Theoretical Modal Analysis of Freely and Simply Supported RC Slabs

M. S. Ahmed, F. A. Mohammad

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***Abstract*—**This paper focuses on the dynamic behavior of reinforced concrete (RC) slabs. Therefore, the theoretical modal analysis was performed using two different types of boundary conditions. Modal analysis method is the most important dynamic analyses. The analysis would be modal case when there is no external force on the structure. By using this method in this paper, the effects of freely and simply supported boundary conditions on the frequencies and mode shapes of RC square slabs are studied. ANSYS software was employed to derive the finite element model to determine the natural frequencies and mode shapes of the slabs. Then, the obtained results through numerical analysis (finite element analysis) would be compared with the exact solution. The main goal of the research study is to predict how the boundary conditions change the behavior of the slab structures prior to performing experimental modal analysis. Based on the results, it is concluded that simply support boundary condition has obvious influence to increase the natural frequencies and change the shape of the mode when it is compared with freely supported boundary condition of slabs. This means that such support conditions have the direct influence on the dynamic behavior of the slabs. Thus, it is suggested to use free-free boundary condition in experimental modal analysis to precisely reflect the properties of the structure. By using free-free boundary conditions, the influence of poorly defined supports is interrupted.

***Keywords*—**Natural frequencies, Mode shapes, Modal analysis, ANSYS software, RC slabs.

I. Introduction

REINFORCED concrete (RC) is common construction materials that frequently used as structural members. Moreover, RC elements such as columns, beams and slabs are the main structural members that used in concrete structures. Predominantly, RC structures are subjected to extreme dynamic loads conditions due to any reason such as an impact, cyclones or earthquake [1], [2]. In such situation, the structures respond differently than statically loaded, it can be said that the structure is subjected to a combination of static and dynamic loads simultaneously. When the RC structures expose to various extreme loading conditions, they will start to shake and vibrate. Therefore, it is expected that the structural elements might initiate to be damaged. As a consequence, local reduction in stiffness of RC structural elements occurs due to crushing of concrete material. This turn leads to change the dynamic behavior of structural elements during their serviceability process [3].

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Understanding the behavior of the structural element to dynamic loads is essential to protect structural members from collapse and make them be sustained. Furthermore, in order to ascertain a reliable dynamic resistant design procedure of structural elements, a series tests are required [4]. The dynamic properties of a structure can be generally determined by theoretical or experimental or a combined theoretical and experimental approach. Analytical analysis using an exact solution and numerical analysis using finite element method are the main examples of the theoretical approach whereas doing experiments is a good example of the experimental approach. There is no doubt that an experimental approach is better to be conducted in order to identify the behavior of the structure. Determining the dynamic response of RC structures through full-scale tests is expensive in terms of providing the required test material, test equipment, and time to perform. It is worthwhile to mention that little attention has been paid to full-scale structure by researchers. For example, [5] and [6] tested full-scale four and seven RC structure respectively to study the dynamic behavior. It is economical when conducting experiments on small specimens instead of full-scale tests owing to aforementioned factors. Recently, more researchers have paid attention to study the effect of different parameters of RC members based on laboratory specimens. Before making laboratory specimens, it should be better ones has some theoretical analysis information on the intended research to prematurely identify and simulate the dynamic behavior of RC structures. Therefore, concentrating on theoretical analysis is highly acceptance by researches. However, the theoretical analysis method such as modelling technique still requires a wide range of exploration and discussion.

RC slabs are essential components of many civil engineering structures; therefore, their vibration analyses are mandatory for safe design to guarantee the life safety of mankind. The RC square slabs have been commonly used in such fields for different purposes. Modal analysis is primarily a tool for deriving reliable models represent the dynamic behavior of structures [7]. It is noteworthy to mention modal analysis has grown steadily in civil engineering realm. Therefore, all parametric studies have not been well addressed. Most of the early research studies concentrated on beam specimens, and none of them was concentrated on RC slabs.

Thus, theoretical modal analysis was used to determine the effect of the freely and simply supported boundary conditions on natural frequencies and mode shapes of RC square slabs. For this purpose, freely (F.F) and simply (S.S) supported RC slabs of dimensions 1000 mm x 1000 mm x 50 mm have been

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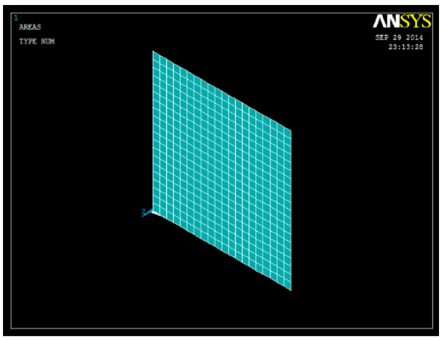
modelled using ANSYS software. After that, the first six numerical natural frequencies were compared with the natural frequencies which are acquired analytically using the exact solution.

II. Numerical Analysis

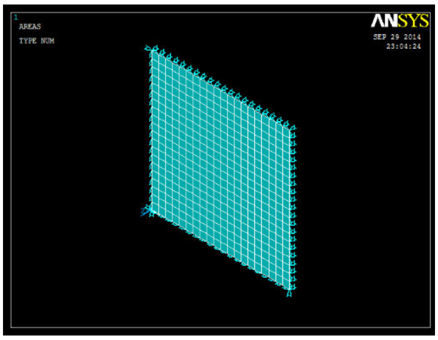
Theoretical modal analysis of RC slab relies on the description of physical properties of the system which is referred as the spatial model. Spatial model description usually contains the mass and stiffness of the system. Calculating the so-called modal model can be made from the spatial model by using Eigenvalue problem [8]. In almost every case, the employment of analytical methods becomes very tedious if not impossible due to the increasing complexity of the geometries, boundary conditions and material of the system. At this point, the use of computational approaches, namely the finite element analysis (FEA), for such cases comes into the picture. FEA can be defined as a computerized procedure for structural analysis.

Numerical technique such as finite element (FE) method should be adopted for more general cases in terms of geometry and boundary conditions [9], [10]. The basic concept of finite element method is that the model is split into a mesh of finite sized elements whose behavior is assumed to be given by applying mathematical expressions. The use of dynamic characteristics in the verification of FE models has become commonly owing to the global nature of the vibration properties. The dynamic properties were obtained from a FE model through solving an Eigenvalue problem of the system. This provides the natural frequencies (eigenvalues) and mode shapes (eigenvectors) of the structural system, which is a central subject in vibration analysis.

Therefore, square RC slabs of dimension 1000mm×1000 mm×50mm with freely and simply supported boundary conditions were created using ANSYS software. It is known that, there are many types of elements in this software. However, some of which are suitable for modelling Reinforced Concrete (RC) slabs. The most favorable element for RC slabs case; theoretical modal analysis of RC slabs is shell element 63. The slabs, in numerical analysis, were split into 20x20 elements along the length and width with Free-free and simply supported boundary conditions as shown in Fig. 1 A and B respectively.



F.F RC slab (A)



S.S RC slab (B)

Fig. 1 Mesh models of F.F and S.S RC slabs A and B

III. Analytical Analysis

Analytic closed form expressions for estimating the six natural frequencies of linear elastic square plates with freely and simply supported boundary conditions are defined by (1) and (2) as follows, [11]:

$$f = \frac{\lambda^2}{2\pi \cdot l \cdot b}\sqrt{\frac{D}{\mu}} \qquad (1)$$

where f is the natural frequency in Hz. λ is a dimensionless natural frequency factor for freely and simply supported slabs, which is tabulated in Table I. l is the length of the longer side of the plate. b is the length of the shorter side of the plate. μis the mass density per unit area of plate (ρh).

$$D = \frac{Eh^3}{12\left(1 - 0.24\upsilon^2\right)} \qquad (2)$$

where D is the plate flexural rigidity. E is modulus of elasticity. υ is the Poisson's ratio of the material of the plate. h is the thickness of the plate.

The first six natural frequencies of RC freely and simply supported square slabs can be obtained using above equations. It can be said that analytical solution has some important benefits. First, relatively small computational time and effort are required to get an idea of the behavior of the structure. Second, analytical method can be used to know the reliability of the numerical results.

TABLE I
Dimensionless Natural Frequency Factor [11]

Mode No.	λ^2	
	F.F	S.S
1	*13.49*	*19.74*
2	*19.79*	*49.35*
3	*24.43*	*49.35*
4	*35.02*	*78.96*
5	*35.02*	*98.70*
6	*61.53*	*98.70*

IV. Results And Discussion

In this section, the six theoretical and numerical natural frequencies of both freely (F.F) and simply (S.S) supported boundary conditions of RC slabs were tabulated in Table II. To determine the accuracy of the numerical modal analysis six numerical natural frequencies were matched with analytical frequencies which are shown in the second and third columns of Table III. In the fourth column of the same able, the relative error between six analytical natural frequencies of freely and simply supported slabs were determined to find the effect of support on natural frequencies analytically. Whereas in the fifth column of that table and for the same purpose, the relative error between six numerical natural frequencies of freely and simply supported slabs were found. The relative error is a function of absolute natural frequency difference between theoretical and numerical model, which is expressed by (3).

$$Error = \frac{\omega_{Num.} - \omega_{Anal.}}{\omega_{Num.}} \quad (3)$$

where ω_{num} is the numerical natural frequency. $\omega_{Anal.}$ is the analytical natural frequency obtained.

TABLE II
COMPARISON BETWEEN ANALYTICAL AND NUMERICAL NATURAL FREQUENCIES OF RC SLABS

Mode No.	Numerical		Analytical	
	F.F	S.S	F.F	S.S
1	*126.08*	*177.68*	*121.50*	*175.36*
2	*182.29*	*443.94*	*178.25*	*444.49*
3	*215.88*	*443.94*	*220.04*	*444.49*
4	*322.33*	*709.20*	*315.42*	*715.19*
5	*322.33*	*887.45*	*315.42*	*888.99*
6	*552.54*	*887.45*	*554.20*	*888.99*

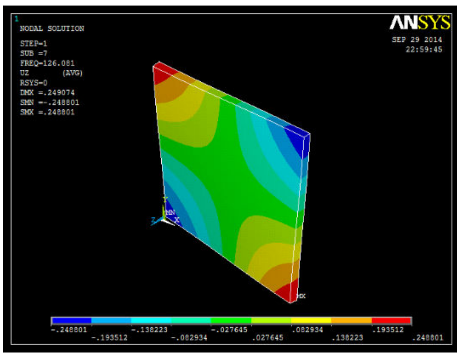
In the second and third columns of Table III, the maximum obtained error between analytical and numerical natural frequencies of RC slabs for both freely and simply supported conditions were obtained which are 3.63% and 1.31% respectively. This means that there is a good correlation between the analytical and numerical results in the whole frequencies range. This convergence was obtained due to the fact that in both methods the concrete slab is assumed as a linear, elastic, homogenous and isotropic material.

TABLE III
ERROR PERCENT BETWEEN ANALYTICAL AND NUMERICAL NATURAL FREQUENCIES OF RC SLABS WITH FREELY AND SIMPLY SUPPORTED CONDITIONS

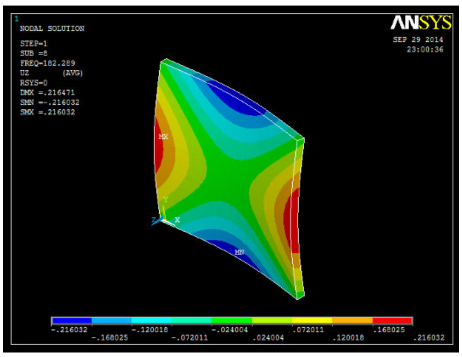
Mode No.	Error%		Difference%	
	Analytical Vs Numerical		Analytical	Numerical
	F.F Vs F.F	S.S Vs S.S	S.S Vs F.F	S.S Vs F.F
1	*3.63*	*1.31*	*30.71*	*29.04*
2	*2.21*	*0.12*	*59.90*	*58.93*
3	*1.93*	*0.12*	*50.49*	*51.37*
4	*2.14*	*0.28*	*55.65*	*54.55*
5	*2.14*	*0.17*	*64.51*	*63.67*
6	*0.3*	*0.17*	*37.66*	*37.73*

The analytical compression between freely and simply supported RC slabs was determined with percentage of difference ranging between 30.71 and 64.51%, which is shown in the fourth column of Table III. While, the numerical compression between freely and simply supported RC slabs was obtained with percentage of difference ranging between 29.04 and 63.67%, which is shown in the fifth column of Table III. This correlation means that simply support boundary condition has obvious influence to increase the natural frequencies of RC slabs when it is compared with freely supported boundary condition. Therefore, it is concluded that much more attentions is required to be paid when the slab is tested experimentally due to the effect of the supports. Furthermore, if the slab or any other structure is tested in the laboratory it is important to take the rigidity of the support into account which has the direct effect on dynamic behavior of a structure.

In addition to natural frequencies, the mode shapes of the slabs are also changed because of the boundary conditions freely and simply supported. The first six numerical mode shapes of the slabs were extracted and depicted in Figs. 2 and 3. It can be observed through these two figures that how changing the support condition will change the mode shapes of the F.F and S. S slabs.



First mode shape (1)



Second mode shape (2)

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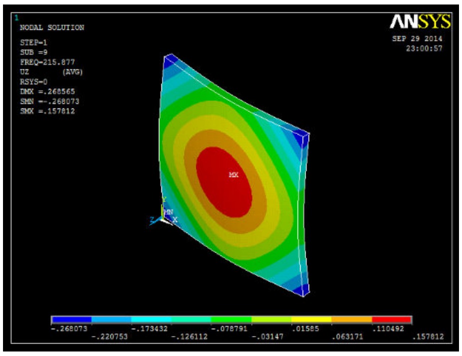
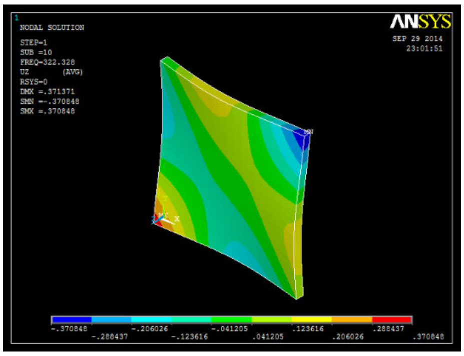
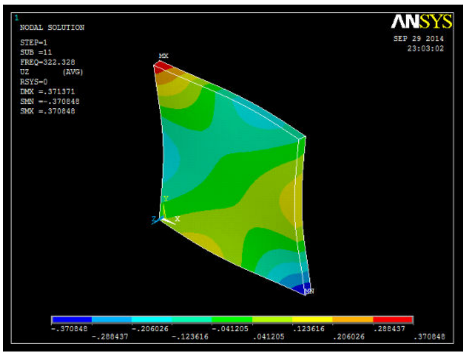


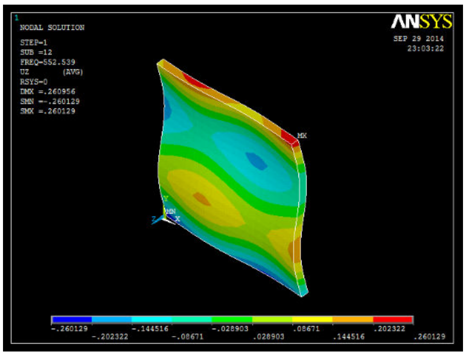
Third mode shape (3)



Fourth mode shape (4)

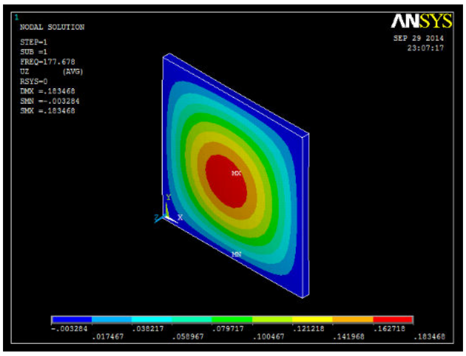


Fifth mode shape (5)

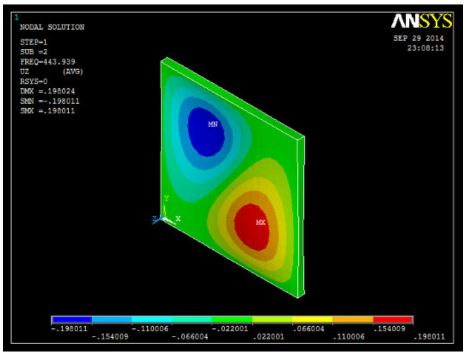


Sixth mode shape (6)

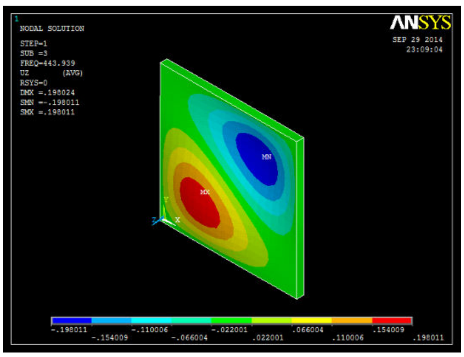
Fig. 2 Six mode shapes of F.F RC slabs



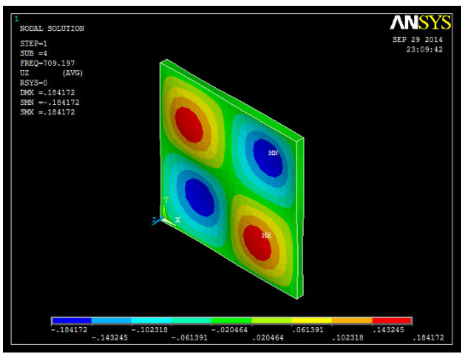
First mode shape (1)



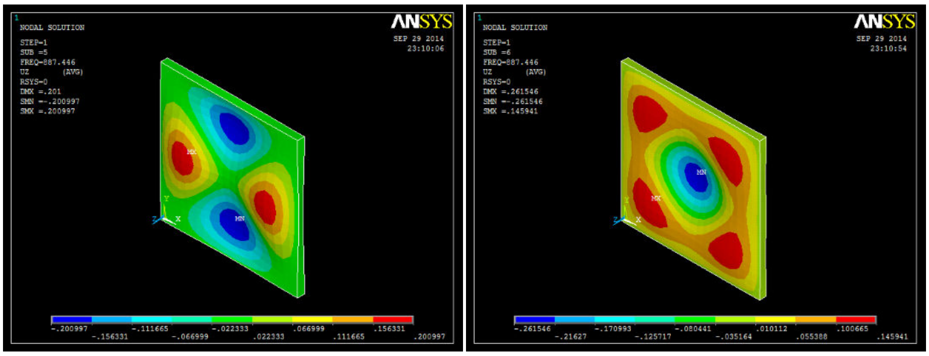
Second mode shape (2)



Third Mode shape (3)



Fourth mode shape (4)



Fifth mode shape (5) Sixth Mode shape (6)

Fig. 3 Six mode shapes of S.S RC slabs

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V. CONCLUSION

In this work, the dynamic characteristics of freely and simply supported RC slabs have been analyzed by theoretical modal analysis. The main aim of this paper is to find the effect of boundary conditions on natural frequencies and mode shapes of the slabs. To judge the reliability of the numerical outcomes, a compression between numerical and analytical natural frequencies has been made. A good correlation was obtained between the numerical and analytical results. This is because RC slab in both analyses was assumed as a linear, elastic, homogeneous and isotropic.

It is important to mention that in this study the analytical and numerical natural frequencies of freely, and simply supported RC slabs were compared. The analytical compression between freely and simply supported RC slabs was determined with percentage of difference ranging between 30.71 and 64.51. While, the numerical compression between freely and simply supported RC slabs was obtained with percentage of difference ranging between 29.04 and 63.67. Based on the results, it is concluded that simply support boundary condition has obvious influence to increase the natural frequencies and change the shape of mode of slabs when it is compared with freely supported boundary condition. This means that such support conditions have direct influence on the dynamic behavior of the slabs. Thus, it is suggested to use free-free boundary condition in experimental modal analysis to precisely reflect the properties of the structure. By using free-free boundary conditions, the influence of poorly defined supports is interrupted.

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