Boundary Layer Flow of a Casson Nanofluid past a Vertical Exponentially Stretching Cylinder in the Presence of a Transverse Magnetic Field with Internal Heat Generation/Absorption

G. Sarojamma, K. Vendabai

Abstract—An analysis is carried out to investigate the effect of magnetic field and heat source on the steady boundary layer flow and heat transfer of a Casson nanofluid over a vertical cylinder stretching exponentially along its radial direction. Using a similarity transformation, the governing mathematical equations, with the boundary conditions are reduced to a system of coupled, non –linear ordinary differential equations. The resulting system is solved numerically by the fourth order Runge – Kutta scheme with shooting technique. The influence of various physical parameters such as Reynolds number, Prandtl number, magnetic field, Brownian motion parameter, thermophoresis parameter, Lewis number and the natural convection parameter are presented graphically and discussed for non – dimensional velocity, temperature and nanoparticle volume fraction. Numerical data for the skin – friction coefficient, local Nusselt number and the local Sherwood number have been tabulated for various parametric conditions. It is found that the local Nusselt number is a decreasing function of Brownian motion parameter Nb and the thermophoresis parameter Nt.

Keywords—Casson nanofluid, Boundary layer flow, Internal heat generation/absorption, Exponentially stretching cylinder, Heat transfer, Brownian motion, Thermophoresis.

I. INTRODUCTION

THE study of boundary layer flow and heat transfer over an exponentially stretching cylinder has attracted many researchers due to its applications in fiber technology, flow meter design, piping and casting systems etc. Lin and Shih [1], [2] analyzed the laminar boundary layer and heat transfer along horizontally and vertically moving cylinders with constant velocity and found that the similarity solutions could not be obtained due to the curvature effect of the cylinder. Wang [3] investigated the steady flow of a viscous and incompressible fluid outside a stretching hollow cylinder in an ambient fluid at rest. Ishak et al. [4] studied the flow and heat transfer of an incompressible electrically conducted viscous fluid outside of a stretching cylinder in the presence of a constant transverse magnetic field. Elbashbeshy et al. [5] studied laminar boundary layer flow of an incompressible viscous fluid along a stretching horizontal cylinder embedded in a porous medium in the presence of a heat source or sink with suction/injection.

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Casson fluid flow in cylindrical geometry has important applications in blood flow. Fredrickson [6] studied the flow of a casson fluid in a tube. Nagarani and Sarojamma [7] investigated the effect of body acceleration on the flow of blood modeling as a Casson fluid through a stenosed artery. Sarojamma et al. [8] analyzed the flow of blood through a stenosed catheterized artery modeling blood as a Casson fluid.

Heat transfer characteristics of nanofluid mainly depend on the thermo physical properties viscosity, thermal conductivity, specific heat and density. To model the viscosity of nanofluid, several researchers have studied the rheology of nanofluid. On the other hand researchers believed that nanofluid behaves as Newtonian fluid [9], [10] others observed non – Newtonian behavior [11], [12]. The study pertaining to the rheology of nanofluids as a part of International Nanofluid Property Benchmark Exercise (INPBE) revealed that nanofluid has the Newtonian and non – Newtonian behavior. It is reported by [13] that shear thinning nature of nanofluids depends upon volume fraction, shear rate and viscosity of the base fluid. No shear thinning behavior was observed when volume fraction is less than 0.001 and shear thinning was observed when the volume fraction was less than 0.05 at low shear rates. However, the volume fraction exceeds 0.05 the shear thinning nature is observed for all values of shear rates. In this paper, a boundary layer flow of a Casson nanofluid over a vertical exponentially stretching cylinder in the presence of magnetic field and non uniform heat source is analyzed. Using a similarity transformation, the governing mathematical equations are transformed into a system of coupled nonlinear ordinary differential equations which are then solved numerically using shooting technique with fourth order Runge – Kutta method. The variation of the different parameters is shown graphically.

II. MATHEMATICAL FORMULATION

Consider the problem of natural convection boundary layer flow of a Casson nanofluid flowing over a vertical circular cylinder of radius 'a'. The cylinder is assumed to be stretched exponentially along the radial direction with velocity U_w. The temperature at the surface of the cylinder is assumed to be T_w and the uniform ambient temperature is taken as T_∞ such that the quantity $T_w - T_\infty > 0$ in case of the assisting flow, while $T_w - T_\infty < 0$ in case of the opposing flow, respectively. Under

World Academy of Science, Engineering and Technology
International Journal of Mathematical and Computational Sciences
Vol:9, No:1, 2015

these assumptions, the continuity, momentum, energy and nanoparticle concentration are written as [14].

$$u_r + \frac{u}{r} + w_z = 0 \tag{1}$$

$$uw_r + ww_z = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \nu\left(1 + \frac{1}{\beta}\right)\left(w_{rr} + \frac{1}{r}w_r\right) + g\beta(T - T_\infty)(1 - \varphi_\infty) +$$
$$\frac{1}{\rho}(\rho^* - \rho)(\varphi - \varphi_\infty) - \frac{\sigma B_0^2 w}{\rho} \tag{2}$$

$$uT_r + wT_z = \alpha\left(T_{rr} + \frac{1}{r}T_r\right) + \frac{\rho^* c_p^*}{\rho c_p}\left(D_T T_r\varphi_r + \frac{D_T}{T_\infty}T_r^2\right) + \frac{q'''}{\rho c_p} \tag{3}$$

$$w\varnothing_z + u\varnothing_r = D_B\left(\varphi_{rr} + \frac{1}{r}\varphi_r\right) + \frac{D_T}{T_\infty}\left(T_{rr} + \frac{1}{r}T_r\right) \tag{4}$$

The pertinent boundary conditions for the problem are given by

$$u(a, z) = 0,\ w(a, z) = U_w,\ w(r,z) \to 0\ \text{as } r \to \infty, \tag{5}$$
$$T(r, z) = T_w(z),\ T(r, z) \to T_\infty \quad \text{as } r \to \infty, \tag{6}$$
$$\varnothing(r, z) = \varnothing_w(z),\ \varnothing(r, z) \to \varnothing_\infty\ \text{as } r \to \infty, \tag{7}$$

where the velocity components along the (r, z) axes are (u, w), ρ is the density (Kg/m^3), ν is the kinematic viscosity, p is the pressure, g is the gravitational acceleration along the z direction, β is the coefficient of thermal expansion, σ is the electrical conductivity (1Sm^{-1}), T is the temperature and α is the thermal diffusivity, D_B and D_T are the Brownian diffusion coefficient and the thermophoresis diffusion coefficient, respectively. $\rho^* c_p^*$ and ρc_p are the heat capacity of the base fluid and the effective heat capacity of the nanoparticles material, respectively. We also assume that the uniform magnetic field with intensity of B_0(1T) acts in the radial direction, q''' is the rate of internal heat generation (> 0) or absorption (< 0) coefficient, C_p is the specific heat at constant pressure.

The internal heat generation or absorption term q''' is modeled according to the following equation

$$q''' = \frac{kU_w(z)}{a\nu}[A^*(T_w - T_\infty)f' + B^*(T - T_\infty)] \tag{8}$$

where A^* and B^* are the coefficient of space and temperature dependent heat source / sink respectively. It is to be noted that the case $A^* > 0$, $B^* > 0$ corresponds to internal heat generation while for both $A^* < 0$, $B^* < 0$, corresponds to internal heat absorption. Where $U_w = 2ake^{z/a}$ (k is the dimensional constant) is the fluid velocity at the surface of the cylinder.

Introduce the following similarity transformations

$$u = -\frac{1}{2}U_w\frac{f(\eta)}{\sqrt{\eta}},\ w = U_w f'(\eta), \tag{9}$$

$$\theta = \frac{T - T_\infty}{T_w - T_\infty}, \eta = \frac{r^2}{a^2},\ h = \frac{\phi - \phi_\infty}{\phi_w - \phi_\infty} \tag{10}$$

where the characteristic temperature and nano concentration difference are calculated from the relations $T_w - T_\infty = c\,e^{z/a}$ and $\phi_w - \phi_\infty = e^{z/a}$. With the help of transformations (8) and (10), (1) – (4) take the form

$$\left(1 + \frac{1}{\beta}\right)(\eta f''' + f'') + Re(ff'' + f'^2) + Re\lambda(1 - \phi_\infty)(\theta + N_r h) - M^2 f' = 0 \tag{11}$$

$$\eta\theta'' + \theta' + Re\,\Pr(f\theta' - f'\theta) + \eta\theta'(N_b h' + N_t\theta') + Re(A^* f' + B^*\theta) = 0 \tag{12}$$

$$\eta h'' + h' + \frac{N_t}{N_b}(\eta\theta'' + \theta') + Re\,Le(fh' - f'h) = 0 \tag{13}$$

where $\lambda = g\beta a(T_w - T_\infty)/U_w^2$ is the buoyancy (natural convection) parameter, $\Pr = \nu/\alpha$ is the Prandtl number, $Le = \nu/D_B$ is the Lewis number, $N_r = (\rho^* - \rho)(\phi_w - \phi_\infty)/\rho\beta(T_w - T_\infty)(1 - \phi_\infty)$ is the buoyancy ratio, $N_b = \rho^* c_p^* D_B(\phi_w - \phi_\infty)/\rho c_p\alpha$ is the Brownian motion parameter, $N_t = \rho^* c_p^* D_T(\phi_w - \phi_\infty)/\rho c_p\alpha T_\infty$ is the thermophoresis parameter and $Re = aU_w/4\nu$ is the Reynolds number, $M^2 = \sigma B_0^2 a^2/4\mu$ is the magnetic parameter. The corresponding boundary conditions in non dimensional form become

$$f(1) = 0,\ f'(1) = 1,\ f' \to 0,\ \text{as } \eta \to \infty \tag{14}$$
$$\theta(1) = 1,\ \text{as } \theta \to 0, as\ \eta \to \infty \tag{15}$$
$$h(1) = 1,\ \text{as } h \to 0, as\ \eta \to \infty \tag{16}$$

The important physical quantities such as the shear stress at the surface τ_w, the skin friction coefficient C_f, and the local Nusselt number Nu_z, the local Sherwood number Sh_z are given by

$$\tau_w = w_{rz/r = a}, q_w = -kT_r/r = a,\ q_m = -D_B C_r/r = a \tag{17}$$

$$C_f = \frac{\tau_w}{\rho U_w^2},\ Nu_z = \frac{ae^{z/a}q_w}{k(T_w - T_\infty)},\ Sh_z = \frac{e^{z/a}q_m}{D_B(C_w - C_\infty)} \tag{18}$$

where q_w and q_m are the heat and mass fluxes at the surface of the cylinder

The solution of the present problem is obtained numerically by using the fourth order Runge – Kutta method with shooting technique.

III. RESULTS AND DISCUSSION

The problem of boundary layer flow of a Casson nanofluid past a vertical exponentially stretching cylinder subject to a transverse magnetic field in the presence of heat generation / absorption is analyzed. The governing non – linear ordinary differential equations are solved using fourth order Runge – Kutta shooting method. The objective of this investigation is to understand the effect of various parameters viz. natural convection parameter λ, Casson parameter β, Hartmann number M, buoyancy ratio parameter Nr, Brownian motion parameter Nb, thermophoresis parameter Nt, Reynolds number Re, Lewis number Le, and the Prandtl number Pr on the velocity, temperature, nanoparticle volume fraction that are graphically presented. The validity of the numerical results is verified by comparing the results with those of [14] in the absence of magnetic field (M = 0), space dependent (A*) and temperature dependent (B*) heat sources. The results are found to be in excellent agreement. The tabular values of –θ'(1), -f"(1) are tabulated in Table I.

Figs. 1–3 indicate the effect of the yield stress parameter/ Casson parameter. It is clear that the velocity decreases with β. It may be noted that increased values of β implies a decrease in the yield stress of the Casson fluid and thus facilitates the flow of the fluid. In other words it accelerates the boundary layer flow in the vicinity of the stretching cylinder. It is observed that increasing values of the Casson parameter

World Academy of Science, Engineering and Technology
International Journal of Mathematical and Computational Sciences
Vol:9, No:1, 2015

enhance the temperature as well as the nanoparticle volume fraction.
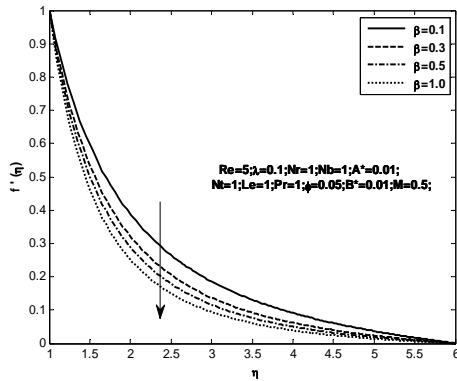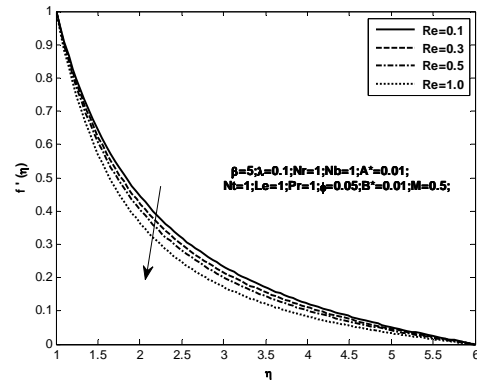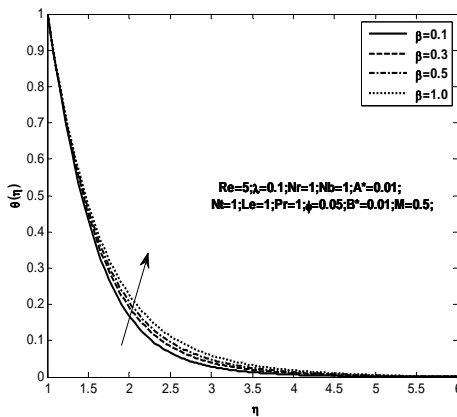

Fig. 1 Variation of f '(η) with η for different values of β

Fig. 2 Variation of θ(η) with η for different values of β

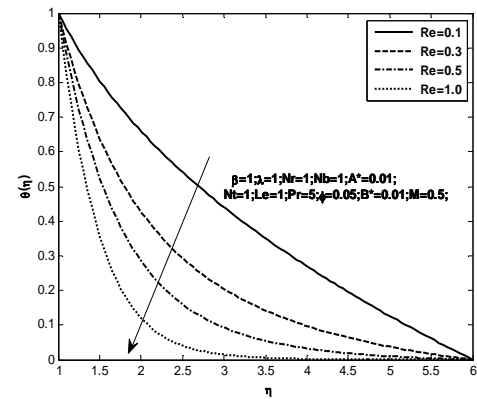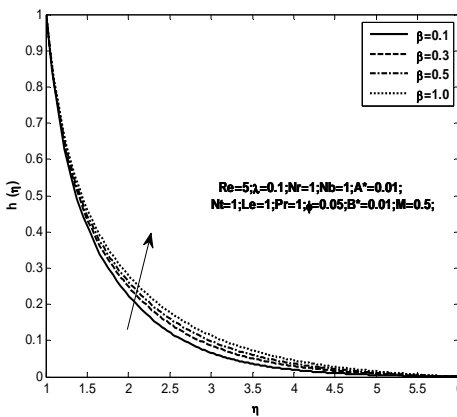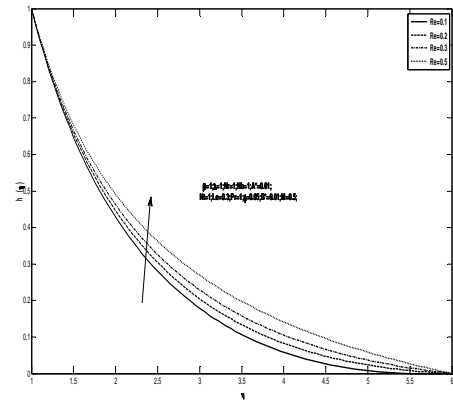Fig. 3 Variation of $h(\eta)$ with η for different values of β

In Figs. 4–6 the influence of the Reynolds number Re on velocity, temperature and nanoparticle volume fraction is shown. For increasing values of Re the velocity and temperature in the boundary layer are observed to decrease while the concentration increases. The increased values of Reynolds number correspond to the enhancement in viscosity of the fluid and consequently it contributes for the reduction in the thickness of the hydrodynamic and thermal boundary

layers. The effect of Re on temperature is more significant than on the velocity.

Fig. 4 Variation of f '(η) with η for different values of Re

Fig. 5 Variation of θ(η) with η for different values of Re

Fig. 6 Variation of h(η) with η for different values of Re

Fig. 7 presents on velocity. The natural convection parameter λ is directly proportional to the velocity profile for constant values of other parameters. Increased values of λ produce an enhancement in the velocity while the temperature and concentration decrease.

Fig. 8 illustrates the influence of the Brownian motion parameter on temperature. It is reported that the enhancement of the thermal conductivity of a nanofluid is owing to Brownian motion Nb which facilitates micromixing.

World Academy of Science, Engineering and Technology
International Journal of Mathematical and Computational Sciences
Vol:9, No:1, 2015

Therefore, we may predict that temperature is an increasing function of Brownian motion. It is clear that the increased values of Nb are found to increase temperature.
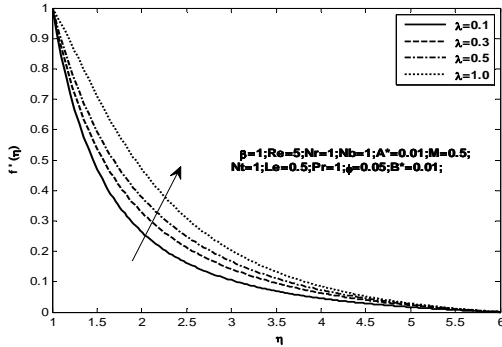

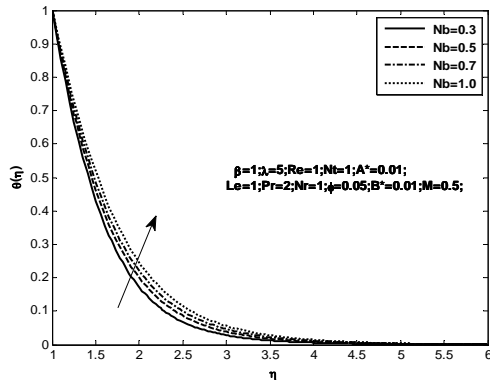Fig. 7 Variation of $f'(\eta)$ with η for different values of λ

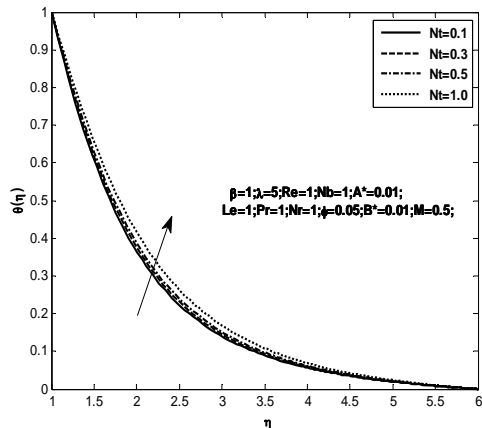Fig. 8 Variation of $\theta(\eta)$ with η for different values of Nb

Fig. 9 Variation of $\theta(\eta)$ with η for different values of Nt

It is observed from Fig. 9 that for higher values of thermophoresis parameter Nt, temperature is observed to enhance.

From Figs. 10-12 we observed that the stronger the magnetic field strength lesser the velocity while the temperature and nanoparticle concentration increase. The Lorentz force acts as a resistive force and as a result the hydrodynamic boundary layer decreases.
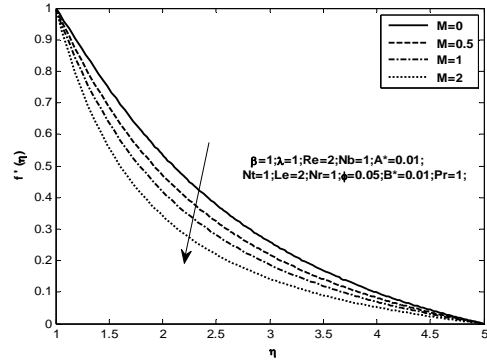

Fig. 10 Variation of $f'(\eta)$ with η for different values of M
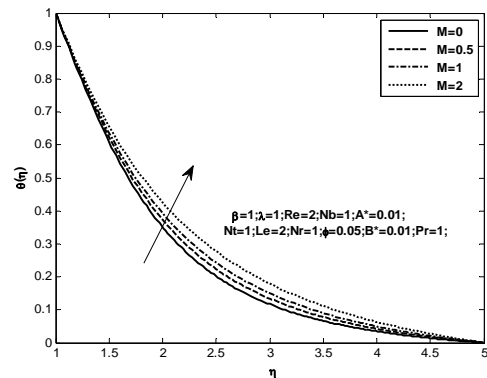

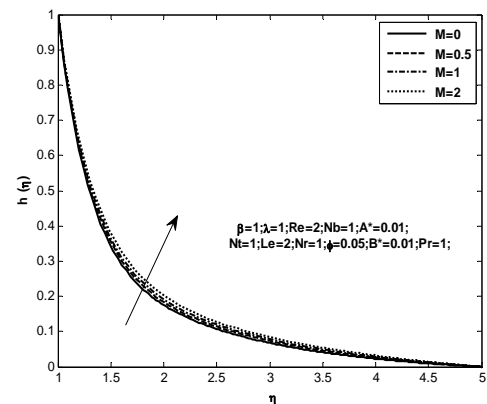Fig. 11 Variation of $\theta(\eta)$ with η for different values of M

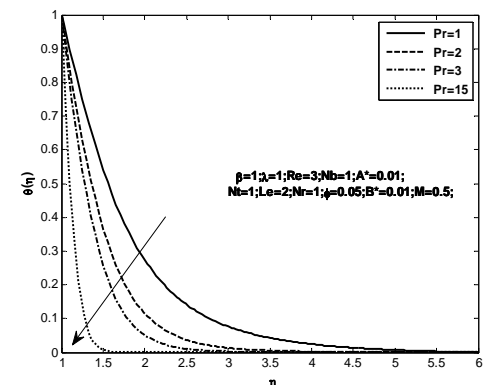Fig. 12 Variation of $h(\eta)$ with η for different values of M

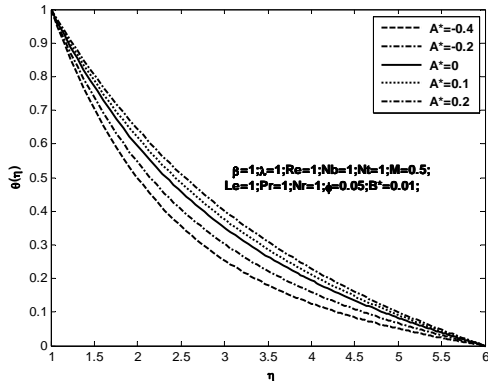Fig. 13 Variation of $\theta(\eta)$ with η for different values of Pr

World Academy of Science, Engineering and Technology
International Journal of Mathematical and Computational Sciences
Vol:9, No:1, 2015

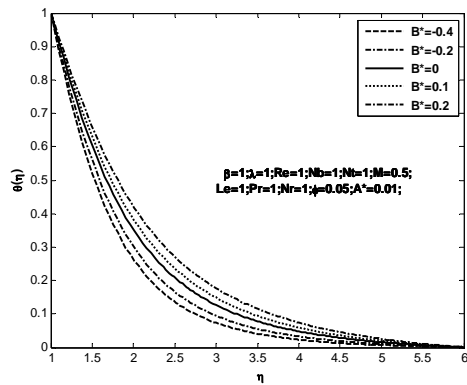Fig. 14 Variation of $\theta(\eta)$ with η for different values of A^*

Fig. 15 Variation of $\theta(\eta)$ with η for different values of B^*
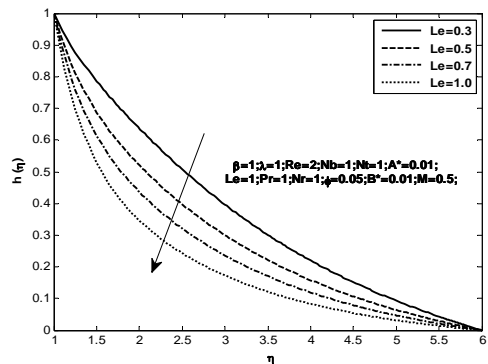

Fig. 16 Variation of $h(\eta)$ with η for different values of Le

Fig. 13 depicts the effect of Prandtl number Pr on the temperature. It is clear that increasing values of Prandtl number produce an enhancement in the temperature. Since Pr is the ratio of the rate of viscous diffusion to the rate of thermal diffusion, increased values of Pr correspond to reduction in the thermal diffusivity which leads to the decreasing of energy ability resulting in the shrinking of the thermal boundary layer.

Fig. 14 illustrates the influence of the spatial – dependent internal heat generation ($A^* > 0$) or absorption ($A^* < 0$) in the boundary layer on temperature. It is evident that increasing values of A^* produce an enhancement in the velocity, temperature and concentration of the fluid. This may be attributed to the fact that the presence of a heat source ($A^* > 0$)

would generate energy resulting a rise in temperature. On the contrary when $A^* < 0$ (heat sink) has the opposite effect, namely cooling of the fluid occurs. Consequently the thickness of the thermal boundary layer is reduced. Fig. 15 demonstrates the effect of temperature – dependent heat source on temperature. The impact of the temperature – dependent internal heat generation ($B^* > 0$) or absorption ($B^* < 0$) in the boundary layer on temperature is similar to that of the spatial – dependent internal heat generation or absorption. In the presence of heat source ($B^* > 0$) the temperature increases while a reduction is noticed in the case of heat sink ($B^* < 0$). Fig. 16 is the plots of concentration versus Lewis number. It is evident that nanoparticle concentration reduces significantly with Le.

Table I shows that results of the present investigation that are in excellent agreement with those of [15] in the absence of A* & B* and M. It is observed from Table II that in the presence of β viz., for a Casson nanofluid the skin friction coefficient produces higher friction on the cylinder than the Newtonian fluid. Further increase in the values of β increases the skin friction coefficient. Increased values of natural convection parameter produce a reduction in the skin friction coefficient. From Table III, it is evident that the local Nusselt number is increased with Prandtl number as well as Reynolds number.

IV. CONCLUSIONS

The boundary layer flow of a Casson nanofluid past a vertical exponentially stretching cylinder under the influence of a transverse magnetic field with internal heat generation/ absorption is analyzed. The model considers the effects of buoyancy ratio, Brownian motion, thermophoresis, Prandtl number, Lewis number, natural convection parameter, Magnetic field, spatial dependent and temperature dependent heat sources. Using similarity variables the resulting non-linear equations of momentum, energy and nanoparticle volume fraction are simplified. The effect of the physical parameters on the flow variables is presented graphically.

- ❖ The Casson fluid parameter and Hartmann number produce a reduction in the velocity and the boundary layer thickness.
- ❖ The influence of Hartmann number and Casson fluid parameter on temperature and nanoparticle volume fraction is to enhance temperature and concentration.
- ❖ Increasing values of the spatial dependent internal heat generation (A^*) as well as the temperature dependent internal heat generation (B^*) results in the enhancement of temperature.
- ❖ The natural convection parameter has an increasing influence on the velocity.
- ❖ The Brownian motion Nb and thermophoresis Nt has the effect of increasing the temperature.
- ❖ The Lewis number reduces the nanoparticle concentration.
- ❖ The Newtonian nanofluid has higher values of skin friction than the Casson nanofluid.

World Academy of Science, Engineering and Technology
International Journal of Mathematical and Computational Sciences
Vol:9, No:1, 2015

TABLE I
COMPARISON OF NUMERICAL VALUES FOR SKIN FRICTION COEFFICIENT AND LOCAL NUSSELT NUMBER IN THE ABSENCE OF M, A* AND B*

-f''(1)	Malik et al. [15]	Present study	Malik et al. [15]	Present study	-θ'(1)	Malik et al. [15]	Present Study	Malik et al. [15]	Present study
λ/β	0.1		0.3		Pr/Re	0.1		0.2	
0.2	0.8494	0.849336	1.1283	1.128340	1	0.2442	0.244205	0.3094	0.309400
0.4	0.7873	0.787383	0.9860	0.986953	3	0.3826	0.382587	0.5627	0.562756
0.6	0.7263	0.726417	0.8490	0.849073	5	0.5118	0.511778	0.7844	0.784403
0.8	0.6662	0.666156	0.7162	0.716398	7	0.6327	0.632698	0.9814	0.981388
1.0	0.6070	0.607086	0.5869	0.586522	15	1.0499	1.049939	0.6121	1.612127

TABLE II
SKIN FRICTION COEFFICIENT AT THE SURFACE FOR VARIOUS VALUES OF λ AND β

-f''(1)					
λ/β	0.1	0.3	0.5	0.7	1.0
0.2	0.846697	1.124540	1.275747	1.374674	1.473923
0.4	0.782383	0.980323	1.081149	1.144861	1.207223
0.6	0.719417	0.842373	0.899148	0.929006	0.958181
0.8	0.657656	0.709398	0.721072	0.723235	0.721521
1.0	0.596986	0.580522	0.551273	0.525290	0.494311

TABLE III
LOCAL NUSSELT NUMBER FOR VARIOUS VALUES OF PR AND RE

-θ'(1)					
Pr/Re	0.1	0.2	0.3	0.4	0.5
1	0.242757	0.306688	0.366193	0.421402	0.472030
3	0.381251	0.560433	0.714219	0.846834	0.962676
5	0.510540	0.782359	1.003225	1.187321	1.345462
7	0.631541	0.979550	1.255536	1.476314	1.668382
15	1.049014	1.610744	1.610744	2.374199	2.671646

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