

# Statistical Description of Counterpoise Effective Length Based On Regressive Formulas

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*Abstract*—This paper presents a novel statistical description of the counterpoise effective length due to lightning surges, where the (impulse) effective length had been obtained by means of regressive formulas applied to the transient simulation results. The effective length is described in terms of a statistical distribution function, from which median, mean, variance, and other parameters of interest could be readily obtained. The influence of lightning current amplitude, lightning front duration, and soil resistivity on the effective length has been accounted for, assuming statistical nature of these parameters. A method for determining the optimal counterpoise length, in terms of the statistical impulse effective length, is also presented. It is based on estimating the number of dangerous events associated with lightning strikes. Proposed statistical description and the associated method provide valuable information which could aid the design engineer in optimising physical lengths of counterpoises in different grounding arrangements and soil resistivity situations.

*Keywords*—Counterpoise, Grounding conductor, Effective length, Lightning, Monte Carlo method, Statistical distribution.

## I. INTRODUCTION

COUNTERPOISE wire is seen as an efficient and cost effective mean of creating extended grounding systems for high voltage transmission line towers [1], [2] and GSM base station towers, as well as for improving impulse response of wind turbine grounding systems [3], [4], particularly in relation to lightning-surge transients. It is basically a grounding conductor of some length, buried at some depth, and attached to the base of the tower, or to the already present (e.g. ring-type) grounding system.

Long grounding conductor (i.e. counterpoise wire) has an effective length under impulse currents, e.g. [5]–[11], which means that during lightning current dissipation (following any particular lightning strike to the associated object), depending on the conductor's actual length, only a certain portion of the conductor length participates in carrying and dissipating this current. The rest of the conductor length is buried in the ground without any effect and, in-fact, represents a needless waste of the conductor material, increasing the total costs associated with a construction of the grounding system (the costs of digging trenches for laying counterpoise wires of some length in high-resistivity soil can be quite high). In other words, when the physical length of a grounding conductor exceeds the effective length (which is imaginary), the grounding conductor will not be utilised effectively. The portion of the grounding conductor's actual length which participates in dissipating the lightning current depends on the particular lightning current

amplitude and front duration, soil resistivity at the location of the grounding system, counterpoise geometry (e.g. single conductor, four-star configuration, etc.), and injection point of the lightning current, e.g. [12, Ch. 8]. This means that the impulse effective length of the grounding conductor (i.e. counterpoise effective length), for a particular geometry and injection point, is in functional relationship with these three aforementioned parameters, viz. it can be stated that

$$\ell_e = f(\rho, I, t_f) \text{ in (m)} \quad (1)$$

where  $\rho$  is the average soil resistivity at the location of the grounding system ( $\Omega m$ ),  $I$  is the lightning-current amplitude (kA) and  $t_f$  is the lightning-current front duration ( $\mu s$ ).

It has been confirmed by several different researchers, e.g. see [12, Ch. 8] as well as [5]–[7], that the impulse effective length is small in the low-resistivity soils and increases with the increase of the soil resistivity in a non-linear manner. It also increases with the increase of the lightning current amplitude associated with the lightning strike to the object at hand, again in the non-linear manner. However, impulse effective length decreases with the increased steepness of the lightning-current front, i.e., for short lightning wave-front duration lightning strikes impulse effective length is short and increases with the increase in the wave-front duration, again in the non-linear manner. Considering all these facts, design engineer is confronted with the problem of choosing the “right” lightning-current parameters for introducing in (1), which is in-fact a pointless task considering the stochastic nature of lightning current.

The existence of the effective length, in the first place, is due to complex interplay which exist between soil parameters and frequency-dependent grounding conductor parameters, during dissipation of lightning currents, including effects of the soil ionization. Namely, the inductive effect of the grounding conductor is notable for high frequency of the impulse current, which in-turn causes a rapid transient voltage drop along the conductor length, thus obstructing the flow of current toward the distant end of the conductor. Hence, if the grounding conductor is physically long, the current leaking from its distant end is severely limited and only the first portion (up to some length) effectively contributes to the dissipation of the current into the soil.

An exact mathematical definition of the effective length varies between different researchers, further complicating the matter. Gupta and Thapar in [5] define it, somewhat arbitrarily, as a length of the grounding electrode in which the voltage wave at the terminal end of the electrode has little effect on the head end. Grcev in [6], on the other

hand, defines the effective length in terms of the impulse coefficient (a dimensionless coefficient obtained from the quotient of the grounding conductors impulse impedance and the low-frequency resistance) as a length of the conductor up to the point in which the impulse coefficient is equal to unity. He *et al.* in [7], and some other researchers, define the effective length of the grounding conductor as the length of the conductor up to the point in which the derivative of the impulse grounding resistance (i.e. impulse impedance) is smaller than a certain value, viz.  $-dR_i/dl \leq tg\alpha$ , where  $R_i$  is the grounding conductor impulse impedance and  $\alpha$  is the steepness of the impulse impedance curve at the point of the effective length definition; see [12, Ch. 8] for more information. This last definition provides a basis for the effective length regressive formulas utilised in this paper.

Up to now, various authors provided graphical depictions of effective length as a function of soil resistivity for different fixed values of lightning-current amplitude, or for different fixed values of lightning-current front duration, e.g. [6], [7]. However, considering the fact that the lightning-current parameters (i.e. amplitude and front duration) are stochastic in nature, and in view of the non-linear relationship provided by (1), it can be advantageous and beneficial to the design engineer, if one could describe the effective length in statistical terms. This has not been done so far—as far as the authors are informed.

Hence, this paper will provide a statistical distribution fit—by means of the Maximum Likelihood Estimation of its parameters—to the counterpoise effective length data generated from the regressive formulas, which have been obtained elsewhere from the extensive numerical analyses and simulation tests of counterpoise impulse behaviour. The parameters of this statistical distribution (mean, median, variance, quartiles, etc.) provide insight and offer valuable aid to the designer in optimising physical counterpoise lengths—in terms of the needed effective length—for different grounding arrangements and soil resistivity situations. Moreover, statistical depiction of the counterpoise effective length will be tied to the keraunic level and the electric shadow area of the object for which the counterpoise is being designed, yielding a method for selecting (in statistical terms) optimal impulse effective length.

## II. LIGHTNING CURRENT STATISTICAL PARAMETERS

Lightning current is fully described, along with the polarity, with the following three parameters: (1) amplitude, (2) front duration, and (3) tail duration. However, due to the fact that lightning is stochastic in nature, above-mentioned parameters can only be described in terms of statistical distributions. In power system studies involving lightning strikes mainly negative downward lightning strikes are of engineering interest, due to the fact that they constitute around 90 % of all lightning strikes to power system installations. Hence, only parameters of this lightning current type will be presented hereafter. These parameters are provided, among others, in the following publications [13]–[16].

The lightning-current parameters (amplitude, front, and tail duration) individually follow a log-normal distribution. The

probability density function (PDF) of the random variable  $x$ , signifying any of the lightning-current parameters individually, can be depicted by the following expression [14]:

$$f(x) = \frac{\exp\left[-\frac{(\ln x - \ln \mu)^2}{2\sigma_{\ln x}^2}\right]}{\sqrt{2\pi x}\sigma_{\ln x}} \quad (2)$$

where  $\mu$  represents the median value and  $\sigma_{\ln x}$  represents the associated standard deviation of the  $\ln x$ . Each of the three lightning-current parameters can be individually depicted by a log-normal distribution, featuring appropriate parameters (median value and standard deviation). However, situation is complicated by the fact that there is a statistical correlation between measurements data depicting lightning current amplitudes and front durations. This necessitates usage of the joint (i.e. bivariate), as well as conditional, probability distributions in their representation [14]. The joint (i.e. bivariate) probability density function, in case of the lightning current amplitude ( $I$ ) and front duration ( $t_f$ ), can be described by the following equation [14]:

$$f(I, t_f) = \frac{\exp\left[-\frac{f_1 - f_2 + f_3}{2 \cdot (1 - \rho_c^2)}\right]}{2\pi \cdot I \cdot t_f \cdot \sigma_{\ln I} \sigma_{\ln t_f} \cdot \sqrt{1 - \rho_c^2}} \quad (3)$$

with

$$f_1 = \left(\frac{\ln I - \ln I_\mu}{\sigma_{\ln I}}\right)^2 \quad (4)$$

$$f_2 = 2\rho_c \cdot \left(\frac{\ln I - \ln I_\mu}{\sigma_{\ln I}}\right) \cdot \left(\frac{\ln t_f - \ln t_{f\mu}}{\sigma_{\ln t_f}}\right) \quad (5)$$

$$f_3 = \left(\frac{\ln t_f - \ln t_{f\mu}}{\sigma_{\ln t_f}}\right)^2 \quad (6)$$

where  $I_\mu$ ,  $\sigma_{\ln I}$  represent median value and standard deviation of the lightning current amplitudes,  $t_{f\mu}$ ,  $\sigma_{\ln t_f}$  represent median value and standard deviation of the lightning current front durations, and  $\rho_c$  is the coefficient of correlation between the lightning current amplitudes and front durations. If the statistical variables are independently distributed, which is the case with the lightning current amplitude ( $I$ ) and tail duration ( $t_h$ ), then associated  $\rho_c = 0$  and (3) reduces to

$$f(I, t_h) = f(I) \cdot f(t_h) \quad (7)$$

with  $f(I)$  and  $f(t_h)$  obtained from (2) by introducing relevant median values and standard deviations.

Following parameters for the statistical distribution of (negative downward) lightning current parameters will be utilised (hereafter termed the original set):  $I_\mu = 31.1$  (kA),  $\sigma_{\ln I} = 0.484$ ;  $t_{f\mu} = 3.83$  ( $\mu$ s),  $\sigma_{\ln t_f} = 0.55$ ;  $\rho_c(I, t_f) = 0.47$ , as recommended in [14]. As an alternative, following lightning-current parameters are provided as well (hereafter termed the alternative set):  $I_\mu = 30.1$  (kA),  $\sigma_{\ln I} = 0.76$ ;  $t_{f\mu} = 2.0$  ( $\mu$ s),  $\sigma_{\ln t_f} = 0.494$ ;  $\rho_c(I, t_f) = 0.5$ . This is in order to account for the fact that there are differences between lightning-current parameters provided by different researchers, [14]–[16].

### III. REGRESSIVE FORMULAS FOR COUNTERPOISE EFFECTIVE LENGTH COMPUTATION

Different authors provided different regressive formulas for the counterpoise effective length computation, e.g. [5]–[7], [17], usually derived from the extensive numerical analyses and simulation test results of counterpoise impulse behaviour, where some of those neglected to explicitly provide for the influence of the lightning current amplitudes. Validity of the formulas have been tested against simulation data (and occasionally against measurements data) with various degrees of success. In general, quality of the regressive formula depends on the level of sophistication of the underlying numerical model on which it is based.

The formulas are, in the most general case, of the following type [12, Ch. 8]:

$$\ell_e = A \cdot (\rho \cdot t_f)^\alpha \cdot I^\beta \quad (8)$$

with  $A, \alpha, \beta$  being coefficients determined by means of the regression analysis carried-out on the extensive results obtained from applying complex numerical models of counterpoise transient behaviour due to lightning surges. If  $\beta = 0$  in (8), the formula reduces to the well-known Gupta and Thapar expression

$$\ell_e = A \cdot (\rho \cdot t_f)^{0.5} \quad (9)$$

with  $A$  being a geometry-dependent coefficient and  $t_f$  a lightning-current front duration, [5].

For the purpose of this paper, a sophisticated regressive formulas of the type (8), derived by He *et al.* in [7], will be utilised. The regressive formulas for the counterpoise effective length will be provided for three different geometry and injection point configurations, as follows:

- Type A: single conductor with end-point current injection,
- Type B: single conductor with middle-point current injection,
- Type C: four-arm star configuration with centre-point current injection.

In the case of Type C configuration, effective length is given for a single arm of the star. Interested reader is at this point advised to consult [12, Ch. 8] for more information on the subject. In addition to that, grounding conductors, featuring same geometry, could be treated with low-resistivity material (LRM), as presented in [17].

From the analysis presented in [7], [12], [17], a unique set of parameters for the formula (8) has been found, for all of the above mentioned geometries and injection point counterpoise configurations (including LRM treatment). Table I conveniently presents these parameters. They are valid for the soil resistivity  $\rho \in [100 - 3000]$  ( $\Omega m$ ) and conventional burial depths usually found in grounding systems design.

By introducing these parameters in (8), and by using the statistical treatment of lightning-current parameters, one can build a statistical depiction of the impulse effective length for different soil resistivity and different counterpoise configurations, which is seen as the principal contribution of this paper.

TABLE I  
 PARAMETERS FOR THE COUNTERPOISE EFFECTIVE LENGTH COMPUTATION FORMULA

Counterpoise configuration	$A$	$\alpha$	$\beta$
Type A	6.528	0.379	-0.097
Type B	7.683	0.379	-0.097
Type C	8.963	0.379	-0.097
Type A (LRM)	5.222	0.379	-0.097
Type B (LRM)	6.531	0.379	-0.097
Type C (LRM)	8.067	0.379	-0.097

### IV. STATISTICAL DESCRIPTION OF COUNTERPOISE EFFECTIVE LENGTH

Statistical description of the counterpoise effective length implies finding its appropriate statistical distribution—for different soil resistivity and different geometries and injection point counterpoise configurations. Computational procedure starts by drawing a large number of random lightning current amplitudes and front durations (forming two random variates) from the associated bivariate log-normal distribution (which includes statistical correlation between them). This is accomplished by means of transforming variates drawn from the standardised bivariate normal distribution [18, Ch. 4]. Namely, if the two-dimensional statistical variable  $\mathbf{Y} = [Y_1, Y_2]^T$  is drawn from the standardised bivariate normal distribution  $\mathbf{Y} \sim N(\boldsymbol{\mu}, \boldsymbol{\Sigma})$  where  $\boldsymbol{\mu} = [0, 0]^T$  is the mean vector and  $\boldsymbol{\Sigma}$  is the variance-covariance matrix

$$\boldsymbol{\Sigma} = \begin{bmatrix} 1 & \rho_c \\ \rho_c & 1 \end{bmatrix} \quad (10)$$

then the associated two-dimensional statistical variable from the bivariate log-normal distribution (denoting a lightning current amplitude and a front duration) can be determined as follows

$$\mathbf{X} = \begin{bmatrix} I_\mu \cdot \exp(\sigma_{\ln I} \cdot Y_1) \\ t_{f\mu} \cdot \exp(\sigma_{\ln t_f} \cdot Y_2) \end{bmatrix} \quad (11)$$

where  $I_\mu, t_{f\mu}$  are median values of lightning current amplitudes and front durations;  $\sigma_{\ln I}, \sigma_{\ln t_f}$  are their standard deviations, while the variable  $\rho_c$  stands for the correlation coefficient between them, in accordance with (3) – (6). Preservation of the correlation coefficient between standardised bivariate normal and the appropriate bivariate log-normal distribution is tested-for by means of the Spearman’s correlation coefficient. These variates are then—in a Monte–Carlo type of simulation procedure—introduced in (8), along with the appropriate parameters from the Table I (depending on the counterpoise configuration at hand), yielding a large data pool of impulse effective lengths. At least ten-thousand random samples are utilised. Different statistical distributions can then be fitted—by means of the Maximum Likelihood Estimation (MLE) of their parameters—to this effective length data pool, e.g. [18, Ch. 6].

It has been found, through numerical experiments, that the log-normal distribution depicts the generated effective length data extremely well for all above mentioned counterpoise

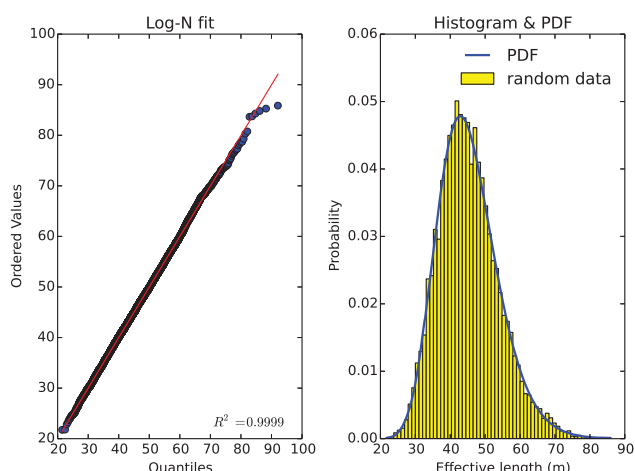


Fig. 1. Log-normal distribution of the counterpoise effective length for the  $\rho = 100 (\Omega m)$  and Type A configuration (left: probability plot, right: histogram and PDF fit)

geometry and injection point configurations, within the span of the soil resistivity for which the formulas are applicable. This can be readily observed from the Fig 1, which is provided for the counterpoise configuration of Type A in a soil with  $\rho = 100 (\Omega m)$ . The lightning-current parameters from the original set have been employed in the computation. On the left side of Fig 1 is the probability plot for the log-normal distribution (which holds the same information as the quantile-quantile plot) and on the right side is the histogram of the data with a probability density function (PDF) of the log-normal distribution superimposed on the histogram [18, Ch. 1]. The log-normal distribution  $\text{Log-N}(\mu, \sigma)$  for this particular case has following parameters:  $\mu_{\ell_e} = 44.6 (m)$ ,  $\sigma_{\ln \ell_e} = 0.188$ . The quality of the fit is clearly evident on the Fig. 1 and need not be further elaborated upon (the correlation coefficient equals 0.9999).

It ought to be emphasised at this point that, due to the space limitations, results will be presented only for the case of bare grounding conductors (although method holds for the case of LRM treated conductors as well).

Statistical description of the counterpoise effective length for the Type A configuration is graphically depicted in Fig. 2 with a so-called “violin plot.” A violin plot holds all of the data provided by the so-called “box plot” (often employed in statistical description of data) and in addition provides “outliers” (blue ticks) and the probability density function of the statistical distribution superimposed on the box plot. The bottom and top of the box—which are part of the box plot and, hence, the violin plot—are the first and the third quartiles of the distribution, the band inside the box (red line) is the second quartile (i.e. median), and the “whiskers” represent the lowest datum still within 1.5 IQR (interquartile range) of the lower quartile, and the highest datum still within 1.5 IQR of the upper quartile. From the violin plot one can visualise the PDF of the log-normal distribution, assess the interquartile range, obtain the median value, etc. With this approach, a complete lightning-current statistical depiction, including correlation,

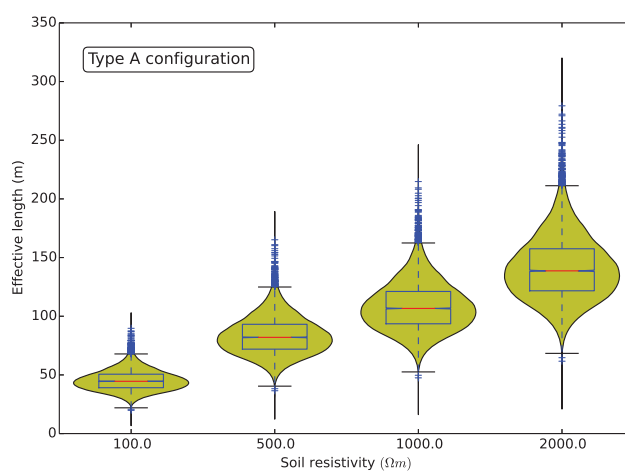


Fig. 2. Violin plot of the counterpoise effective length statistical properties, for the Type A counterpoise configuration and four different soil resistivity values

went into the computation of the effective length.

By examining the Fig. 2 it becomes clear that the statistical depiction of the effective length has much more sense than the “traditional” depiction, in which a family of curves would have been provided for different fixed values of lightning-current amplitude or front duration. This will become even more evident in a moment. Namely, it would be very difficult to choose a particular value of the effective length based on the traditional graphical representations. In order to make a point, Fig. 3 presents a typical “traditional” depiction of the counterpoise effective lengths, for the Type A configuration and several different lightning-current parameters.

Additionally, Fig. 4 provides a comparison between statistically determined effective lengths—for the counterpoise configuration of the Type A—and measurements data obtained for the same configuration and following lightning-current parameters: 10 kA, 2.6/50  $\mu s$ . Statistical data is now presented

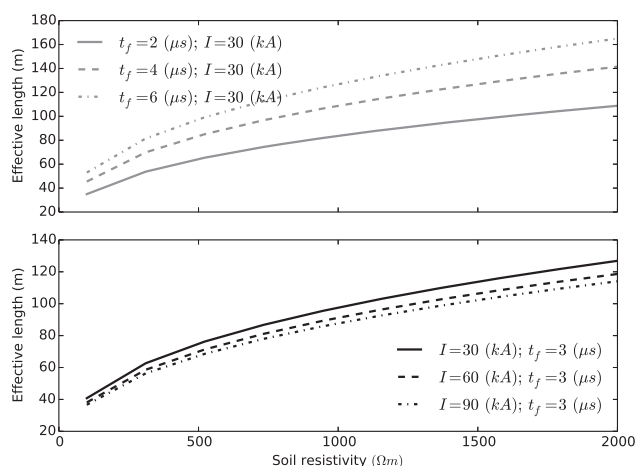


Fig. 3. Traditional depiction of the effective length for the Type A counterpoise configuration and several different lightning-current values

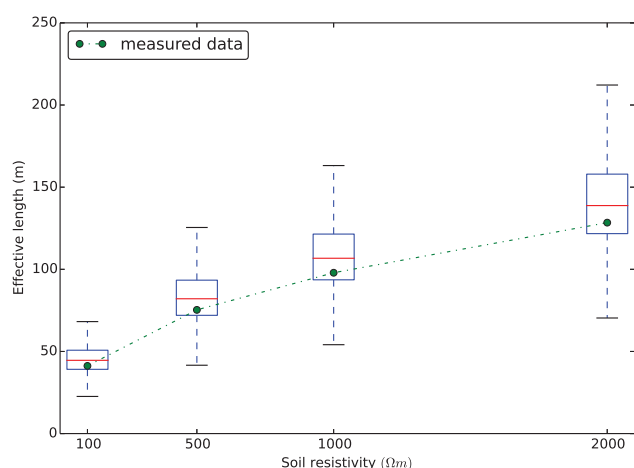


Fig. 4. Measurements data and a box plot of statistical distribution of the effective length for the Type A counterpoise configuration

in terms of the box plot. It can be readily observed from the Fig. 4 that the measurements data fit quite nicely within the statistical depiction of the effective length; they are somewhat below the median value of the distributions, which is expected, since the measurements were produced by the 10 kA amplitude and the 2.6/50  $\mu$ s wave-shape. Both, amplitude and front duration of this wave-shape are below the median values of the lightning data (see Section II); the effective length, as already stated, is lower for shorter front durations and smaller amplitude lightning currents.

Parameters of the effective length Log-N( $\mu, \sigma$ ), for different treated counterpoise geometry and injection point configurations are conveniently presented in Table II; they have been obtained with lightning-current statistical parameters from the original set and are given for several different values of the soil resistivity, where the inter-range values could be obtained by interpolation. It can be seen from the Table II that—for the same soil resistivity—the effective length of Type A configuration is the smallest of the three, followed by the Type B configuration and, finally, by the Type C configuration, which features longest effective length (assumed, of course, for a single arm of the four-star configuration). This is in agreement with the findings reported in e.g. [6], [7].

Since the effective length has been described by the statistical (log-normal) distribution one can easily obtain its probability density function (PDF), cumulative distribution function (CDF), and complementary cumulative distribution function (CCDF), where  $CCDF = 1 - CDF$ . The CCDF function estimates the probability by which some value of the effective length will be attained or exceeded for a particular counterpoise geometry and soil resistivity. Additionally, inverse CCDF function, given probability level, provides the associated effective length; modern computational tools provide all of these functions for different statistical distributions (e.g. Scipy library in Python), [19]. Hence, by using this approach the effective length ceases to be in a formal functional relationship with the lightning-current

TABLE II  
 PARAMETERS OF THE LOG-N( $\mu, \sigma$ ) FOR DIFFERENT TREATED COUNTERPOISE GEOMETRY AND INJECTION POINT CONFIGURATIONS

Counterpoise configuration	$\rho$ ( $\Omega m$ )	Effective length	
		$\mu_{\ell_e}$	$\sigma_{\ln \ell_e}$
Type A	100	44.6 (m)	0.188
	500	82.1 (m)	0.188
	1000	106.8 (m)	0.188
	1500	124.3 (m)	0.188
	2000	138.6 (m)	0.188
Type B	100	52.4 (m)	0.189
	500	96.4 (m)	0.189
	1000	125.4 (m)	0.189
	1500	146.2 (m)	0.189
	2000	163.1 (m)	0.189
Type C	100	61.1 (m)	0.190
	500	112.5 (m)	0.190
	1000	146.3 (m)	0.190
	1500	170.6 (m)	0.190
	2000	190.3 (m)	0.190

parameters and (although statistical in nature) depends only on the geometry and soil resistivity. Fig. 5 graphically depicts the complementary cumulative distribution function of counterpoise effective lengths for the Type A configuration, for three different soil resistivity values. The same figures could be readily obtained for other counterpoise geometry and injection point configurations (including cases of LRM treated conductors).

Fig. 5 can be of immediate use to the design engineer for determining the appropriate (i.e. optimal) length of the counterpoise—in terms of its effective length—for this particular configuration, for different values of soil resistivity. For example, if one chooses that it is acceptable (in terms

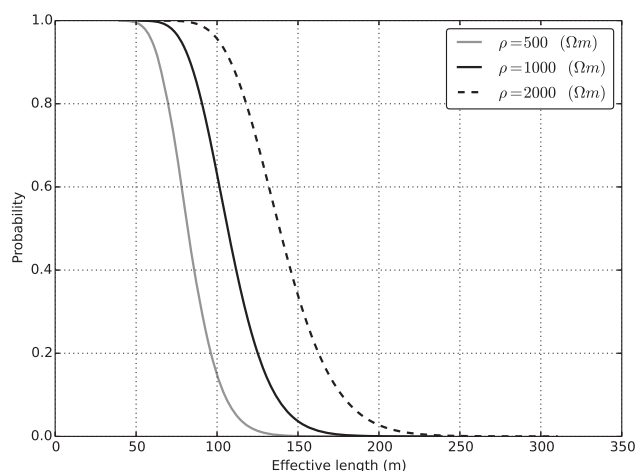


Fig. 5. Complementary cumulative distribution functions of counterpoise effective lengths for the Type A configuration, for different soil resistivity

of technical and economical feasibility) to base the effective length on the 90 % probability level, then for the  $\rho = 1000 (\Omega m)$  Fig. 5 would yield a value of 83 m. If this is selected as the actual (i.e. physical) counterpoise length, it means that the counterpoise would be, in these circumstances, optimally utilised in the 90 % of cases of lightning strikes. For the sake of comparison, for example, for the 50 % probability level (median effective length), the counterpoise length from the Table II would be cca 24 m longer (29 % relative difference).

Furthermore, since the number of lightning strikes to any object depends on the keraunic level of the site, orographic factors and exposure of the object (i.e. its lightning attractiveness), these factors would be instrumental in determining the acceptable level of probability for the counterpoise effective length distribution—in line with the expressions utilised for estimating the number of dangerous events due to lightning strikes in general, e.g. see [20]. In other words, an estimated number of lightning strikes for which the counterpoise actual length (selected equal to its effective length) would *not* be utilised effectively could be obtained from the expression

$$N = \text{CDF} \cdot N_g \cdot A_e \cdot \tau \quad (12)$$

where CDF is the effective length cumulative distribution function,  $N_g = 0.04T_d^{1.25}$  in ( $km^{-2}year^{-1}$ ) is the annual average ground flash density ( $T_d$  is the long-term average annual number of thunderstorm days),  $A_e$  is the exposure area (i.e. electric shadow area) of the associated object in ( $km^2$ ), and  $\tau$  is the time window in (*years*). This expression features the  $\text{CCDF} = 1 - \text{CDF}$  value for any object in any terrain, given  $N$  tolerable (but unfavourable) events in  $\tau$  years. From the CCDF one can obtain the inverse CCDF and from this function, given probability level, calculate the effective length for the particular scenario considered. This analysis might provide the basis for the selection of counterpoise length, based on the effective length for lightning surges. The counterpoise length, of course, features in obtaining the low-frequency resistance of the associated grounding system as well, so this is another selection criteria to be considered.

In order to demonstrate the procedure let one assume an object (e.g. tower) with the exposure area of  $A_e = 0.5 (km^2)$  situated in the terrain having annual average ground flash density of  $N_g = 1 (km^{-2}year^{-1})$  and relative soil resistivity  $\rho = 1000 (\Omega m)$ . The Type A of the counterpoise geometry is assumed here, for convenience, although other mentioned types could be treated in the exactly the same manner. Furthermore, let one assume that a *single* unfavourable event within the time window spanning some  $\tau$  (years) is tolerable. Consequently, based on the described input data, Table III is obtained as the result of the presented computational procedure. It ought to be stated that the same results could have been obtained by using the inverse CDF in place of the inverse CCDF but, considering the relationship between the actual and effective counterpoise length, using the inverse CCDF seems to be slightly more appropriate.

It can be observed from the Table III that the counterpoise lengths that will *not* be effectively utilised only once within the

TABLE III  
 COUNTERPOISE EFFECTIVE LENGTH BASED ON TOLERATING A SINGLE EVENT IN  $\tau$  YEARS

$\tau$ (years)	CCDF	Effective length
5	0.600	102 (m)
10	0.800	91 (m)
15	0.867	86 (m)
20	0.900	83 (m)

time window of  $\tau$  years decreases with the lower probability events, meaning here longer time windows. This means that, in general, if the probability of lightning-associated events (based on the keraunic levels and object exposure, along with the time window and the tolerable number of unfavourable events) is low, the counterpoise length could be short, and *vice versa*.

Finally, in order to assess the influence of different lightning statistical parameters on the counterpoise effective length, complete statistical computational procedure is repeated with the alternative set of lightning-current statistical parameters. Comparative partial (statistical) results are provided in Table IV. It can be seen from the Table IV that the application of lighting data statistics from the alternative set provides lower values of the effective length for the same counterpoise configuration and soil resistivity. This is expected and is due to the fact that the parameters of the bivariate log-normal distribution of lightning-current data from the alternative set feature shorter front duration (median of  $2 \mu s$ ) than the parameters of the original set (median  $3.83 \mu s$ ).

TABLE IV  
 COUNTERPOISE EFFECTIVE LENGTH  $\text{Log-N}(\mu, \sigma)$  STATISTICS OBTAINED FROM TWO DIFFERENT LIGHTNING-CURRENT STATISTICAL PARAMETERS

Counterpoise configuration	$\rho$ ( $\Omega m$ )	Effective length			
		Original set		Alternative set	
		$\mu \ell_e$	$\sigma_{\ln \ell_e}$	$\mu \ell_e$	$\sigma_{\ln \ell_e}$
Type A	500	82.1 (m)	0.188	64.3 (m)	0.165
	1000	106.8 (m)	0.188	83.6 (m)	0.165
	2000	138.6 (m)	0.188	108.8 (m)	0.165
Type B	500	96.4 (m)	0.189	75.7 (m)	0.165
	1000	125.4 (m)	0.189	98.4 (m)	0.165
	2000	163.1 (m)	0.189	128.0 (m)	0.165
Type C	500	112.5 (m)	0.190	88.3 (m)	0.165
	1000	146.3 (m)	0.190	114.8 (m)	0.165
	2000	190.3 (m)	0.190	149.3 (m)	0.165

It is interesting to note here that the simulation tests carried-out by the authors revealed that the correlation coefficient between lightning-current amplitude and front duration has negligible effect on the obtained effective length, regardless of the soil resistivity and counterpoise configuration. This is interesting finding, since this correlation has noticeable influence in other lightning-related phenomena, e.g. in backflashover analysis on high-voltage transmission lines.

## V. CONCLUSION

This paper presented a novel view of the counterpoise impulse effective length—obtained from the statistical perspective. Statistical treatment of the impulse effective length seems natural, owing to the fact that lightning-current parameters, which feature prominently in its realisation, are stochastic in nature and can be determined only in statistical terms. It has been shown that the impulse effective length can be depicted by the log-normal distribution function, regardless of the counterpoise configuration or soil resistivity.

This combination of sophisticated regressive formulas for computing counterpoise effective length (derived from the extensive numerical analyses and simulation test results of counterpoise impulse behaviour) and statistical depiction of lightning-current parameters provide powerful instruments at the disposal of the design engineer for the optimisation (in statistical terms) of the actual length of the counterpoise wires in different situations. The (complementary) cumulative distribution function of the counterpoise effective length statistical (log-normal) distribution features prominently in this procedure. Moreover, due to the fact that the number of lightning strikes to any object depends on its lightning attractiveness in terms of exposure to lightning (featuring actual geometry and orographic factors), along with the keraunic level of the site, all of these factors are instrumental in determining the counterpoise effective length.

This novel view of the counterpoise impulse effective length could be seen as beneficial in designing economically feasible grounding systems for wind turbines at wind farm sites featuring high soil resistivity, as well as for improving the backflashover performance of high-voltage transmission line towers. This is a reasonable claim, considering the fact that digging trenches for laying grounding wire of any considerable length, at remote locations, in high-resistivity (often meaning rocky) soil can be quite expensive.

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