

# Designing Intelligent Adaptive Controller for Nonlinear Pendulum Dynamical System

R. Ghasemi, M. R. Rahimi Khoygani

**Abstract**—This paper proposes the designing direct adaptive neural controller to apply for a class of a nonlinear pendulum dynamic system. The radial basis function (RBF) neural adaptive controller is robust in presence of external and internal uncertainties. Both the effectiveness of the controller and robustness against disturbances are importance of this paper. The simulation results show the promising performance of the proposed controller.

**Keywords**—Adaptive Neural Controller, Nonlinear Dynamical, Neural Network, RBF, Driven Pendulum, Position Control.

## I. INTRODUCTION

HISTORICALLY pendulums were used as gravimeters to measure the acceleration of gravity in geophysical surveys, and even as a standard of length. Generally, they used to regulate pendulum clocks, in scientific instruments such as accelerometers, seismometers and widely in aerospace technology, robotics and etc. [1]. Many researches on modern control theory and intelligent control theory it as study object and many new control theories and methods are developed, because it consists of some physical subjects such as the simple harmonic motion, the period of oscillation, the acceleration of gravity, the center of mass, the moment of the inertia, momentum, etc. [1]. The Nonlinear Driven Pendulum Dynamical system poses a challenging control problem. However, the comparatively complex system is hard for low cost microcontrollers to process in many control systems. Intelligent computational techniques such as Artificial Neural Network (ANN), Fuzzy Logic theory (FL), Genetic Algorithm (GA) and etc., which have given novel solutions to the control nonlinear system problems [2]-[6].

Neural Networks (NNs) play a significant role in control engineering, especially in nonlinear system control. Due to their public approximation abilities, learning, and adaptation abilities, they are used to estimation unknown nonlinear functions. This makes them one of the effective tools in nonlinear controller system design. Active research has been carried out in NN controller by using the fact that NNs can estimate a wide range of nonlinear functions to any desired degree of accuracy under certain conditions.

The real capability of RBF Neural Networks for modeling and learning has rapidly increased in recent years. The weight update laws of the RBF networks are derived in such a way that the closed loop system is stable and the output tracking

error converges to 0 with time. When the approximation error of the RBF networks are taken into account, the weight update laws are modified in [7] for closed loop stability. But in this paper we have shown that the same update law can maintain the closed loop stability with a bounded tracking error. Though, the proposed adaptive control scheme is implementable only for a class of nonlinear pendulum dynamical system, however there are many practical systems for utilization.

There are literatures present which have taken linearization model of nonlinear Driven Pendulum dynamical System for implementing the various control schemes [8]-[12]. The paper aims to studying Position control the nonlinear Driven Pendulum dynamical system under the step input. To solve this problem, proposed a direct adaptive control scheme to achieve output tracking. The main advantage of a direct adaptive control scheme over an indirect adaptive control scheme is that in a direct adaptive control scheme there is no need for explicit system identification. In indirect adaptive control scheme, the system is generally identified off-line from its input-output data and the controller is designed based on the identified system model. The identified model should be accurate enough for better performance of the controller. Moreover stability is a critical issue in indirect adaptive control. But in direct adaptive control, the controller is designed in such a way that the closed loop stability is maintained while the tracking error converges to 0 with time. Theory of direct adaptive control is analyzed to demonstrate the effectiveness of the control scheme simulation is developed with in Simulink and Matlab for evaluation of the control strategy.

The contents of this paper are organized as follows: Section II Provided Mathematical modeling. Section III direct adaptive neural controller by RBF neural network; Section IV simulation and results; finally, in Section V presented a brief conclusion.

## II. MATHEMATICAL MODELING

It is obvious that in order to analysis and control of a physical system, it is necessary to construct the mathematical modeling. In the above picture, there is a DC motor with a propeller on the lead of a suspended stick. After applied voltage, the propeller spins and generates torque  $T$  to pull up the pendulum. It is the most benefits of driven pendulum that enables us controlling its behavior with regulating the applied voltage. Therefore, the control variable for this system is the angle of the pendulum settled and the manipulated variable is the voltage fed to the motorized-propeller. The schematic

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picture of the suspended pendulum control system is given in Fig. 1.

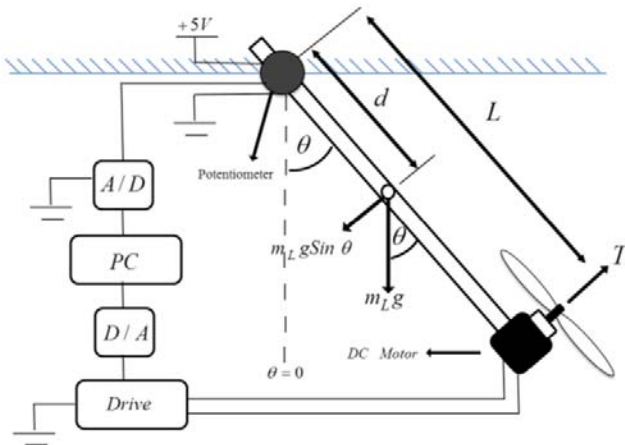


Fig. 1 The Driven Pendulum system

The rational equation between  $V$  voltage which applied to DC motor and thrust  $T$ , can be written as follows:

$$T = K_m \cdot V \quad (1)$$

According to Newton's laws and angular momentum, the motion equation of driven pendulum is derived as:

$$J \ddot{\theta} + c \dot{\theta} + m_L \cdot g \cdot d \cdot \sin \theta = T \quad (2)$$

The generated thrust  $T$  in above equations is not manipulated variable for control system since the pendulum is adjusted by applied voltage.

$$\ddot{\theta} = \frac{K_m V - c \dot{\theta} - m_L \cdot g \cdot d \cdot \sin \theta}{J} \quad (3)$$

where,  $m$ =weight of the pendulum,  $L$ =Length of pendulum,  $J$ =inertia moment,  $g$ =acceleration of gravity,  $V$ =voltage DC motor,  $T$ =the thrust which is provided by DC motor,  $\theta$ =angular position,  $c$ =viscous damping coefficient,  $d$ =the distance from suspending point to center of mass. For numerical simulation of the nonlinear model for the driven pendulum system, it is required to represent the nonlinear equations (4) into standard state space form:

$$\frac{d}{dt} X = F(x, u) \quad (4)$$

Now these equations may be represented into state space form by considering the state variables as following:

$$x_1 = \theta, \quad x_2 = \dot{\theta} = \dot{x}_1, \quad \dot{x}_2 = \ddot{\theta} \quad (5)$$

Then, the final state space equation for the driven Pendulum system may be written as:

$$\frac{d}{dt} X = \frac{d}{dt} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \frac{d}{dt} \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix} \quad (6)$$

where

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = f(x) + g(x)u + d(x) \\ y(x) = h(x) = x_1, u = V \\ f(x) = -J^{-1}(cx_2 + m_L \cdot g \cdot d \cdot \sin(x_1)) \\ g(x) = K_m \cdot J^{-1} \end{cases} \quad (7)$$

$d(x)$  is disturbance The pendulum angle  $\theta$  is the variable of interest, and then the output equation may be written;

$$Y = [x] = CX = [1 \quad 0] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad (8)$$

### III. DIRECT ADAPTIVE NEURAL CONTROLLER

Artificial Neural Networks (NNs) which are inspired by biological nervous systems are promising and powerful tools in identification and modeling of non-linear systems due to their effective suitability in learning capabilities and data processing. Radial basis function (RBF) networks are feedforward networks; they are typically configured with a single hidden layer of units whose activation function is selected from a class of functions called basis functions. While similar to back propagation in many respects, radial basis function networks have several advantages. They usually train much faster than back propagation networks. They are less susceptible to problems with non-stationary inputs because of the behavior of the radial basis function hidden units. The structure RBF neural network is shown in Fig. 2.

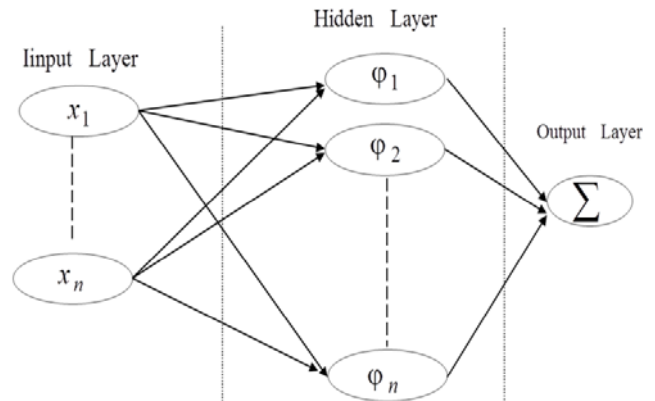


Fig. 2 The RBF neural network

The problem arises with Feedback Linearization Control (FLC) techniques when the nonlinear functions  $f(x)$  or  $g(x)$  or both are unknown. For such cases different function approximators like neural networks or fuzzy systems can be used to estimate these nonlinear functions.

#### A. $f(x)$ Is Unknown: Using FLC

In this section we have assumed that the function  $f(x)$  is

unknown while the function  $g(x)$  is known.  $f(x)$  is approximated by a RBF network. Feedback linearization is a common approach used in controlling nonlinear systems that has attracted lots of research in recent years. The central idea is to algebraically transform nonlinear systems dynamics into (fully or partly) linear ones. The approach involves coming up with a transformation of the nonlinear system into an equivalent linear system through a change of variables and a suitable control input, so that linear control techniques can be applied. As can be seen from Fig. 2, the RBFN has a single output thus the network weight constitutes a vector  $W$  in this case. Then the control law;

$$u = \frac{1}{g(x)} \left[ -\hat{f}(x) - k_v r + \dot{x}_{2d} + \lambda e^{(1)} \right] \quad (9)$$

where  $e$  is output tracking,  $\hat{f}(x)$  is approximated  $f(x)$ ,  $r$  is power denotes respective derivatives,  $k_v$  and  $\lambda$  are positive design parameters, the closed loop error dynamics becomes;

$$\dot{r} = -k_v r \quad (10)$$

where

$$r = e^{(1)} + \lambda_1 e^{(1)} = \dot{e} \quad (11)$$

Using (7), the output tracking error is defined as;

$$e = y_d - y = x_{1d} - x_1 \quad (12)$$

where  $y_d$  is ideal output system, and  $y$  is real output system. There exist weights such that;

$$f(x) = W^T \varphi(x) + \varepsilon \quad (13)$$

$\varepsilon$  is the estimation error,  $W$  is the ideal network weights, that can be approximated  $\hat{f}(x)$ , therefore, network weights are constant. Here have assumed that the estimation error  $\varepsilon=0$ . Therefore, Let  $f(x)$  be approximated as

$$\hat{f}(x) = \hat{W}^T \varphi(x) \quad (14)$$

when the structure of the neural network is known, a suitable learning rule should be appointed to educate the network. This weight-updating mechanism is usually defined in such a way that the stability of the observer is guaranteed. Where  $\hat{W}$  is updated using the update law

$$\dot{\hat{W}} = -F \varphi(x) r \quad (15)$$

where parameter matrix  $F$  is taken as the diagonal matrix of appropriate dimension. The structure direct adaptive neural controller by RBF neural network for this section is shown in Fig. 3.

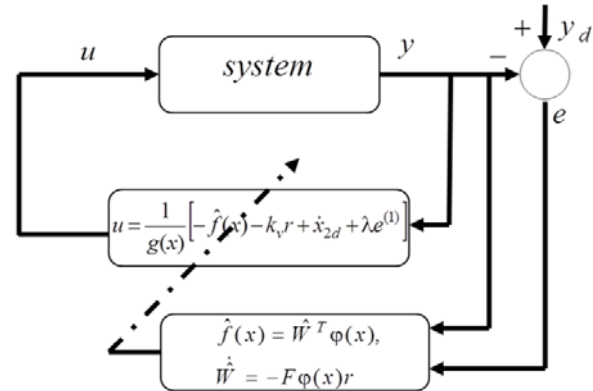


Fig. 3 The structure direct adaptive neural controller

#### B. When $f(x)$ and $g(x)$ Are Unknown: Using FLC and SMC

It is difficult to achieve closed loop stability when both  $f$  and  $g$  are unknown. It is assumed that  $|g(x)| > 0$ . An additional sliding mode term is added with the control input to maintain the closed loop stability. Sliding Mode Control (SMC) is a kind of control technique of variable structure control systems. Sliding mode control technique is based on forcing the error vector into desired dynamic and holding in this dynamic. The line whereof was discussed was called as surface, switching surface, sliding surface over time. The basic philosophy at the sliding mode control is to generate control strategy at the sliding surface. The control law when both  $f$  and  $g$  are unknown:

$$u = u_1 + u_2 \quad (16)$$

where

$$u_1 = \frac{1}{\hat{g}(x)} \left[ -\hat{f}(x) - k_v r + \dot{x}_{2d} + \lambda e^{(1)} \right] \quad (17)$$

Therefore, Let  $f(x)$  and  $g(x)$  be approximated as;

$$\begin{aligned} \hat{f}(x) &= \hat{W}^T \varphi(x) \\ \hat{g}(x) &= \hat{P}^T \psi(x) \end{aligned} \quad (18)$$

where  $\hat{f}(x)$  and  $\hat{g}(x)$  are estimated,

$$u_2 = \frac{|\hat{g}(x)|}{g_1} |u_1| \text{sgn } r \quad (19)$$

where  $g_1$  is lower bound of  $g$ , the structure direct adaptive neural controller by RBF neural network for this section is shown in Fig. 4.



In Fig. 8 observed the output tracking responses and output error with disturbances to power 0.001 and Frequency signal 1000 stays stable.

#### V. CONCLUSION

The strategy for adaptive neural network control of driven pendulum system nonlinearity and the input nonlinearity have been assumed to be unknown and RBF networks have been used to approximate those nonlinearities. Feedback linearization techniques have been successfully used over the past decades to design controllers for nonlinear systems. The controller design becomes difficult when the functions of system are unknown completely. Direct adaptive control with the update laws for the networks used to establish stability of the closed loop system. A sliding mode term was added to the adaptive control law to maintain the boundedness of the closed loop system. The simulation results show the promising performance of the proposed controller.

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