Flow Transformation: An Investigation on Theoretical Aspects and Numerical Computation

Abhisek Sarkar, Abhimanyu Gaur

***Abstract*—**In this report we have discussed the theoretical aspects of the flow transformation, occurring through a series of bifurcations. The parameters and their continuous diversion, the intermittent bursts in the transition zone, variation of velocity and pressure with time, effect of roughness in turbulent zone, and changes in friction factor and head loss coefficient as a function of Reynolds number for a transverse flow across a cylinder have been discussed. An analysis of the variation in the wake length with Reynolds number was done in FORTRAN.

***Keywords*—**Attractor, Bifurcation, Energy cascade, Energy spectra, Intermittence, Vortex stretching.

I. Introduction

THE word flow, as described in the phrase flow transformation, can be defined as the set of curves of a dynamical system produced from all feasible initial conditions linked with a specific attractor [1]. In this document, we have taken into account the involvement of the flow prediction, computational method solutions, and the parameters, such as Reynolds number causing a discontinuous qualitative change in system behavior during the period when flow is transformed from laminar to turbulent via periodic and quasi-periodic states, bypassing a critical value. Others are drag coefficient, pressure gradient, viscous force, streamlines, dynamic density. The sequence of transitions (bifurcations) that a flow will undergo as the Reynolds number is increased to arrive at a frenzied state, are, namely, steady to periodic, periodic to quasi-periodic, quasi-periodic to turbulent have analysed theoretically.

II. Theory

A. Laminar Flow

In laminar flow, we visualize the fluid particles to move along parallel path in layers or laminae. The locus of individual fluid particles does not cross those of neighboring particles. Existing literature describes laminar flow to achieve only at low Reynolds number [2].

To define a low Reynolds number flow we have conceptualised a new parameter called 'angle parameter', denoted by θ in which θ is the angle between initial stream-direction (say horizontal) to streamline and it can be calculated at any point in flow field called 'instantaneous angle parameter'. It can be applied to both attached and detached (Wake region) flows. Taking the special case of a transverse flow across a cylinder, (considering external laminar flow), as fluid approaches body (freestream condition), θ will be zero, and at downstream when fluid will follow the contour of the body θ firstly will increase, becomes maximum 'θ_{max}' at a point and then decreases to zero again nearby the shoulders of cylinder (considering location of separation point at the shoulders). Each streamline is characterized by a particular value of 'θ_{max}'. Considering the upper half from the center streamline, this maximum value decreases as we move towards the upper streamlines. The concept of angle parameter can be validated wherever and whenever the streamlines are defined in the flow field. The above graph shows the value of θ_{max} for different streamlines considered in the flow field. Several streamlines were picked in serial order taken in the outward direction away from the body and results have been plotted for θ_{max}. It was found that for streamline no. 1 has maximum value of θ_{max} and as we move away its value decreases. Streamline no. 19 is the boundary streamline which does not feel the presence of obstacle in the flow field. Streamline no. 20 has θ_{max} equal to zero and beyond that streamlines have constant value of zero for θ_{max}.

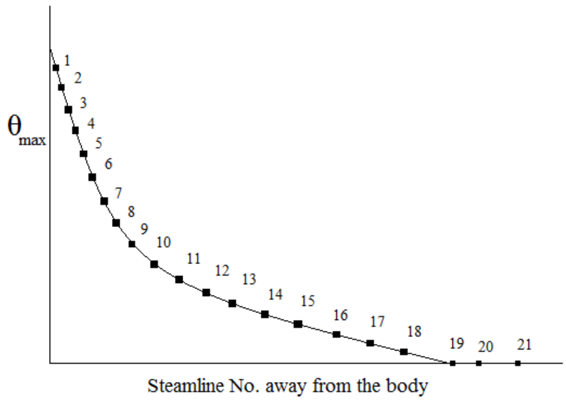

Fig. 1 Variation of θ_{max} for different streamlines in outward direction away from the body

B. Flow Stability

Flow stability depends on the growth and decay of the existing perturbations that a system contains/gains. As such, it can be expected that the occurrence of the transformation would depend on the intensity and structure of existing disturbances, and this has often been found to be the case. Linear stability theory can predict when a flow is unstable

Abhisek Sarkar is with the Department Of Mechanical Engineering, National Institute of Technology, Agartala, Tripura 799046, India (corresponding author to provide phone: +91- 8414054073; e-mail: abhiseks.nita@gmail.com).

Abhimanyu Gaur is with the Department of Mechanical Engineering, National Institute of Technology, Agartala, Tripura 799046, India (e-mail: gaur.abhimanyu_001@yahoo.com).

under infinitesimal disturbances. It thus gives no estimation about transition assisted by considerably large disturbances; this may happen when the theory can well indicate stability. It should point out the situation in which it will essentially transform under a given set of disturbances. The linear stability theory for a particular dynamical flow system starts with an initial approximate solution of the basic flow equations, which are then superimposed with the disturbances present. It is then decided if the disturbances are intensified or abated. We neglect all terms involving the square of its amplitude of the disturbances. This simplification through linearization limits the theory to infinitesimal disturbances. This linearization of the disturbances simplifies this theory to be applicable to many different forms of disturbances. Fourier analysis can be used to study any pattern of disturbance. The linearization makes various harmonics to be independent of each other. The theory for a single Fourier component considers a disturbance whose general form is directly proportional to exp(ik.r + σt) [3] where σ is a complex parameter, as $\sigma = \sigma_r + i\sigma_i$ where σ_r is the real part σ_i is the imaginary part. The sign of σ_r determines the intensification/ decay and thus transformation of the flow system. When σ_r is positive, the Fourier component under consideration gets amplified which will lead to the transformation of the initial flow. If σ_r is negative, the Fourier component (disturbance) vanishes eventually and the original flow gets sustained. Hence, if we can make σ_r negative for all values of k, the original flow will remain stable to all existing infinitesimal disturbances [3]. So, this makes a necessary condition for stability. If σ_r is positive for some values of k, the corresponding disturbance will be spontaneously intensified. Hence, this is a sufficient condition for instability. Another situation called over-stability will occur when $\sigma_i \neq 0$ where the disturbance amplifies sinusoidally with time. As such, if non-linear effects are taken into account, the resulting motion will be an oscillatory motion. The actual name is a misnomer, pointing that the system has diminishing disturbances over time while it is not. Such a situation is exhibited when the Prandtl number of the fluid flow is low enough and the system has a quality or component that can give rise to wave motions. In case of stratified fluid flows, both the mean velocity and mean density varies vertically. Velocity and density gradients perpendicular to the flow at a point are thus the parameters which govern the stability in such cases. In one hand, the velocity gradient can lead to generation of turbulence in the usual way through the action of inertia forces while the role of density gradient, on the other hand, is variable. If the density increases upwards, then buoyancy forces support and provide energy for the turbulence and hence called destabilizing form of density gradient. If the density decreases upwards, then turbulent work must be expended against buoyancy forces, which therefore produces a loss of turbulent energy; as such, turbulence cannot persist when the density gradient is too large and hence called stabilizing form of density gradient. Quantitatively, the extent of stability in such flows is determined by dimensionless Richardson number [3].

$$Ri = -\left[\frac{g\left(\frac{d\rho}{dz}\right)}{\rho\left(\frac{dU}{dz}\right)^2} \right]$$

As evident from its expression, the quantitative value of the stability depends on the sign of the density gradient but not on the velocity gradient. Negative Richardson number corresponds to a destabilizing density gradient; where both shear and buoyancy affect to generate turbulence. The relative dominance between these two competing factors depends on the value of this negative Ri. When – Ri is small, the former is dominant and the motion is essentially of the type we have considered in the foregoing discussion. When – Ri is large, the latter becomes more effective and the turbulence may be more like the free convection turbulence. Positive Ri corresponds to a stabilizing density gradient; turbulent motion cannot be sustained when Ri is positive and large. In such cases, the turbulence changes in a way that makes it relatively less efficient as a heat transfer mechanism then as a momentum transfer mechanism.

C. Transformation

Fluid behavior has been observed to change massively at moderate Re. The smooth, steady behavior switches to an erratic one. The primary and essential parameter on which this switchover depends is the Reynolds number. The fluctuations, typically ranging from 1 to 20% of the average velocity, are not strictly periodic but are random and encompass a continuous range of frequencies [3]. In a typical wind tunnel flow at high Re, the turbulent frequency ranges from 1 to 10,000Hz [3]. Though at higher Re, flow is unsteady, disordered and irregular but, but when properties are averaged overtime, become steady and predictable. The following plot is a general observation of the erratic behavior of fluid flow at appoint observed at intermediate Re. As shown, the average velocity fluctuates randomly at a very high frequency when the flow system undergoes transition.

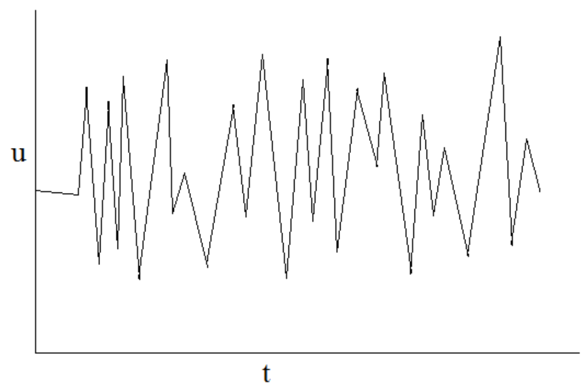

Fig. 2 Flow velocity as a function of time in the regime of viscous flow in transition at intermediate Reynolds number [1]

In case of pipe flows, the following approximate flow regimes have been observed to occur: 0<Re<2000: viscous laminar, 2000<Re<4000: laminar, Re>4000: turbulent flow regime. For open channel flows, the ranges are: $0<Re<500$: viscous laminar, $500<Re<1000$: transition, $Re>1000$ [4].

D. Intermittency and Related Effects

Turbulence is mostly found to occur in regions having high shear properties. Often, turbulent jets and wakes are enveloped by non-turbulent fluid. The interface between the turbulent and non- turbulent regions has been observed to be quite distinct and sharp in such cases. High detailing of it however has a highly irregular shape with bulges and indentations of various sizes. The bulges and indentations are carried downstream by the flow. The composition and details of the boundary also keep on varying; there is a varying time for the indentation and bulges to survive in their flow path. At a fixed point, we observe repeated and random transformation between turbulent and non-turbulent motion. The fraction of the time that the motion is turbulent at a certain point can be defined by the intermittency factor γ. At the center of the wake γ is 1; the motion is always turbulent. Outside the wake γ is 0; turbulent motion is never sustained there. But over a substantial fraction of the wake width, γ is greater than zero but less than 1. Another effect that is observed due to the flow of turbulent stream is that at the interface between turbulent and non-turbulent fluid, the turbulence growth takes place. Fluid in the turbulent region will a short time later engulf the fluid just in the non-turbulent zone. In particular, the entrainment is related to energy dissipation. Turbulent flows have much higher entrainment rates than the corresponding laminar flows, due to the additional dissipation by the velocity fluctuations. The process by which entrainment of new fluid so as to become turbulent is the spreading at the interface. Velocity fluctuations of turbulent motion are rotational in nature. If the flow in the non-turbulent region is irrotational, as is usually the case, the initially irrotational fluid can become rotational through the action of viscosity. Thus the spreading process essentially involves the action of viscosity and must be affected by the small eddies for which the viscosity is significant. The shape of the interface is, on the other hand, affected by eddies of all sizes. The largest bulges and indentations are produced by the large eddies. Even the non-turbulent flow outside the interface, involve velocity fluctuations due to the neighboring turbulent region. However, as opposed to the turbulent region the velocity fluctuations are purely irrotational and are dynamically quite different from turbulent fluctuations. The intensity of such fluctuations reduces rapidly with distance from the interface and thus value of intermittence factor also decreases rapidly as the fluctuations. Thus, from the foregoing discussion, it is revealed that the intermittency factor is most appropriately defined as the fraction of the time that vorticity fluctuations exist. The total turbulence spreading area of the interface is determined by the larger eddies.

E. Turbulence

Next comes the most complicated, non-intuitive, intriguing and important kind of fluid motion, known as turbulence. Turbulence was, and still remains as one of the great unsolved mysteries of science. The subject of turbulent flow is very deep, complicated and even though we make thorough studies, we are yet to call it precise. The basic characteristic of turbulence, and therefore our ability to predict its nature, is still an unsolved problem in classical physics. Pure theory of turbulent flow doesn't exist. The analysis of turbulent flows always requires empirical data in order to obtain a particular solution. Despite the broad occurrence of fluid flow, and the ubiquitous nature of turbulence, the "problem of turbulence" remains to this day the last unsolved problem of classical mathematical physics. The problem of turbulence has been studied by many of the greatest physicists and engineers since decades, and yet we do not comprehend in complete detail how or why turbulence occurs, nor can we predict turbulent behavior with any degree of reliability, even in very simple engineering flow situations. Thus, study of turbulence is motivated both by its inherent intellectual challenge and by the practical utility of a thorough understanding of its nature. Perhaps, the best brief encapsulation of it is a state of persistent instability. We can rather more easily introduce turbulence as: each time a flow modifies as the result of disturbance intensification, our predictability of the details of the motion gets reduced. The immediate effect of instability may not necessarily result in turbulent motion. Consider the Karman vortex street in the wake of an obstacle. The velocity varies periodically and roughly sinusoidal at a point in the street fixed relative to the obstacle. The phase of this variation is arbitrary, and depends on the small disturbances at the time the flow started. Thus, without making observations, the prediction of the instantaneous velocity within defined limits cannot be achieved. This lack of predictability arises due to the instability producing the vortex street; while in case of steady flow, such a prediction could be made in the developed vortex street. The degree of unpredictability though exists, is small but one can determine all the details of the flow by requiring only a single observation indicating the phase of the fluctuations. When we increase Reynolds number, a further instability causes loss of regularity in the series of vortices, and so the unpredictability is increased. One can, for example, no longer say that, if one has made an observation of the velocity, then the velocity is justified for one period later. However, other systematic features may still exist – regions of high vortices passing a point in a sequence, although not a completely periodic one. Hence, we instead go for describing systematic features rather than the fluctuating ones. The persistent instability keeps on reducing the systematic features till the random features get dominance. A flow may be called turbulent when due to persistent instability; the level of predictability gets so reduced that we need to describe the flow statistically. Turbulence may be seen to occur suddenly in the transition zone. For example, turbulent spots appearing in boundary layer transition. The difference arises through one stage of the sequence occurring on a small length scale; the time scale is correspondingly small and the developments leading to local randomness are rapid compared with other stages of transition.

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1. Statistical Description of Turbulent Motion:

A statistical description is formulated in terms of average quantities which are repeated from one experiment to another. Turbulent flow is achieved at higher Reynolds number due to fluctuation in various parameters. It is neither feasible nor desirable to consider in detail all of the small-scale fluctuations that occur in the turbulent flows. Also as turbulent fluctuations are random and varying with time so no deterministic approach is valid so certain properties or information averaged over time can be derived from statistical approach. We have to introduce averaging or smoothing operators, and attempt to describe only the averaged state of the flow system. Experimental results are not reproducible in detail as they are highly varied and dispersed. But statistical properties are reproducible and can be comprehensively understood e.g. energy cascade and correlation are well understood statistical phenomenon. For this reason, we have to introduce averaging or smoothing operators, and attempt to describe only the averaged state of the flow system, following the approach of "Reynolds Averaging."[5] The governing equations are essentially deterministic; deterministic implies predictable, at least for short times. Any behavior described by differential and/or algebraic systems possessing no random coefficients or forcing can be expected to be deterministic. Predictability may, in fact, be for only a short time. Indeed, deterministic chaos is of precisely this nature—predictable, but not for very long, as the systems are sensitive to initial conditions; sensitivity to initial conditions.

2. Turbulence Equations

In turbulent flow the velocity components of fluid fluctuates at any point in the flow field. The other parameters such as density and temperature also fluctuate if flow is incompressible and has non-uniform temperature distribution respectively. Since turbulence is the most complex phenomenon in fluid dynamics, we need to define certain parameters in order to understand it properly.

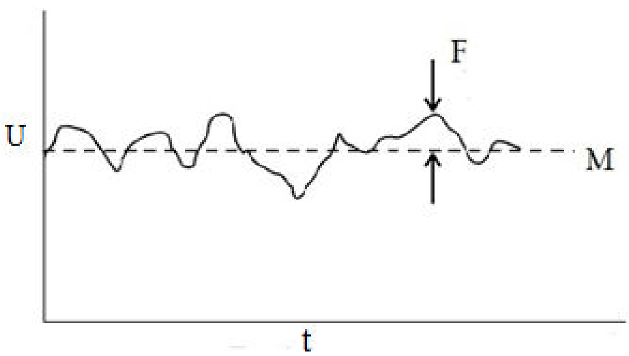

Fig. 3 Turbulent velocity as a function of time

In case of velocity two components are present; one is mean component of velocity and another one is the fluctuating components.

Let Mean Velocity component is designated by M, Fluctuating velocity Component by F, and total Velocity at a point be U. Then,

$$U = M + F.$$

From continuity equation we can write

$$\nabla(M + F) = 0 \quad (1)$$

Navier Stokes Equation of Differential form can be written as

$$\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial x} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \upsilon\frac{\partial^2 U}{\partial x^2} \quad (2)$$

On considering total velocity as component of both mean and fluctuating the differential form of Navier Stokes equation becomes

$$\frac{\partial(M+F)}{\partial t} + (M+F)\frac{\partial(M+F)}{\partial x} = -\frac{1}{\rho}\frac{\partial(p+dp)}{\partial x} + \upsilon\frac{\partial^2(M+F)}{\partial x^2} \quad (3)$$

Using Reynolds Averaged Navier-Stokes (RANS) Method fluctuating components of velocity can be manipulated as follows

$$\frac{\partial M}{\partial t} + M_i\frac{\partial M}{\partial x_i} + F_i\frac{\partial \overline{F}}{\partial x} = -\frac{1}{\rho}\frac{\partial p}{\partial x_i} + \upsilon\frac{\partial^2 M_i}{\partial x_i^2} \quad (4)$$

Since turbulent is always three dimensional, but in modelling we do not apply three dimensional boundary condition so that in effect, its effect in third direction (z direction) is uniform. Considering two dimensional components only, equations can be written as follows

$$\frac{\partial M}{\partial t} + M\frac{\partial M}{\partial x} + \upsilon\frac{\partial M}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \upsilon\frac{\partial^2 U}{\partial y^2} - \frac{\partial \overline{Fv}}{\partial y} \quad (5)$$

Now, taking the statistical average of above equation we get

$$\frac{\partial M}{\partial t} + M\frac{\partial M}{\partial x} + v\frac{\partial M}{\partial y} = -\frac{1}{\rho}\frac{\partial}{\partial y}(\mu\frac{\partial M}{\partial y} - \overline{\rho Fv}) \quad (6)$$

The term avg. (ρFv) referred as Reynolds Stress; this generally arises due to velocity collision at same point in the flow field. This value correlates the two velocity components. If its value is zero then two components becomes independent. In real it have non zero values. Since in any example either flat plate or flow through pipe, these velocity components are correlated and Reynolds stress plays a vital role in their relation.

3. Homogeneous Isotropic Turbulence: A Simplification

Till now, turbulence has mostly been treated by the concept of homogeneous, isotropic turbulence where the statistical properties are uniform spatially and directionally. The isotropic consideration is valid when rotational and buoyancy effects are neglected which otherwise restricts the vertical motion and there is also no mean flow. From the equation,

$$\frac{1}{2}\frac{\partial \overline{F}_i^2}{\partial t}+\frac{1}{2}M_j\frac{\partial \overline{F}_i^2}{\partial x_j}=-\overline{F_iF_j}\frac{\partial M_i}{\partial x_j}-\frac{1}{2}\frac{\partial \overline{F_i^2F_j}}{\partial x_j}-\frac{1}{\rho}\frac{\partial \overline{pF_i}}{\partial x_i}+\upsilon \overline{F_i\frac{\partial^2 F_i}{\partial x_j^2}} \quad (7)$$

we can see that the energy production term in equation is zero in isotropic turbulence and so the motion must decay through and that by viscous dissipation. We suppose that turbulence (assumed homogenous isotropic in many cases) is generated at an initial instant and then decays as time passes.

a. Energy Cascade

Experimental observations say that viscous dissipation occurring in turbulent flows is greater than in the corresponding laminar flow. Turbulent flows normally occur at high Reynolds number, and the viscous dissipation is associated with and brought about by small eddies. Hence, turbulence requires the development of local regions of high shear; that is the presence of small length scales. The small dissipative eddies must be generated from larger ones. This interpretation allows the development of a model of turbulence which has relevance not just too homogeneous isotropic turbulence but to most turbulent flows. Energy flown into the turbulence goes primarily into the larger eddies. From these, smaller eddies are generated, and then still smaller ones. The process continues until the scale of the vortices is small enough for viscous action to be important and dissipation to occur. This sequential transfer of energy is called the energy cascade. As we keep increasing the Reynolds number, the length of cascade becomes longer; i.e. there is a large difference in the sizes at its extreme ends. Then the direct interaction between the large eddies governing the energy transfer and the small dissipating eddies governing the energy dissipation becomes less and less pronounced. The dissipation is determined by the rate of supply of energy to the cascade by the large eddies and becomes independent of the dynamics of the small eddies in which the dissipation actually takes place. The rate of dissipation then becomes independent of the magnitude of the viscosity. Increasing the Reynolds number to a still higher value by decrease of viscosity keeping other variables constant only extends the cascade at the small eddy end. Still smaller eddies get generated before the dissipation can occur. All other aspects of the dynamics of the turbulence remain unaltered. Thus, there is no effect of variation in the viscosity on the turbulent dynamics at high Reynolds number.

b. Mechanism of Energy Cascade

The mechanism of energy cascade is a very complex one. It entails the interaction of the velocity field with itself. The interaction can be described by three processes. Firstly, the recurrent instability in which each stage may give rise to motions not only of greater complexity but also involving smaller scales than the previous stages. One stage may produce local regions of high shear that can themselves be unstable. Secondly, turbulence in these smaller scales may extract energy from larger scale motions. Thirdly, due to vortex stretching, the random nature of turbulent motion gives a diffusive action; two fluid particles that happen to be close together at some instant are likely to be very farther apart at any later time. The turbulence will have carried them over very different paths. This can be applied to two particles on the same vortex line. This produces intensification of the vorticity, but because of continuity the cross-section of the vortex tube also reduces. There is thus also an intensification of the motion on a smaller scale; that is a transfer of energy to smaller eddies. The distance up to which this hypothesized turbulent eddy can retain its identity due to the intensification of the vorticity is called the mixing length. This intensification at a region however depends on the value of the vorticity already present; places where existing vorticity happens to be large intensifies the vorticity greater than where vorticty is weak.

F. Turbulence Modeling

It is necessary to accept a model to include the effects of turbulence in analysis of turbulence in real life applications. The Baldwin-Lomax model is one such widely accepted model which has been observed to give reasonable results for a wide range of computational fluid dynamic analyses in a turbulent flow system. Each trial of computation made contains many empirical relations. The molecular and turbulent values are both taken into consideration each for viscosity and thermal conductivity while using this model to compute eddy viscosity as a function of local boundary layer velocity profile. This model is more suitable for high speed flows with thin attached boundary layers, typically found in aerospace and turbo-machinery applications. It has good agreement with the experiments for attached flows or near the wall. But it is found to be unsuitable for cases with broadly separated regions [6].

G. Reverse Transition

Reverse transition also called relaminarisation is the process of turbulent flow switching over back to the laminar flow. The laminar properties start to appear while turbulent properties start vanishing. It can take place when the viscous dissipation starts increasing as a result of which turbulent energy goes down. Further in case of wall flows, the inhibition of formation of large eddies emanated from laminar/viscous sub layer may also reduce gradually the turbulent energy. While in case of boundary layers formed in region of favorable pressure gradient, it is the region of intermittent turbulence that becomes so large that it extends to the wall. Examples include boundary layer entering a region of stable stratified zone, boundary layers that enter a region of strongly favorable pressure gradient; pipe and channel flow in which the Reynolds number is reduced either by a change of geometry; by increase of viscosity is increased and/or by decrease of density due to heat transfer. Though reverse transition usually leading to a decrease in the turbulent energy follows effects with which viscous dissipation starts increasing leading to a reduction in the turbulent energy but this transformation possesses its own inherent mechanism. The change in velocity becomes less correlated with this transformation as turbulence decays itself in a way so as to convert original turbulent structures faster into laminar ones.

III. Key Points

1. In laminar flow, the fluid moves slowly in layers, without much mixing among the layers. It typically occurs when the velocity is low or the fluid is very viscous. Whereas, opposite of laminar, in turbulent flow, considerable mixing occurs, Reynolds number is higher than a critical value.
2. Laminar and turbulent flows can be characterized and quantified using dimensionless Reynolds Number established by Osborne Reynolds and is given as –

$$Re = \rho VL/\eta = VL/\nu$$

where ρ is fluid density, V is fluid velocity, L is characteristic length, η is dynamic viscosity, ν is kinematic viscosity. More viscous fluid (lower Reynolds number) will tend to exhibit laminar flow characteristics for a given flow velocity.

3. Although the final result of turbulent mixing is the same as that of diffusive mixing, the physical mechanisms are very different. Intermixing due to turbulence arises due to dominance of macroscopic transport over molecular diffusion effects. Diffusive mixing involves predominantly molecular transport [7].
4. If ν is small, advective, nonlinear behavior becomes dominant, and this happens in a turbulent flow. While if ν is relatively large molecular diffusion will be dominant and the flow will be laminar.
5. The friction losses in pipes depend on whether the flow is laminar or turbulent. The head loss in pipes is directly proportional to the friction factor, f. For laminar flows, f is inversely proportional to Reynolds number (f= 64/Re) where Re is Reynolds number. For turbulent flows, Swami and Jain estimated f within 1% of error given by-

$$f = \frac{0.25}{\left[\log\left(\frac{\varepsilon}{3.7D}\right) + \frac{5.74}{Re^{0.9}}\right]^2}$$

where ε denotes the average height of the surface projections on the inside of pipe and D is the diameter of pipe. In the laminar zone – f decreases as Re increases. In transition zone, we can't predict the value of f, as it shows intermittent and fluctuating values.

6. In case of pipe flows, beyond 4000, for a given Re, as the relative roughness term ε/D increases (less rough), friction factor decreases. For a given relative roughness, friction factor decreases with increasing Reynolds number till the zone of complete turbulence. Within the zone of complete turbulence, Reynolds number has no effect. As relative roughness increases (less rough), the boundary of the zone of complete turbulence shifts (increases). If we know the value of relative roughness, Reynolds number, we can compute the friction factor from the Moody diagram.
7. The surface roughness has an effect on friction resistance; for laminar flow, however, this effect is negligible. The turbulent flow is strongly affected by roughness, as viscosity is very powerful near contour. Nikuradse [7] simulated roughness by gluing grinded sand grains on to the inner walls of the pipes. He then calculated the pressure drops and flow rates computationally and correlated friction factor versus Reynolds number.
8. The laminar friction is unaffected, and turbulent friction after an onset point, increases monotonically with the roughness ratio, ϵ/d, friction factor becomes constant (fully rough) at high Reynolds numbers, fluid gets highly energetic and its capability of energy transfer also gets high [8].
9. For flow over a body, if $(dV/dy)_{y=0}$ denotes the velocity gradient in cross-stream direction at the wall, $(dV/dy)_{y=0}$ for laminar flow is less than $(dV/dy)_{y=0}$ for turbulent flow. Hence, at wall, laminar shear stress is less than turbulent shear stress [9].
10. Fluid/Aerodynamic heating in laminar flow is less than that in turbulent flow. So, at wall, temperature gradient for laminar flow is less than that for turbulent flow [10].
11. Angle parameter as described in this article is very much effective in flow analysis and visualization in external flow past a blunt body.

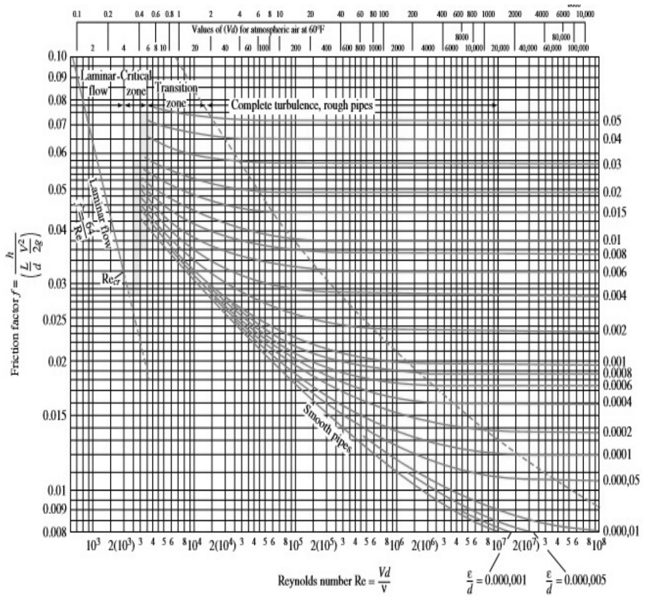

Fig. 4 Variation of Reynolds number with friction factor [9]

IV. Experimental Results

Through our experimental results, we have noticed that for each Reynolds number there is a unique shape of vortex. Ultimately, we have analyzed the variation in wake length with change in Reynolds number computationally. Within the experimental range of Reynolds number, we obtained almost linear relationship between the two variables. The data are tabulated in Table I and plotted in Fig. 5. In creeping flow keeping very low Reynolds number as shown in the table, approaching fluid will follow the body contour; vortices will not form and hence almost no wake, or wake length will be zero. These results are also well established in the existing literature [11]. On increasing Reynolds number the flow will still be laminar, but we can see that at a Reynolds number 10 there is a formation of wake and further it varies linearly with wake length. On further increase in Reynolds number wake length increases and when flow becomes turbulent (in continuous turbulent), Re= 50000, the flow gets separated and we can't tell about wake length.

TABLE I
VARIATION IN WAKE LENGTH WITH REYNOLDS NUMBER

S.No	REYNOLDS NO.	Wake Length
1	1	0
2	2	0
3	3	0
4	4	0
5	5	0
6	10	1.286802
7	15	1.958898
8	20	2.634998
9	25	3.310703
10	30	3.985503
11	35	4.661173
12	40	5.335324

The results can be graphically plotted as shown below.

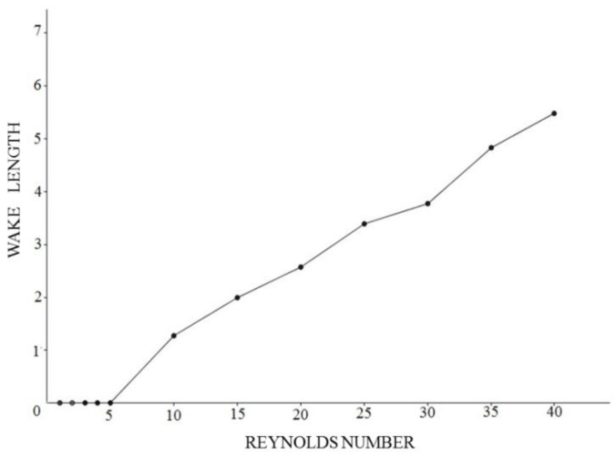

Fig. 5 Variation of Reynolds number with wake length

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