ b1+\alpha,\ \lambda,\ \lambda;\frac{y-1}{y+1},xt,-t\right] \quad (45)$$

Next, some more generating functions are expressed by using (2), which in conjunction with Lauricella's triple hypergeometric function $F_C^{(S)}$ and $F_D^{(S)}$.

$$\sum_{n=0}^{\infty}\frac{(\lambda)_n(\mu)_n}{(1+b)_n\ n!}A_n^{(-b-n,c)}(x)$$

$$\times F_C^{(S)}\left[\lambda+n,\ \mu+n\ ;\rho_1,...,\rho_s\ ;z_1,...,z_s\right]t^n$$

$$=F_C^{(S+2)}\left[\lambda,\mu;\rho_1,...,\rho_s,c,1+b;z_1,...,z_s,-t,xt\right] \quad (46)$$

and

$$\sum_{n=0}^{\infty}\frac{(\lambda)_n(1+c)_n}{(\mu)_n\ n!}A_n^{(b,-c-n)}(x)$$

$$\times F_D^{(S)}\left[\lambda+n\ ,\nu_1,....,\nu_s\ ;\mu+n\ ;z_1,....,z_s\right]t^n$$

$$=F_D^{(S+2)}\left[\lambda,\nu_1,...,\nu_s,b,1+c;\mu;z_1,...,z_s,-t,xt\right] \quad (47)$$

where in (46) and (47) $F_C^{(S+2)}$ and $F_D^{(S+2)}$ denote the Lauricella's triple hypergeometric function defined by (14) and (15) with $n=s+2$.

REFERENCES

[1] B.L. Sharma, *Integrals involving hypergeometric functions of two variables,* Proc. Nat. Acad. Sci. India. Sec., A-36, 713-718, 1966.
[2] D.T. Haimo, *Expansion in terms of generalized heat polynomials and their Appell transform,* J. Math. Mech., 15 , 735-758, 1966 .
[3] E.D. Rainville, *Special Functions,* MacMillan, New York 1960.
[4] H.M. Srivastava and H.L. Manocha, *A Treatise on generating functions,* Halsted press (Ellis Horwood Limited, Chichester), John Wiley and sons, New York, Chichester Brisbane, Toronto, 1984.
[5] I.K. Khanna and V. Srinivasa Bhagavan, *Lie Group-Theoretic origins of certain generating functions of the generalized hypergeometric polynomials,* Integeral transform and Special function, Vol-11, No.2, 177-188, 2001.
[6] M. Singh, M.A. Khan, A.H. Khan and S. Sharma, *Some generating functions for the Gauss' hypergeometric polynomials,* Research Today: Mathematical and Computer Sciences,Vol.1, 3-13, 2013.
[7] P. Appell and J. Kampé de Fériet, *Fonctions hypérgeométriques et hyperspheriques,* Polynômes d' Hermite Gauthier-Villars, Paris, 1926.
[8] S. Saran, *Hypergeometric functions of three variables,* Ganita, India, Vol.1, No.5, 83-90, 1954.
[9] S.D. Bajpai and M.S. Arora, *Some -orthogonality of a class of the Gauss' hypergeometric polynomials,* Anna. Math. Blasic Pascal, Vol-1, No.1, 75-83 (1994).