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Experimental Modal Analysis of Reinforced Concrete Square Slabs

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***Abstract*—**The aim of this paper is to perform experimental modal analysis (EMA) of reinforced concrete (RC) square slabs. EMA is the process of determining the modal parameters (Natural Frequencies, damping factors, modal vectors) of a structure from a set of frequency response functions FRFs (curve fitting). Although, experimental modal analysis (or modal testing) has grown steadily in popularity since the advent of the digital FFT spectrum analyzer in the early 1970's, studying all types of members and materials using such method have not yet been well documented. Therefore, in this work, experimental tests were conducted on RC square slab specimens of dimensions 600mm x 600mmx 40mm. Experimental analysis was based on freely supported boundary condition. Moreover, impact testing as a fast and economical means of finding the modes of vibration of a structure was used during the experiments. In addition, Pico Scope 6 device and MATLAB software were used to acquire data, analyze and plot Frequency Response Function (FRF). The experimental natural frequencies which were extracted from measurements exhibit good agreement with analytical predictions. It is showed that EMA method can be usefully employed to investigate the dynamic behavior of RC slabs.

***Keywords*—**Natural frequencies, Mode shapes, Modal analysis, RC slabs.

I. Introduction

CIVIL engineering structures could be damaged due to various man-made and natural hazards such as explosions, cyclones and earthquakes. In case of severe damages, a possible catastrophic and cascading engineering failure might occur. However, this could be prevented or minimized if much more attention has been paid to structural health monitoring. As a consequence, structural health monitoring is an attracting area of study in various fields of engineering especially for civil engineering research community [1]. Experimental modal analysis could be employed as a basis for health monitoring of structures.

Experimental modal analysis is generally defined as the process of an effective mean for identifying, understanding, and simulating the dynamic characteristics and response of a structure through determining the modal parameters such as natural frequencies, mode shape and damping factors of a linear, time-invariant system. This is due to the fact that modal parameters of a structure have an important role on its behavior under dynamic loading [2]. It is noteworthy to mention that one common reason for EMA is the correction of the results of the theoretical (numerical and analytical) approach. Predominately, EMA is employed to explain a dynamics problem (vibration or acoustic) whose solution is not obvious from intuition, analytical models, or previous experience [3]. In EMA, the most reliable way to test the structure is in the free-free condition, which is freely-supported, the tested structure in space and it is not attached at any of its coordinates. This is achieved by supporting the tested structure by very soft springs such as light elastic cords. Such support conditions have the advantage that the dynamic measurements have the capability to reflect the properties of the structure precisely when the influence of poorly defined supports is interrupted [4].

In EMA, the frequency response functions (FRFs) are used to extract the modal parameters of a structure which is based on measuring both input and output in a test simultaneously. Over the past decades, several modal parameter estimation techniques have been proposed and studied for modal parameter identifications. Among these are the frequency domain spectrum analysis methods including the peak picking (PP) method [5]. In this study, PP method is adopted where the details of the processing the measured excitations and responses and extracting the modes of the RC square slabs will be discussed and presented in due course.

EMA has grown steadily in popularity since the advent of the digital FFT spectrum analyzer in the early 1970's [6]. However, studying all types of members and materials using such methods have not yet been well documented. Therefore, in this work, experimental tests were conducted on RC square slab specimens of dimensions 600mm x 600mm x 40mm. Experimental analysis was based on freely suspended boundary conditions. Moreover, impact testing as a fast and economical means of finding the modes of vibration of a structure was used during the experiments.

II. Equipment and Instrumentation

In order to achieve the subsequent steps of EMA, the following main equipment and instrumentation were used and therefore they are important to be identified.

A. Impact Hammer

An impact hammer DeltaTron® version 8208 was used to excite and provide a known input force to the RC slabs. It was equipped with a force transducer, which allowed the measurement of the applied impulse. When the slab was excited by the impact hammer with impact tip, in a very short period of time energy was transferred to the slab with giving a typical input force signal. The mass of the hammer head is

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1.4kg (3lb) and the nominal force range is 22.1kN (5000lb) for full scale of ±5V, and its reference sensitivity is 0.22mV/N (1.00mV/lb).

B. Accelerometers

Three accelerometers DeltaTron® model were used in the analysis, accelerometers with a sensitivity of about 1.005mV/ms-2 (9.86mV/g) and frequency range between 1 Hz-10 kHz. One important characteristic of accelerometer is that it includes integrated amplifier that converts the movement of the seismic mass into acceleration signal in the form of voltage. In order to measure the acceleration response of the RC square slabs, steel studs were glued first on the measuring grid of each slabs using super glue. Then, the accelerometers were mounted to the studs by which the studs and the screwed accelerometers were normally affixed to the surface of the tested slab.

C. Amplifier

A conditioning amplifier NEXUSTM type 2690 was employed during this study. In EMA, the amplifier is needed as the necessary device to boost the very small electrical charges, which are generated by the accelerometer into a signals. The signals were fed to the data acquisition card after filtering.

D. Data Acquisition Card

The data acquisition card Pico Scope 4424 was employed to convert the captured analogue signal to a digital signal (A/D conversion) with giving less quantization error. Data files can then be transferred to the PC and they can be copied and pasted using Pico scope software.

E. Personal Computer

Personal computer was used to show the experimental data on Pico scope software. Moreover, personal computer with signal processing software such as MATLAB® was also used to analyse the signals.

III. The Measurement Procedure

Two RC square slab specimens of side length 600mm and thickness 40mm were cast, cured for 28 days and then dynamically tested. EMA was employed to find the dynamic behaviour of a linear vibrating structure by measuring its natural frequencies, mode shapes and modal damping coefficients. To achieve this aim each slab was suspended, which is commonly designated as free-free conditions. More specifically, it was floating in the air, free in space with no holding points whatsoever. The flexible elastic ropes (bungee cords) were used for the simulation of free-free conditions of a slab. The reason behind this is that the dynamic measurements have the capability to precisely reflect the properties of the slab when the influence of poorly defined supports is interrupted. After the slab was properly suspended as free-free boundary condition in the laboratory testing environment as shown in Fig. 1, both impulse and excitations were used. The input dynamic forces were applied through impact hammer which is equipped with a force transducer. While, the response of the slab was measured with a lightweight piezoelectric accelerometer attached at a selected point on the slab's surface. Roving accelerometer test was conducted in this study. It means that the output position changes whereas the input location remains fixed. As a result, the hammer was fixed at response point 1 while the accelerometers were placed at all the 25 equally spaced response points, which is shown in Fig. 2.

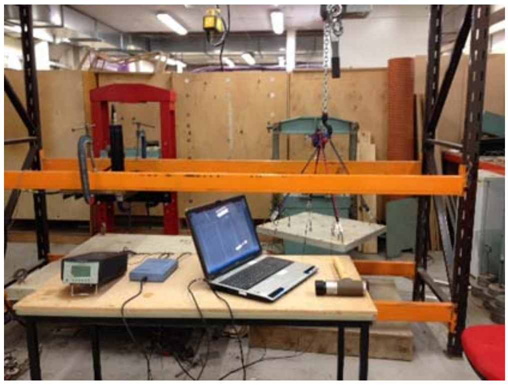


Fig. 1 Experimental test set-up

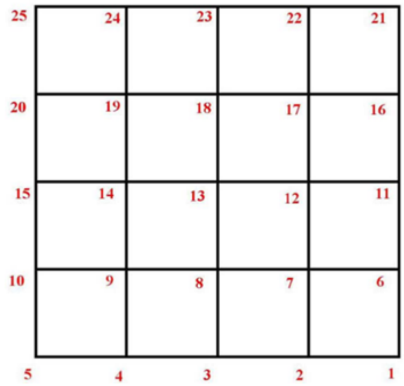
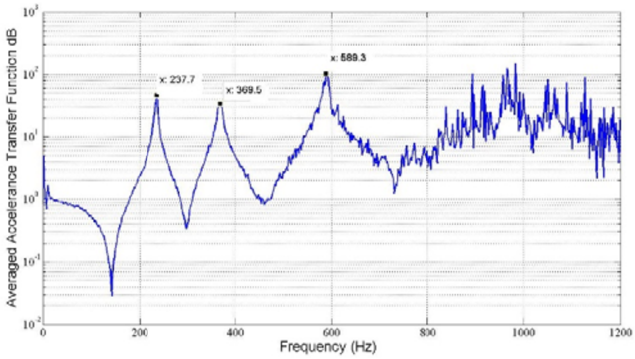


Fig. 2 Vibration grid measurement points

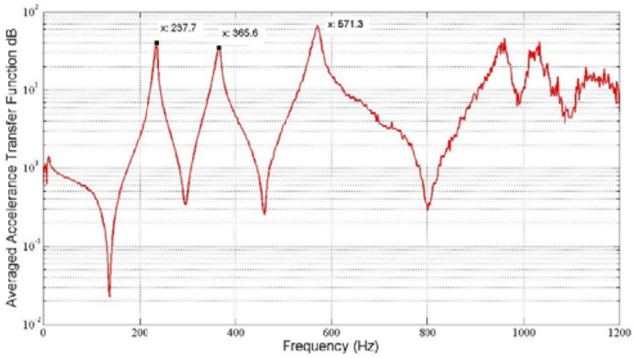
Then, the time signals from both impact hammer and accelerometers were recorded for a short period of time and collected as well as saved on file. By using Matlab software, the Fast Fourier Transform (FFT) of the two signals and dividing the two spectra as the frequency response curve which is plotted in the next section. Averaging of the results has been done over five impacts to obtain high quality frequency response curves. Such plot shows sharp peaks which is a rough reference of modal parameters estimation. As mentioned earlier, Peak Picking (which is called Quadrature Picking) method is employed to extract modal parameters.

IV. Modal Parameter Estimation

From a set of frequency response function measurements, modal parameters can be extracted from a number of measurements. It is worth mentioning that natural frequency and damping of the structure can be extracted from any FRF measurement where the resonance is clearly presented. To extract the natural frequency, the magnitude of averaged FRF was drawn as shown in Fig. 3. Then, at resonance, the magnitude of FRF rapidly approaches a sharp maximum value, which gave the desired natural frequencies of the slabs.



(slab A)



(slab B)

Fig. 3 Extracting natural frequencies of RC slabs A and B from FRF

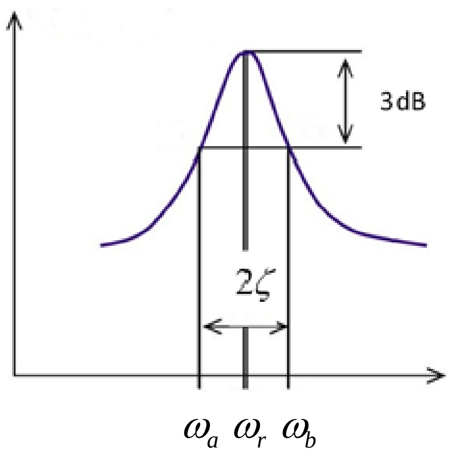


Fig. 4 FRF plot for extracting damping ratio

In the FRF chart, where natural frequency was determined by the maximum value of the amplitude, one of most used method which is called the half power (–3 dB) points of the magnitude of FRF was employed to obtain damping factor of the slabs. Extraction of the modal damping was relatively simple. From the sharp point of natural frequency, one can easily identify the maximum amplitude and size of half-power. At the value of (–3 dB) and on both sides of the peak, a parallel lines were dropped off on the abscissa axis which is the natural frequency axis as seen in Fig. 4. By determining ωr, ωa, and ωb, the modal damping was calculated using (1):

$$\zeta = \frac{\omega_b - \omega_a}{2 \cdot \omega_r} \tag{1}$$

where ζ is the modal damping of the system. ω_r is the resonance frequency. $\omega_b - \omega_a$ is the width of the resonance curve at $\frac{1}{\sqrt{2}}$ of the resonance amplitude.

The extraction of natural frequencies and damping is not relatively difficult for a structure. However, mode shapes extraction are more complicated and complex. It is not possible to easily extract the mode shapes from the chart or from only some FRF measurements. Therefore, in this study, the mode shapes were measured from 25 response points. Peak picking method was employed to extract the mode shapes of the slabs. This method can be employed for the structures which have lightly coupled modes.

V. Analytical Natural Frequencies

Analytic closed form expressions for estimating natural frequencies of linear elastic square plates with free-free boundary condition are defined by (2) and (3) as [7]:

$$f = \frac{\lambda^2}{2\pi \cdot l \cdot b}\sqrt{\frac{D}{\mu}} \tag{2}$$

$$D = \frac{Eh^3}{12\left(1 - 0.24\upsilon^2\right)} \tag{3}$$

where f is the natural frequency in Hz. D is the plate flexural rigidity. E is modulus of elasticity. υ is the Poisson's ratio of the material of the plate. h is the thickness of the plate. l is the length of the longer side of the plate. b is the length of the shorter side of the plate. μ is the mass density per unit area of plate (ρh). λ is a dimensionless natural frequency factor.

The first three natural frequencies of RC free-free square slab can be obtained using above equations. It can be said that analytical solution has some important benefits. First, relatively small computational time and effort are required to get an idea of the behaviour of the structure. Second, without analytical method it is not possible to know the reliability of the experimental results.

VI. Results and Discussion

Three experimental natural frequencies were matched with analytical ones in order to find the accuracy of the experimental modal analysis as shown in Table I. The relative error between three analytical and experimental natural frequencies of the freely supported slabs is reported. The relative error is a function of absolute natural frequency difference between theoretical and experimental model, which is expressed by (4):

$$Error = \frac{\omega_{Exp} - \omega_{Theo}}{\omega_{Exp}} \tag{4}$$

where: ω_{exp} is the experimental natural frequency obtained; ω_{theo} is the theoretical natural frequency obtained .

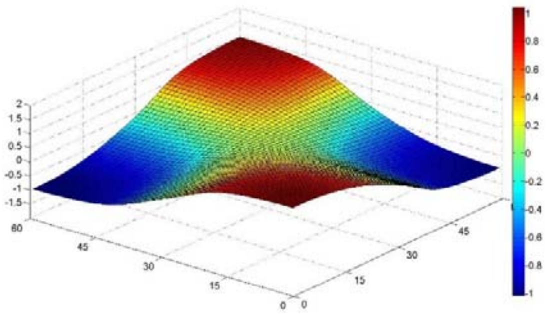
TABLE I
COMPARISON BETWEEN NATURAL FREQUENCIES OF RC SLABS

Mode	Natural frequencies (Hz)			%Error	
	Analytical	Experimental		Theo. Vs Exp. Slab A	Theo. Vs Exp. Slab B
		Slab A	Slab B		
1	*269.5*	*237.7*	*237.7*	*13.37*	*13.37*
2	*395.3*	*369.5*	*365.5*	*6.98*	*8.15*
3	*488.0*	*589.3*	*571.3*	*17.20*	*14.58*

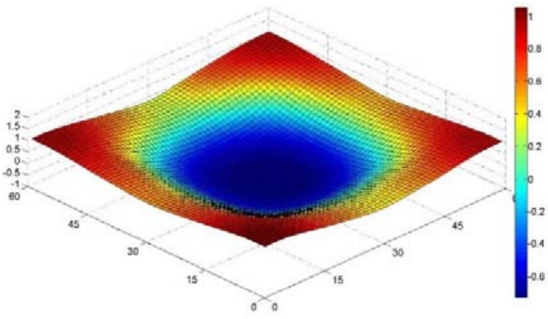
Table II tabulates three modal damping ratio of the slabs, which are extracted from the experimental data. While in Figs. 5, 6, there are the three extracted mode shapes of the slabs A and B respectively. Any of the experimental modal parameters such as natural frequencies, mode shapes and modal damping can be used as description of the dynamic behaviour of the slabs.

TABLE II
MODAL DAMPING RATION OF THE SLABSS

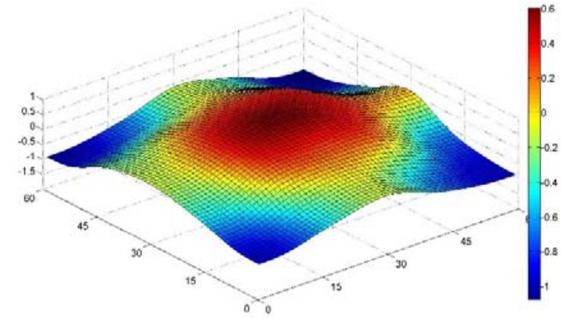
Mode	Modal damping ration ζ %	
	Slab A	Slab B
1	*0.93*	*1.2*
2	*1.50*	*1.41*
3	*0.56*	*1.09*



Mode Shape 1

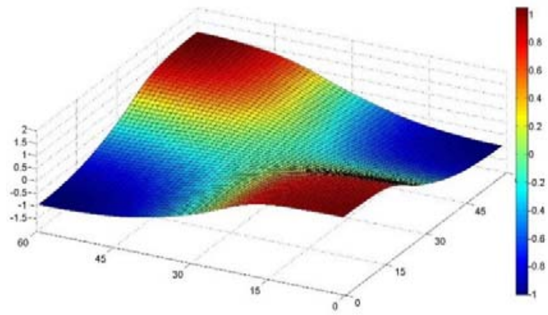


Mode Shape 2

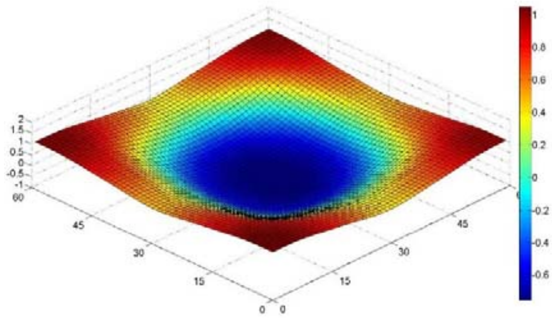


Mode Shape 3

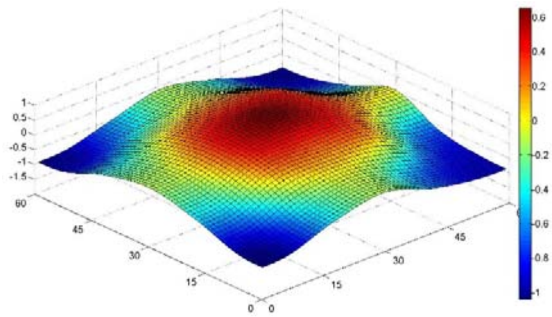
Fig. 5 Experimental mode shape of RC slab A



Mode Shape 1



Mode Shape 2



Mode Shape 3

Fig. 6 Experimental mode shape of RC slab B

In Table I, a comparison of the results clearly reveals that there is a good correlation between the experimental and analytical natural frequencies with percentage of error ranging between 7 and 17%. Such a difference is quite acceptable due to the complex behaviour of RC slabs and characteristics of support idealization. More specifically, the theoretical model assumes the concrete slab to be linear, elastic, homogenous and isotropic. Whereas in reality, concrete is heterogeneous and anisotropic material and the specimens inevitably contain imperfections in forms of crack, defects and steel bars misalignment.

Modal damping ratios of the slabs were also extracted from the experimental data, which are presented in Table II. They have values ranging between 0.6% and 1.5%. Furthermore, as mentioned earlier, the first three mode shapes of the slabs A and B were extracted and depicted in Figs. 5, 6. The comparison between the two slabs had been made through these two figures. It is shown that the slab A is slightly stiffer than slab B. The colour bar of third mode shapes in Fig. 5 shows that the maximum positive response of slab A is about to 0.6 unit while it is about 0.65 of slab B which is shown in Fig. 6.

Generally speaking, there are slight differences of results in terms of natural frequency, modal damping ratios and mode shapes between slab A and B as shown in Tables I, II and Figs. 5, 6. This is quite common and acceptable for reinforced concrete experiment. Such a difference is mainly attributed to the whole process of preparing, making, curing of concrete specimens in lab. For instance, no two batches of concrete mix can be guaranteed of having exactly same ingredients, receiving same degree of compaction or having exactly same dimensions during casting.

VII. Conclusion

In this work, the dynamic characteristics of freely supported RC slabs have been analyzed by EMA. The main goal of this paper is to employ EMA as a powerful technique to efficiently and accurately extract the natural frequencies, mode shapes and modal damping of the slabs. Moreover, impact testing as a fast and economical means of finding the dynamic behaviour of a structure was used during the experiments. To judge the reliability of experimental results of RC slabs, a comparison between theoretical and experimental natural frequencies has been made. The theoretical natural frequencies were obtained analytically using closed form expressions while experimental natural frequencies were determined using EMA. The differences in results are relatively small, hence within acceptable limits for the reasons mentioned in the discussion section above. Overall, the experimental work shows successful extraction of experimental modal parameters when the slabs are excited by impact hammer. Such promising investigation can be deemed as the right step towards the description of dynamic behaviour of the structures.

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