

# Model Order Reduction for Frequency Response and Effect of Order of Method for Matching Condition

Aref Ghafouri, Mohammad Javad Mollakazemi, Farhad Asadi

**Abstract**—In this paper, model order reduction method is used for approximation in linear and nonlinearity aspects in some experimental data. This method can be used for obtaining offline reduced model for approximation of experimental data and can produce and follow the data and order of system and also it can match to experimental data in some frequency ratios. In this study, the method is compared in different experimental data and influence of choosing of order of the model reduction for obtaining the best and sufficient matching condition for following the data is investigated in format of imaginary and reality part of the frequency response curve and finally the effect and important parameter of number of order reduction in nonlinear experimental data is explained further.

**Keywords**—Frequency response, Order of model reduction, frequency matching condition.

## I. INTRODUCTION

MODEL order reduction is a powerful method in science especially in engineering. This method helped the designers for increasing the speed of design and also for modeling of time dependent and time independent dynamical systems and also high dimensional experimental data. This method can be structured in such a form that constraints in modeling and control of system or other characteristics of system such as stability conditions is obtained.

This method is based on decomposition of a given system into a number of subsystems that has general features of main system. The idea for decomposition was first explained and obtained by [1] and totally there are 2 methods for decomposition and construction of this method. These methods are coupled and decoupled approach. In the coupled approach we tradeoff between problems structure and fast and efficient computational speed but in decoupled method, the system categorized to some subsystems and each subsystem is solved independently and also the duration of computation is adjusted with decoupling parameter.

In other clear words and different view a majority of nonlinear model reduction approaches can be considered as a two-step overall procedure as follows. First, the state order and dimension of system is decreased through some functions, and then the phase space of the reduced state is approximated

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and obtained. For investigation in error boundary we can obtain and analyze it in techniques of linear robust method. For example, the small gain theorem can be used for analyzing the bounded and classification uncertainty in system. Also, this formulation is robust with noisy measurement in our data and nowadays the input dimensionality of system is increased and the model reduction have very important tool to overcome this situation [2], [3].

Furthermore, this method can be developed either by statistical performance index [4], [5] or on moment matching conditions [6], [7]. References [8], [9] directly formulate the model reduction problem as a minimizing some norm of system such as H2 and with solving and formulization in nonlinear least squares we can obtain it. In fact those papers solving linear least square iteratively and proving the constraints in system such as passivity can be checked during the computation. Reference [10] uses a different result derived from to check passivity.

Model order reduction approach does not necessarily generate optimal model reduction because both the system and the frequency points should be considered as important factors. Totally, the whole methods want obtain desirable properties such as stability, optimality or others important features in dynamical system.

In this paper, we used rational- approximation-based model reduction framework and also instead of solving the model reduction directly, we solve it with relaxation method. In this paper, these methods are studied for different model reduction problems. The rest of this paper is organized as follows. First, we provided some background and method. Complete step by step of this algorithm is explained in many references and we don't want to describe the whole algorithm here [11], [12]. Then we demonstrate how to modify and choose the order of model reduction and important effect on matching condition in experimental data is investigated and finally practical experimental data for obtaining the limitation of method and need to increase of number of order for better matching at some frequency response is explained.

## II. IMPORTANT NOTE OF ALGORITHM IN THIS APPROACH

In this method, choosing the center of frequency or frequency division of experimental data is important. Although, the choice of this center is arbitrary but it is better that we employ an automatic procedure that chooses the center frequency of data by applying the minimization the maximum slope of the magnitude of the frequency response.

Another important notation in this algorithm is that for some applications, it is desirable that the reduced model

transfer function has positive real part. For doing this constraint, it is sufficient to note the real part of the transfer function because this section is important for analyzing this constraint. It should be said that we can add this constraint in our iterative algorithm, and this statement can check the passivity of constraint too.

As last features of this method we want to express the matching of frequency samples that the program should be able to modify them so that the reduced transfer function matches exactly to original transfer function at desired frequencies. To meet this end, this equality constraints such as (1) is applied [11], [12]:

$$H(e^{j\omega_k})\tilde{a}(\omega_k) - \tilde{b}(\omega_k) - j\tilde{c}(\omega_k) = 0 \quad \forall k \quad (1)$$

Besides the suggestions about the exact matching frequency which are available in references, this modification has the practical meaning of reducing the number of states of the system in algorithm and hence makes better performance of method and reducing the runtime of computational of method significantly [13]-[15].

### III. SIMULATION RESULTS FOR DIFFERENT EXPERIMENTAL DATA

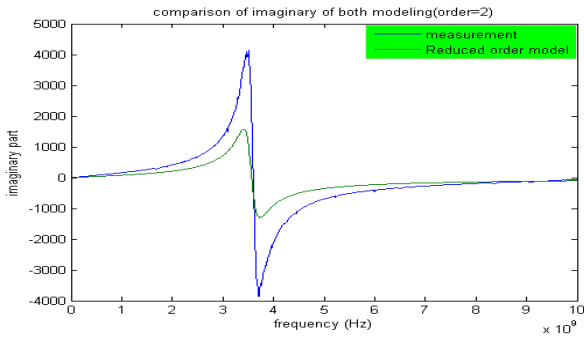


Fig. 1 Comparison of imaginary part of model and data

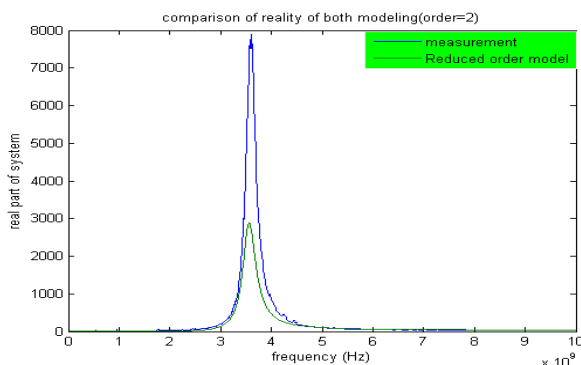


Fig. 2 Comparison of reality part of model and data

In this section we construct a reduced model from measured frequency response of arbitrary suspension system of car. In the first simulation experimental data with reduced model approximation for it on both imaginary and reality part are plotted in Figs. 1 and 2. In these examples the order of the reduced model is two, and the overall behavior of response is

similar in two methods but with this reduced model approximation we cannot obtain and approximate the main amplitude of frequency response. So with this important proof to this model we find that the order of model is higher because of behavior of first model reduction is correct. Then, we try another modeling with increasing the number of order but with evaluation of our previous model we increase it with one degree. Then, the frequency response of experimental data with our model reduction is plotted in Figs. 3 and 4. Now, it is evident the matching between the experimental data and model is better in some ranges of frequencies.

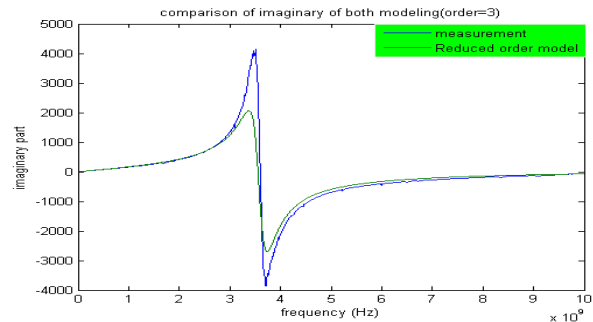


Fig. 3 Comparison of imaginary part of model and data

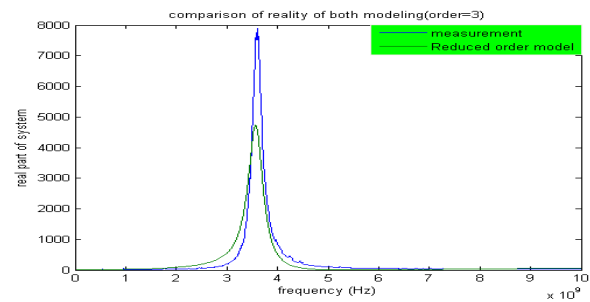


Fig. 4 Comparison of reality part of model and data

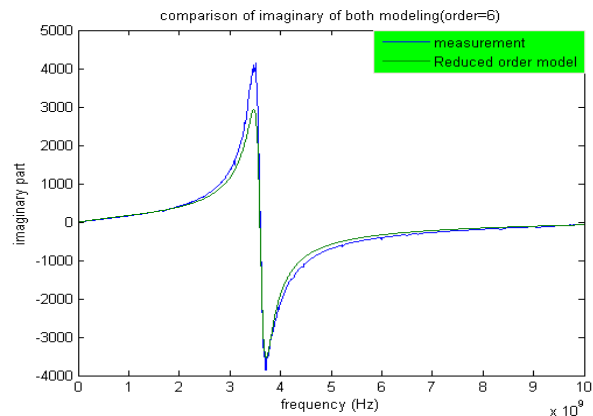


Fig. 5 Comparison of imaginary part of model and data

But in reality part behavior of model is out of the region of frequency response of data so we understand that with adding the order of model this behavior is becoming narrower and then the model reduction with order 6 are plotted in Figs. 5 and 6. This model with ignoring the peak amplitude in

frequency response diagram is best order for experimental data.

#### IV. SECOND EXPERIMENTAL RESULT AND MATCHING OF MODEL REDUCTION

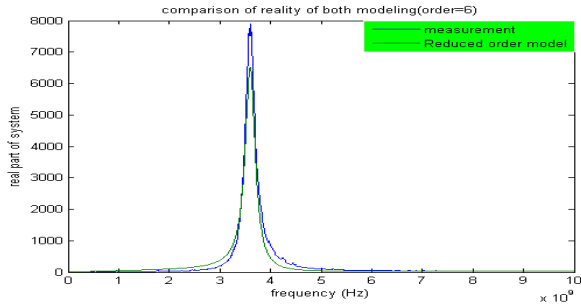


Fig. 6 Comparison of reality part of model and data

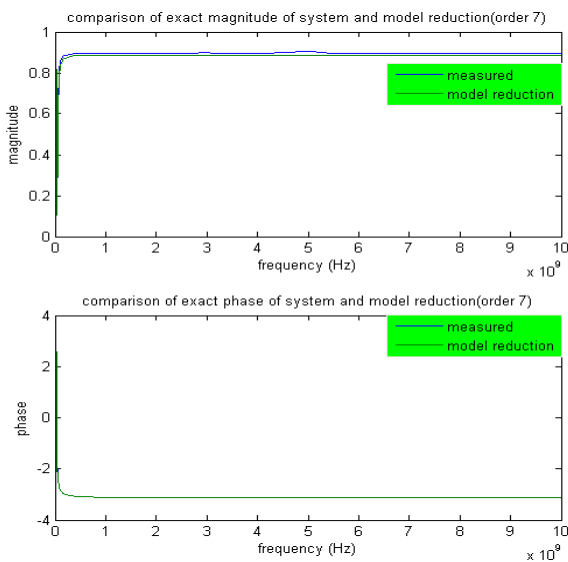


Fig. 7 Comparison of magnitude of system and model reduction (order 7)

In this section we want compare two different series of experimental data with model reduction method. In the first series data we construct our model with order  $n=7$  as is shown in Fig. 7 in which we showed the reality and phase of both model and data. Then, we construct our model with  $n=9$  and exact phase and reality part of system with model are illustrated in Fig. 8. The difference between these two models with two different orders is evident. By increasing the order of magnitude, the matching is getting to oscillate but in narrow band of amplitude but phase of the system is jumped at 5 Hz frequency that is not exist in model with  $n=7$ . So for the best trade off and choosing the exact order of system we should check the system with more order model and compare the accuracy between data and model accurately but it is evident from figures in this simulation that the order  $n=9$  is more exact than  $n=7$ . Finally, we checked this algorithm for reality part of some nonlinear frequency response that is plotted in Fig. 9 and we modeled the system with  $n=10$  as is shown in Fig. 10 with  $n=8$  and at last in Fig. 11 with  $n=15$ . In summary, in situation

with facing more difficult frequency response we should check the different orders of model reduction to decide for choosing the optimal order for modeling because in Fig. 10 with  $n=10$  the best matching is obtained but in other figures depend to frequency the rapid change in our model exists because of lower or upper of model order and its effect on nonlinearity of system. This rapid change makes the behavior instable. Finally, we understood that this method although is important approach for reducing the dimensional of data but it has some limitations and in design of algorithm or in matching the model we should notice so much to the order of model and matching the response in each frequency response.

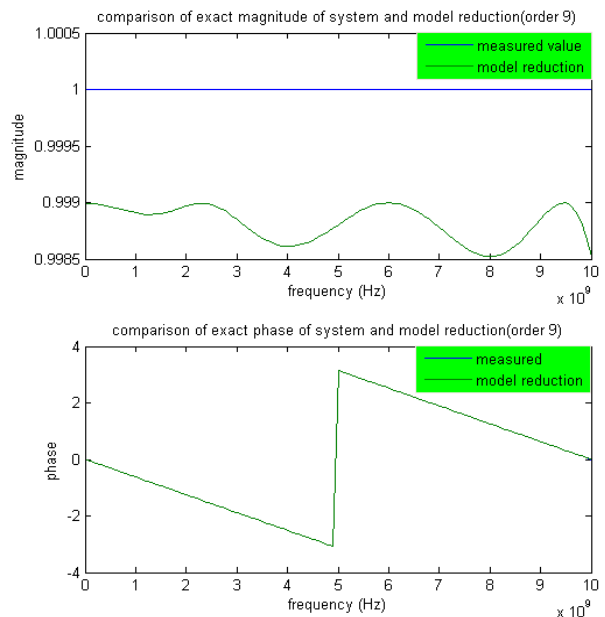


Fig. 8 Comparison of magnitude of system and model reduction (order 9)

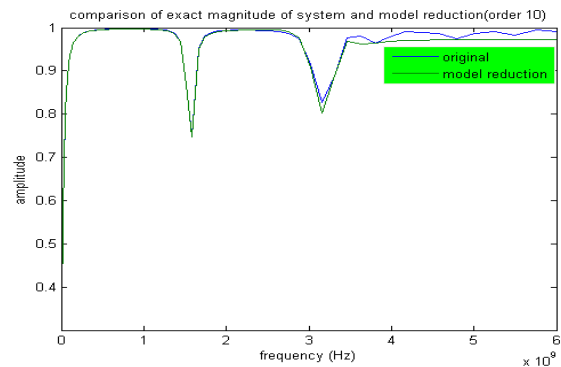


Fig. 9 Comparison of magnitude of system and model reduction (order 10)

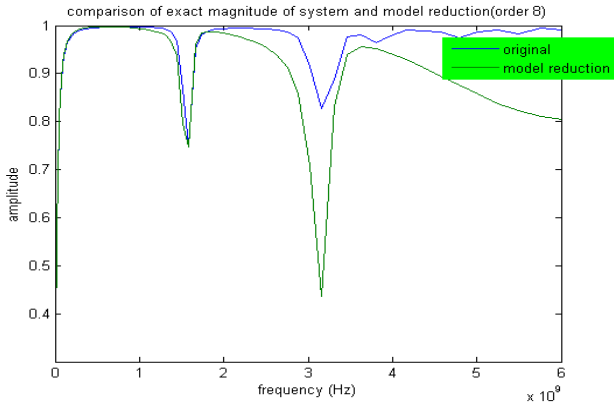


Fig. 10 Comparison of magnitude of system and model reduction (order 8)

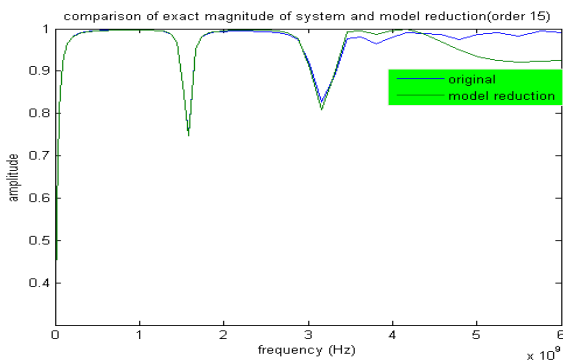


Fig. 11 Comparison of magnitude of system and model reduction (order 15)

#### V. CONCLUSION

In this paper, the model reduction has been demonstrated through different experimental data. The proposed approach can handle stability, and matching in complex frequency response. It has been extended to solve the parameterized model reduction problem as well. Theoretical of this method is small gain theorem which leads to a theoretical statement, as well as a numerical procedure describing the error bound. While the structure is simple, it has the potential to model important nonlinear experimental data.

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