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Parameters Estimation of Multidimensional Possibility Distributions

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Abstract—We present a solution to the Maxmin u/E parameters estimation problem of possibility distributions in m-dimensional case. Our method is based on geometrical approach, where minimal area enclosing ellipsoid is constructed around the sample. Also we demonstrate that one can improve results of well-known algorithms in fuzzy model identification task using Maxmin u/E parameters estimation.

Keywords—Possibility distribution, parameters estimation, Maxmin u/E estimator, fuzzy model identification.

I. Introduction

AMONG few papers devoted to parameters estimation of fuzzy variables one can refer to works [1]– [3]. Cai Kai-Yuan proposed parameters estimation method for normal fuzzy variables in Nahmias's sense [1]. His approach is based on the point estimation, maximum scale likehood estimation and interval estimation methods.

Dug Hun Hong generalized Cai's results and in the paper [2] introduced the parameters estimations of general T-related not necessarily normal fuzzy variables.

Wang Xizhao and Ha Minghu introduced μ/E parameters estimation method [3] for the family of fuzzy numbers with two parameters, location and scale, using results from [4]– [6]. They showed that this estimator is consistent, sufficient and maximum likehood.

In this paper we generalize the results of Wang Xizhao and Ha Minghu to the case of m-dimensional possibility distributions. Firstly we will briefly review the possibility theory and set up notations needed in the paper. Secondly we will present the μ/E parameters estimation method for m-dimensional fuzzy variables and main properties of this estimation. Further we will demonstrate the practical application of the μ/E parameters estimation method on the task of fuzzy model identification.

II. Definitions

According to [7], [8] let Γ be an abstract space of generic elements $\gamma \in \Gamma$, $P(\Gamma)$ be a class of all subsets of Γ, a scale π be a possibility measure on $P(\Gamma)$. Then, $(\Gamma, P(\Gamma), \pi)$ is named possibility space.

Definition 1: The mapping $X : \Gamma \to R^m$ is a fuzzy variable with distribution function, denoted by μ_X and is given by:

$$\mu_X(\mathbf{x}) = \pi\{\gamma \in \Gamma : X(\gamma) = \mathbf{x}\}, \mathbf{x} \in R^m.$$

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Definition 2: The point $\mathbf{a} \in R^m$ is a modal value of fuzzy variable X if $\mu_X(\mathbf{a}) = 1$.
According to [9] we introduce definition of joint distribution function.

Definition 3: Let $X_1, ..., X_n$ be fuzzy variables. Joint distribution function $\mu_{X_1,...,X_n}(\mathbf{x}_1, ..., \mathbf{x}_n)$ of $X_1, ..., X_n$ is defined by:

$$\begin{aligned}&\mu_{X_1,...,X_n}(\mathbf{x}_1, .., \mathbf{x}_n)\\ &= \pi\{\gamma \in \Gamma : X_1(\gamma) = \mathbf{x}_1, ..., X_n(\gamma) = \mathbf{x}_n\}\\ &= \pi\{X_1^{-1}(\mathbf{x}_1) \cap \cdots \cap X_n^{-1}(\mathbf{x}_n)\}, \forall \mathbf{x}_i \in R^m, i \in [1, .., n].\end{aligned}$$

Fuzzy variables $X_1, ..., X_n$ are said to be mutually min-related if for any subset $\{i_1, ..., i_k\}$ of $\{1, 2, ..., n\}$

$$\mu_{X_{i_1},...,X_{i_k}}(\mathbf{x}_1, ..., \mathbf{x}_k) = \min\{\mu_{X_{i_1}}(\mathbf{x}_1), ..., \mu_{X_{i_k}}(\mathbf{x}_k)\}.$$

Definition 4: Let $\boldsymbol{\xi}_1, \boldsymbol{\xi}_2, ..., \boldsymbol{\xi}_n$ be n independent identically distributed fuzzy variables, than $(\boldsymbol{\xi}_1, \boldsymbol{\xi}_2, ..., \boldsymbol{\xi}_n)$ is said to be the sample of the family, and $\boldsymbol{x}_1, \boldsymbol{x}_2, ..., \boldsymbol{x}_n$ are the observed values of the sample.

Definition 5: If any function of the sample $(\boldsymbol{\xi}_1, \boldsymbol{\xi}_2, ..., \boldsymbol{\xi}_n)$ doesn't involve unknown parameters, it is called a statistic. Note, that statistics is a fuzzy variable.

Definition 6: The family of distributions is the set of distributions, that depends on parameter vector $\boldsymbol{\theta}$:

$$\{\mu_\xi(\boldsymbol{x}, \boldsymbol{\theta}) | \boldsymbol{\theta} = (\theta_1, \theta_2, ..., \theta_k) \in \Theta\}, \boldsymbol{x} \in R^m,$$

where Θ is parameter space ($\Theta \subseteq R^k$).

The problem of parameters estimation of possibility distribution is to find the appropriate parameter vector $\boldsymbol{\theta}$ on the base of the values $(\mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_n)$ of the sample $(\boldsymbol{\xi}_1, \boldsymbol{\xi}_2, ..., \boldsymbol{\xi}_n)$ after n observations.

III. The Parameters Estimation Method for m-dimensional Fuzzy Variables

Now we are going to consider a solution of parameters estimation problem of symmetric, strictly decreasing possibility distributions. Generalizing results of Wang Xizhao and Ha Minghu, we introduce μ/E parameters estimation method for the family of fuzzy numbers in the case of m-dimensional possibility distributions.

A. The μ/E Parameters Estimation Method for Multidimensional Fuzzy Variables

Let us consider symmetric, strictly decreasing with distance from zero m-dimensional possibility distribution:

$$\mu_0(\mathbf{x}) = \bar{f}(\|\mathbf{x}\|) = \max\{0, f(\|\mathbf{x}\|)\}, \quad (1)$$

where $\mathbf{x} \in R^m$, f is strictly decreasing on $R^1_+ = [0, +\infty)$ function, $f(0) = 1$.

Let Q be a family of possibility distributions, obtained from (1) with affine transformations. Distribution functions in this family can be represented in the form:

$$\mu_\xi(\mathbf{x}, \mathbf{c}, A) = \max\{0, f(\sqrt{(\mathbf{x}-\mathbf{c})^T A(\mathbf{x}-\mathbf{c})})\}, \quad (2)$$

where $\mathbf{c} \in R^m$, A is $m \times m$ positive definite symmetric matrix. Parameter $\mathbf{c}$ is a location of modal value and defines translation of μ_0, so that $\mu(\mathbf{c}, \mathbf{c}, A) = 1$. Rotation and scaling are defined by matrix A, so that rotation angles are defined by eigenvectors of A, and scale along axes is proportional to square roots of corresponding eigenvalues.

One can see that equipotential surfaces of function μ_ξ defined according to (2) are ellipsoids.

Following [3] we denote a fuzziness measure of distribution μ_ξ by $E(\mu_{\boldsymbol{\xi}}) = \int\limits_{R^m}\cdots\int \mu_{\boldsymbol{\xi}}(t^1, ..t^m, \boldsymbol{\theta})dt^1...dt^m$. Here $\boldsymbol{\theta}$ is a vector of distribution parameters, $\boldsymbol{\theta} = (\mathbf{c}, A)$, $\boldsymbol{\theta} \in \Theta$.

Suppose, that observed values $(\mathbf{x}_1, ..., \mathbf{x}_n)$ of multidimensional sample $(\boldsymbol{\xi}_1, ..., \boldsymbol{\xi}_n)$ of the family Q are collected during n observations. Because observations are min-related, joint distribution of possibility to observe $(\mathbf{x}_1, ..., \mathbf{x}_n)$ is:

$$\mu((\mathbf{x}_1, ..., \mathbf{x}_n), \boldsymbol{\theta}) = \bigwedge_{j=1}^{n} \mu_{\boldsymbol{\xi}}(\mathbf{x}_j, \boldsymbol{\theta}), \quad (3)$$

where $\mathbf{x_j} = (x^1_j, ..., x^m_j)$, $x^i_j \in R^1$.

Note that changes of values of parameters vector $\boldsymbol{\theta}$ directly influence both the possibility, that the observed sample appears, and the value of $E(\mu_{\boldsymbol{\xi}})$. So we should select $\boldsymbol{\theta}$ such that the possibility (3) is as high as possible and the value of $E(\mu_{\boldsymbol{\xi}})$ is as small as possible.

Therefore, we denote by

$$L(\boldsymbol{\theta}) = \bigwedge_{j=1}^{n} \mu_{\boldsymbol{\xi}}(\mathbf{x}_j, \boldsymbol{\theta})/E(\mu_{\boldsymbol{\xi}}). \quad (4)$$

Definition 7: If there exists $\hat{\boldsymbol{\theta}} \in \Theta$ such that $L(\hat{\boldsymbol{\theta}}) = \max\limits_{\boldsymbol{\theta}\in\Theta} L(\boldsymbol{\theta})$, then $\hat{\boldsymbol{\theta}}$ is called Maxmin μ/E estimator of the parameters vector $\boldsymbol{\theta}$.

The following theorem gives the method for definition the estimators of the parameters $\mathbf{c}, A$.

Theorem 1: Let:

1. $f : R^1_+ \to R^1$ be strictly decreasing on $[0, +\infty)$ function, $f(0) = 1$;

2. $Q = \{\mu_{\boldsymbol{\xi}}(\mathbf{x}, \mathbf{c}, A)\}$ be a family of distributions, where $\mathbf{c} \in R^m$, A is $m \times m$ positive definite symmetric matrix, $\mu_{\boldsymbol{\xi}}(\mathbf{x}, \mathbf{c}, A) = \max\{0, f\big(\sqrt{(\mathbf{x}-\mathbf{c})^T A(\mathbf{x}-\mathbf{c})}\,\big)\}$;

3. $X = \{\mathbf{x}_1, ..., \mathbf{x}_n\}$ be observed sample of the family Q, where $\mathbf{x}_i \in R^m, i \in [1, ..., n]$;

4. W be m-dimensional ellipsoid of minimal volume, circumscribed around the set X.

Then the Maxmin parameter estimator of $\boldsymbol{\theta} = (\mathbf{c}, A)$ is $(\hat{\mathbf{c}}, \hat{A})$ such that ellipsoid W is defined by equation $(\mathbf{x}-\hat{\mathbf{c}})^T \hat{A}(\mathbf{x}-\hat{\mathbf{c}}) = 1/q^2$, $q = \arg\max\limits_{t\geq 0} t^m \bar{f}(t)$.

Proof: Consider the distribution $\mu_{\boldsymbol{\xi}}$ of the family Q with parameters: $\mathbf{c}, A$. Let $\tilde{A} = p^2 A$, $p \in R^1$. Let $\int\limits_{R^m}\cdots\int \mu_0(t^1, ..., t^m)dt^1...dt^m = I$. It is easy to compute that $E(\mu_{\boldsymbol{\xi}}) = Ib^1 \cdot ... \cdot b^m/(p)^m$, where $b^i = 1/\sqrt{v^i}$, v^i is i–th eigenvalue of the matrix $\tilde{A}$.

According to (4), we have:

$$L(\mathbf{c}, A) = \bigwedge_{j=1}^{n} \frac{(p)^m \bar{f}(\sqrt{\mathbf{u}}\,)}{Ib^1 \cdot ... \cdot b^m}, \quad (5)$$

where $\mathbf{u} = (\mathbf{x}-\mathbf{c})^T A^{-1}(\mathbf{x}-\mathbf{c})$.

Consider $Z = \{\mathbf{x}_{z_1}, ..., \mathbf{x}_{z_k}\}$ is the set of points of distribution, where (5) attains its minimum. Suppose the following equation is fulfiled for the points in Z: $\mu_{\boldsymbol{\xi}}(\mathbf{x}_{z_i}) = \mu_Z(X, \boldsymbol{\theta}) = \min\limits_{\mathbf{x}\in X} \mu_{\boldsymbol{\xi}}(\mathbf{x})$, $i \in [1, .., k]$. Then

$$L(\mathbf{c}, A) = \bigwedge_{j=1}^{n} \frac{(p)^m \bar{f}(\sqrt{\mathbf{u}}\,)}{Ib^1 \cdot ... \cdot b^m} = \frac{(p)^m \mu_Z(X, \boldsymbol{\theta})}{Ib^1 \cdot ... \cdot b^m}. \quad (6)$$

Equipotential surfaces for distribution, defined according to (2), are ellipsoids. Therefore, all points of the set Z are situated on the one of such ellipsoids (the ellipsoid U) and the other points are inside this ellipsoid. Let $\tilde{\mathbf{c}} = (\tilde{c}^1, ..., \tilde{c}^m)$ and the matrix $\tilde{A}$ defines the ellipsoid U.

To find $\mu_Z(X, \boldsymbol{\theta})$ we consider the point $\mathbf{H} = (h^1, ..., h^m)$ on the end of the longest axis of ellipsoid U. We obtain $\mu_Z(X, \boldsymbol{\theta}) = \mu_{\boldsymbol{\xi}}(\mathbf{H}) = \bar{f}(p)$. Then $L(\tilde{\mathbf{c}}, \tilde{A}) = (p)^m \bar{f}(p)/Ib^1 \cdot ... \cdot b^m$.

Considering that the volume of ellipsoid U is

$$V_U = \frac{2\pi^{\frac{m}{2}}}{m\Gamma(\frac{m}{2})} b^1 \cdot ... \cdot b^m,$$

where Γ is gamma-function [10].

Then

$$L(\tilde{\mathbf{c}}, \tilde{A}) = \frac{2\pi^{\frac{m}{2}}(p)^m \bar{f}(p)}{m\Gamma(\frac{m}{2}) I V_U}.$$

As we are seeking for parameters to maximize $L(\mathbf{c}, A)$, we should take $p = q$, that maximizes the function $(p)^m \bar{f}(p)$, and minimal-volume ellipsoid W as ellipsoid U.

W is defined by equation $(\mathbf{x}-\hat{\mathbf{c}})^T \tilde{A}(\mathbf{x}-\hat{\mathbf{c}}) = 1$. Considering that $A = \dfrac{1}{p^2}\tilde{A}$, we obtain $(\mathbf{x}-\hat{\mathbf{c}})^T \hat{A}(\mathbf{x}-\hat{\mathbf{c}}) = 1/q^2$.This completes the proof of the theorem. ∎

Example 1: Consider μ/E parameters estimation of 2-dimensional distribution of the family $Q_\mu(\mathbf{c}, A)$. We will use $f(t) = e^{-t^2}$ for normal distribution, here parameter $q = \arg\max\limits_{t\geq 0} t^2 \bar{f}(t) = 1$.

Fig.1 shows a plot of distribution function $\mu(\hat{\mathbf{c}}, \hat{A})$, where: ∘ denote the observed values of the fuzzy variable; −− denotes W (circumscribed minimal value ellipse); • denotes the center of W; ellipses shown at the base of the graphs demonstrate the equipotential lines of distribution function μ.

B. Properties of Parameters Obtained with μ/E Estimation

To show the μ/E parameters estimation deserves attention, we investigate its properties. Let's start with extending definitions from [3] to multidimensional case.

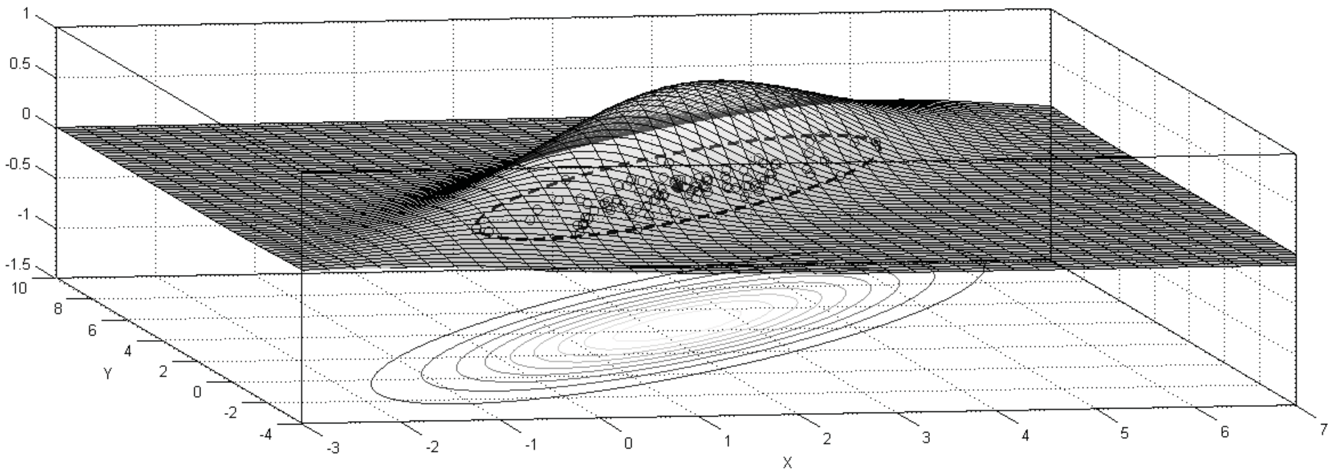

Fig. 1. Parameters estimation of normal distribution

Definition 8: Let F be a family of distribution functions, $\bar{\boldsymbol{\xi}} = (\bar{\boldsymbol{\xi}}_1, \bar{\boldsymbol{\xi}}_2, ..., \bar{\boldsymbol{\xi}}_n)$ be a sample, $\bar{\boldsymbol{\xi}}_i = (\bar{\xi}_i^1, ..., \bar{\xi}_i^m)$, $i = \overline{1,n}$, and let $T(\bar{\boldsymbol{\xi}})$ be a statistic whose distribution function belongs to F. We say $T(\bar{\boldsymbol{\xi}})$ is sufficient with respect to F if the joint distribution function of $(T(\bar{\boldsymbol{\xi}}), \bar{\boldsymbol{\xi}})$, $G(\mathbf{t}, \mathbf{x})$, does not depend on $\mathbf{x}$.

Note 1: Every sample includes a certain amount of information on the family. Definition (8) shows that a sufficient statistic contains same amount of information as the sample with respect to the family. It follows that a sufficient statistic may be used to simplify a sample without losing information.

Definition 9: Let F be a family denoted by $F = \{F(\mathbf{x}, \boldsymbol{\theta}) | \mathbf{x} = (x^1, ..., x^m), \boldsymbol{\theta} = (\theta_1, \theta_2, ..., \theta_k) \subset R^k\}$, $\bar{\boldsymbol{\xi}} = (\bar{\boldsymbol{\xi}}_1, \bar{\boldsymbol{\xi}}_2, ..., \bar{\boldsymbol{\xi}}_n)$ be a sample of the family and $T(\bar{\boldsymbol{\xi}})$ be a statistic. We say:
1) $T(\bar{\boldsymbol{\xi}})$ is a sufficient estimator of $\boldsymbol{\theta}$, if $T(\bar{\boldsymbol{\xi}})$ is sufficient with respect to F;
2) $T(\bar{\boldsymbol{\xi}})$ is a consistent estimator of $\boldsymbol{\theta}$, if $\pi(\{T(\bar{\boldsymbol{\xi}}) = \boldsymbol{\theta}\}) = 1$.
3)$T(\bar{\boldsymbol{\xi}})$ is a maximum likehood estimator of $\boldsymbol{\theta}$ if:

$$M(\mathbf{x}_1, ..., \mathbf{x}_n, T(\bar{\boldsymbol{\xi}})) = \max_{\boldsymbol{\theta} \in \boldsymbol{\Theta}} M(\mathbf{x}_1, ..., \mathbf{x}_n, \boldsymbol{\theta}),$$

where
$M(\mathbf{x}_1, ..., \mathbf{x}_n, \boldsymbol{\theta}) = \bigwedge_{j=1}^{n} F(x_j^1, ..., x_j^m, \boldsymbol{\theta})$,
$\mathbf{x}_i = (x_i^1, ..., x_i^m)$, $i \in [1, n]$.

Note 2: Consistency illustrates the possibility that the estimator takes true value is maximum. Maximum likelihood explains the possibility that the sample appears attains maximum. Therefore, definition (9) may be regarded as a criterion for judging reasonableness of an estimator.

We have proved the following theorem about properties of maxmin μ/E estimation in multidimensional case:

Theorem 2: The estimation $\hat{\boldsymbol{\theta}} = (\hat{\mathbf{c}}, \hat{A})$ as given by the theorem 1 is:
1. a sufficient estimator of parameters $\mathbf{c}, A$
2. a consistent estimator of the parameter $\mathbf{c}$ and eigenvectors of A if eigenvalues of A are fixed.
3. a maximum likehood estimator of the parameter $\mathbf{c}$ and eigenvectors of A if eigenvalues of A are fixed.

We present some practical application of considered theory in the next section.

IV. The Maxmin μ/E Parameters Estimation in Fuzzy Model Identification Task

To verify practical usefulness of the proposed parameters estimation method we evaluate it on the task of fuzzy model identification. Consider clustering-based fuzzy model identification method presented in [11] by Stephen L. Chiu.

This method is summarized in the following four steps:

1. Cluster data points using modification of Mountain clustering method and obtain cluster centers.

2. Consider each cluster center as a prototype of a fuzzy rule. Use appropriate part of center coordinates as a modal value of fuzzy term for the rule premise.

3. For a first estimate of rule consequent use a remainder of cluster center coordinates.

4. Use least-squares estimation to optimize rules consequents and obtain Sugeno order 0 or order 1 model.

Modified Mountain Method used on step 1 requires manual selection of parameter r_a (it defines neighborhood radius of points). The same parameter is used on step 2 to define the width of fuzzy terms for rule premises.

We propose to use the same general strategy for fuzzy model identification, but use Maxmin μ/E parameters estimation to define parameters of terms for rule premises.

We will evaluate this algorithm on chaotic time series prediction as in [11] to verify algorithm performance.

A. Chaotic Time Series Prediction

Let us consider prediction of time series generated by chaotic Mackey-Glass differential delay equation:

$$\dot{x}(t) = \frac{0.2x(t-\tau)}{1 + x^{10}(t-\tau)} - 0.1x(t).$$

The task is to use past values of x up to the time t to predict future value of x at $t + \Delta t$. The standard settings [12] is to predict $x(t+6)$ based on $\mathbf{x} = \{x(t-18), x(t-12), x(t-6), x(t)\}$.

TABLE I
RESULTS OF DIFFERENT METHODS ON CHAOTIC TIME SERIES PREDICTION TASK

Method	Error Measure
ANFIS	0.007
Mountain Method & μ/E-estimation	0.010
Cluster Estimation	0.014
Back-Prop NN	0.02
6-order polynomial	0.04

The dataset consists of 1000 data points extracted from $t = 118$ to $t = 1117$. The first 500 points were used for training and last 500 data points were used for checking generalization ability of the model. The error measure used is RMS error divided by standard deviation of time series.

We use modified Mountain Method implemented by Matlab *subclust* command. After receiving cluster centers from *subclust* algorithm, we divide dataset points between clusters, selecting cluster with nearest center.

Improved algorithm allows us to obtain better results, than from [11]. It is interesting to note, that while authors of Cluster Estimation method have found that their model overfits data if r_a parameter of clustering algorithm is less than 0.3 and number of clusters and rules is 35, we were able reduce r_a to 0.18 and get a model with 53 rules without overfitting. At lower r_a our algorithm was not able to estimate distribution parameters due to singular matrices during calculation of minimal volume ellipsoids.

Table I shows results obtained by different algorithms on this task. Rows 1 and 8 are from Jang [13]; row 3 is from Chiu [11]; rows 4, 5 are from Crowder [14].

V. CONCLUSION

This paper is concerned with finding a solution to the Maxmin μ/E estimation for the family of m-dimensional possibility distributions. The solution for symmetric unimodal distributions with parameters of location, rotation and scale was found. Described estimator is sufficient, consistent and maximum likelihood.

We have studied the practical application of Maxmin μ/E estimation on the task of fuzzy model identification and have found that it improves models created by other methods.

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