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Heat and Mass Transfer for Viscous Flow with Radiation Effect past a Nonlinearly Stretching Sheet

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***Abstract*—**In this study, an analysis has been performed for heat and mass transfer of a steady laminar boundary-layer flow of a viscous flow past a nonlinearly stretching sheet. Parameters n, Ec, k0, Sc represent the dominance of the nonlinearly effect, viscous effect, radiation effect and mass transfer effect which have presented in governing equations, respectively. The similarity transformation and the finite-difference method have been used to analyze the present problem.

***Keywords*—**Nonlinearly stretching sheet, heat and mass transfer, radiation effect, viscous effect.

NOMENCLATURE

A control surface area
B constant
C concentration
D diffusing coefficient
c_p specific heat at a constant pressure
Ec Eckert number
f dimensionless stream function
g dimensionless temperature
k1 surface temperature parameter.
k fluid thermal conductivity
k_0 dimensionless parameter related with thermal radiation
k^* mean absorption coefficient
L reference length
m surface temperature parameter
n parameters related to the surface stretching speed.
N_r radiation parameter
q_r radiative heat flux
T temperature across the thermal boundarylayer
u, v velocity components along x and y directions, respectively
x, y Cartesian coordinates along the plate and normal to it, respectively
α thermal diffusivity
η dimensionless similarity variable
θ dimensionless temperature
μ dynamic viscosity
ν kinematic viscosity
ρ density
Pr Prandtl number
σ^* Stefan-Boltzmann constant
τ shear stress

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I. INTRODUCTION

THE study of visco-elastic fluids had become of increasing importance in the last few years. Qualitative analyses of these studies have significant bearing on several industrial applications such as polymer sheet extrusion from a dye, drawing of plastic films etc. When the manufacturing process at high temperature and need cooling the stretching sheet. The flows maybe need visco-elastic fluids to produce a good effect to reduce the temperature from the sheet. And also, the fluids have processed many types of effects (i.e. magnetic force, buoyancy and mass diffusion) into the problem, and have become a hybrid system need to analysis by many different ways. It is a well-known fact in the studies of non-Newtonian fluid flows by Hartnett [1]. Rajagopal et al. [2] studied a Falkner-Skan flow field of a second-grade visco-elastic fluid. Massoudi and Ramezan [3] studied a wedge flow with suction and injection along walls of a wedge by the similarity method and finite-difference calculations. An excellent review of boundary layers in non-linear fluids was recently written by Rajagopal [4]. These are related studies to the present investigation about second-grade fluids. All of above are dealing with forced convection problems. Recently, Vajravelu and Soewono [5] had solved the fourth order non-linear systems arising in combined free and forced convection flow of a second order fluid, over a stretching sheet. The stretching sheet flow of a non-Newtonian fluid is also one of important flow fields in real world, Raptis [6] had studied heat

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transfer of a visco-elastic fluid. On the other hand, researches in connection with visco-elastic fluid or second grade non-Newtonian fluids, but there are not the mixed convection flow [7]. Recently, the thermal boundary layer over a nonlinearly stretching sheet has become important for many studies toward the related problems. Rafael Cortell [8] studied effects of viscous dissipation and radiation on the thermal boundary layer over a nonlinearly stretching sheet. Kechil and Hashim [9] studied series solution of flow over nonlinearly stretching sheet with chemical reaction and magnetic field. Bataller [10] studied similarity solutions for flow and heat transfer of a quiescent fluid over a nonlinearly stretching surface. Cortell [11] studied viscous flow and heat transfer over a nonlinearly stretching sheet. Vajravelu [12] studied viscous flow over a nonlinearly stretching sheet. Sanjayanand et al. [13], Cortell, Rafael [14] and Seddeek [15] had studied the heat and mass transfer problems about the viscoelastic boundary layer flow over a stretching sheet with magnetic effect, but not consider the mixed convection with radiation effect. In the present investigation, a study for heat and mass transfer problem has been processed.

II. Theoretical and Analysis

A. Flow Field Analysis

We consider the flow of an incompressible viscous fluid past a flat sheet coinciding with the plane $y = 0$, the flow being confined to $y > 0$. Two equal and opposite forces are applied along the x-axis so that the wall is stretched keeping the origin fixed. The steady two-dimensional boundary layer equations for this fluid, in the usual notation, are:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2} \tag{2}$$

The boundary conditions to the problem are:

$$u_w(x)=Bx^n,\ v=0 \text{ at } y=0, \tag{3}$$

$$u\to 0 \text{ as } y\to\infty, \tag{4}$$

Defining new similarity variables as:

$$\eta=y\sqrt{\frac{B(n+1)}{2v}}x^{\frac{n-1}{2}},\ u=Bx^n f'(\eta), \tag{5}$$

$$v=-\sqrt{\frac{Bv(n+1)}{2}}x^{\frac{n-1}{2}}\left[f+\frac{n-1}{n+1}\eta f'\right]$$

and substituting into Eqs. (1) and (2) give:

$$(f')^2\left(\frac{2n}{n+1}\right)-ff''-f'''=0 \tag{6}$$

where a prime denotes differentiation with respect to the independent similarity variable η. The boundary conditions (3) and (4) become:

$$f=0, f'=1 \text{ at } \eta=0 \tag{7}$$

$$f'\to 0 \text{ as } \eta\to\infty \tag{8}$$

For the linearly stretching boundary problem (i.e., n = 1) the exact solution for the velocity field f is:

$$f(\eta)=1-\exp(-\eta) \tag{9}$$

and this exact solution is unique, while for the nonlinearly stretching boundary problem (i.e., $n \neq 1$) there is no exact solution. The shear stress at the stretched surface is defined as

$$\tau_w=\mu\left(\frac{\partial u}{\partial y}\right)_w \tag{10}$$

and we obtain form (5) and (10)

$$\tau_W=B\mu\sqrt{\frac{B(n+1)}{2\upsilon}}x^{\frac{3n-1}{2}}f''(0) \tag{11}$$

B. Heat Transfer Analyses

By using usual boundary layer approximations, the equation of the energy for temperature T in the presence of radiation and viscous dissipation, is given by:

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2}+\frac{\upsilon}{c_p}\left(\frac{\partial u}{\partial y}\right)^2-\frac{1}{\rho c_p}\frac{\partial q_r}{\partial y} \tag{12}$$

Using the Rosseland approximation for radiation [16], the radiative heat flux is simplified as

$$q_r=-\frac{4\sigma^*}{3k}\frac{\partial T^4}{\partial y} \tag{13}$$

We assume that the temperature differences within the flow such as that the term T^4 may be expressed as a linear function of temperature. Hence,

expanding T^4 in a Taylor series about T_∞ and neglecting higher-order terms we get

$$T^4\cong 4T_\infty^3T-3T_\infty^4 \tag{14}$$

In view to Eqs. (13) and (14), Eq. (15) reduces to

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\alpha}{k_0}\frac{\partial^2 T}{\partial y^2}+\frac{\upsilon}{cp}\left(\frac{\partial u}{\partial y}\right)^2 \tag{15}$$

where $\alpha=\frac{k}{\rho cp}$ is the thermal diffusivity; $k_0=\frac{3N_R}{3N_R+4}$ and $N_R=\frac{k_T k^*}{4\sigma^* T_\infty^3}$ is the radiation parameter. It is worth mentioning that the parameter N_R is physically more relevant that the similarity parameter k_0 above introduced. For this reason, although throughout the paper the parameter k_0 will be employed some time to simplify some equations, however, we take N_R instead of k_0 as a governing parameter. If the thermal radiation's effect is not considered in the energy equation, we have $k_0 = 1$ in the above equation. Similarity solutions of Eq. (15) can be found by choosing appropriate boundary conditions. It is of a certain interest to consider separately the characteristics of the following two cases of main practical interest.

Here, the boundary conditions are:

$$T=T_W\left(=T_\infty+Ax^{k1}\right) \text{ as } y=0; \tag{16}$$

$$T\to T_\infty \text{ as } y\to\infty$$

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Defining the non-dimensional temperature $\theta(\eta)=\frac{T-T_\infty}{T_W-T_\infty}$ and using Eqs. (5) and (16) into Eq. (15), we get .

$$\theta''+\sigma k_0 f\theta'-\left(\frac{2k}{n+1}\right)\sigma k_0 f'\theta= -\sigma k_0 E_c x^{2n-k}\left(f''\right)^2 \quad (17)$$

$$\theta(0)=1,\quad \theta(\infty)\to 0, \quad (18)$$

Where $E_C=\frac{C^2}{Ac_p}$, and $Pr=\frac{\upsilon}{\alpha}$

C. Mass Transfer Analyses

The steady boundary-layer equation for this flow, mass transfer, in usual notations, is:

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2} \quad (19)$$

where C is the concentration, D is mass diffusivity, respectively. For the solutions of heat and mass transfer equations, it can be defined non-dimensional temperature and concentration variables as:

$$\phi(\eta)=\frac{C-C_\infty}{C_w-C_\infty} \quad (20)$$

This leads to the non-dimensional form of temperature and concentration equations as follows:

$$\phi''+S_c f\phi'=0 \quad (21)$$

Where Sc=υ/D is the Schmidt number. The corresponding boundary conditions are:

$\phi=1$ at $\eta=0$

$\phi=0$ as $\eta\to\infty$

the heating rate on the wall is:

$$q_w=-k(\frac{\partial T}{\partial y})_{y=0}=-k(T_w-T_\infty)\sqrt{B(n+1)/2\upsilon}x^{(n-1)/2}\theta'(0) \quad (22)$$

Once we know the f (η) and its derivatives, one can calculate the values of the local skin friction at the surface from the following relations:

$$\tau_x=-(\frac{\partial u}{\partial y})_{y=0}=B\mu\sqrt{B(n+1)/2\upsilon}x^{(3n-1)/2}f''(0) \quad (23)$$

In addition, the local Nusselt number Nu_x has defined by:

$$Nu_x=\frac{hx}{k}=\frac{q_w}{T_w-T_\infty}\frac{x}{k} \quad (24)$$

This expression has written as:

$$Nu_x=-\sqrt{B(n+1)/2\upsilon}x^{(n+1)/2}\theta'(0) \quad (25)$$

The Sherwood number has defined by:

$$Sh_x=\frac{hx}{C_w-C_\infty}(\frac{\partial C}{\partial y})_{y=0}=-\sqrt{B(n+1)/2\upsilon}x^{(n-1)/2}\phi'(0) \quad (26)$$

III. Numerical Technique

In the present problem, the set of similar equations (13) to (17) are solved by a finite difference method. These ordinary differential equations have discretized by an accurate central difference method, and a computer program has been developed to solve these equations. To avoid errors in discretization and calculation processing and to ensure the convergence of numerical solutions, some conventional numerical procedures have been applied in order to choose a suitable grid size $\Delta\eta=0.01$ - 0.05, a suitable η range and a direct gauss elimination method with Newton's method [17] is used in the computer program to obtain solutions of these difference equations. Hsiao et al. [18-23] Vajravelu. [24] are also using analytical and numerical solutions to solve the related problems. So, some numerical technique methods will be applied to the same area in the future. In this study, the program to compute finite difference approximations of derivatives for equal spaced discrete data. The code employ centered differences of O(h^2) for the interior points and forward and backward differences of O(h) for the first and last points, respectively. See Chapra and Canale, Numerical Methods for Engineers [25].

IV. Results and Discussion

The model for grade-two fluids is used in this study. The effects of dimensionless parameters are included the nonlinearly number (n), the Prandtl number (Pr), the radiation parameter (k0), the viscous dissipation number (Ec) and the Schmidt number (Sc) which are mainly interested of the study. Flow and temperature fields of the stretching sheet flow are analyzed by utilizing the boundary layer concept to obtain a set of coupled momentum equation, energy equation and mass equation. A similarity transformation is then used to convert the nonlinear, coupled partial differential equations to a set of nonlinear, coupled ordinary differential equations. A second-order accurate finite difference method is used to obtain solutions of these equations.

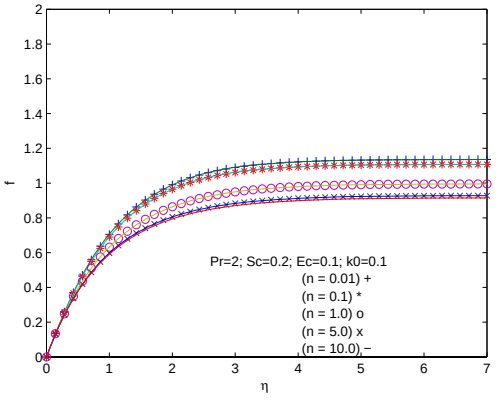


Fig. 1 f vs. η for varies parameters

Fig. 1 depicts dimensionless velocity profiles f vs. η as Pr=2, Sc=0.2, Ec=0.1, k0=0.1 and n=0.01, 0.1, 1.0, 5.0, 10.0. Figure 1 reveals that the increase of n results in the decrease of dimensionless velocity distribution at a particular point of the flow region. This is because there would be a decrease of the momentum boundary layer thickness with the increase of

values of non-linear parameter n. From the result could find the momentum effect decreasing with a larger n.

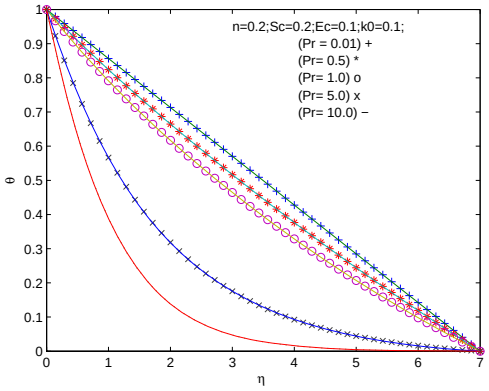


Fig. 2 θ vs. η for varies parameters

Fig. 2 depicts dimensionless temperature gradient profiles θ' vs. η as n=0.2, Sc=0.2, Ec=0.1, k0=0.1 and Pr=0.01, 0.5, 1.0, 5.0, 10.0. Figure 2 reveals that the increase of Prandtl number Pr results in the decrease of temperature distribution at a particular point of the flow region. This is because there would be a decrease of the thermal boundary layer thickness with the increase of values of Prandtl number Pr. From the result could find the thermal effect increasing with a larger Pr.

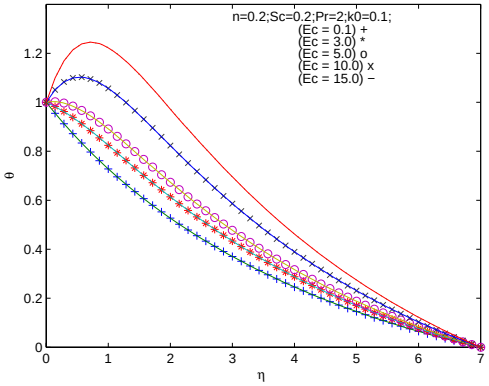


Fig. 3 θ vs. η for varies parameters

Fig. 3 depicts dimensionless temperature gradient profiles θ' vs. η as n=0.2, Sc=0.2, Pr=0.1, k0=0.1 and Ec=0.1, 3, 5.0, 10.0, 15.0. Fig. 3 reveals that the increase of viscous dissipation number Ec results in the increase of dimensionless temperature distribution at a particular point of the flow region. This is because there would be a increase of the thermal boundary layer thickness with the decrease of values of viscous dissipation number Ec. From the result could find the thermal effect decreasing with a larger Ec.

Fig. 4 depicts dimensionless temperature gradient profiles θ' vs. η as n=0.2, Sc=0.2, Ec=0.1, Pr=2 and k0=0.01, 0.5, 1.0, 2.0, 3.0. Fig. 4 reveals that the increase of radiation parameter k0 results in the decrease of temperature distribution at a particular point of the flow region. This is because there would be a decrease of the thermal boundary layer thickness with the increase of values of radiation parameter k0. From the result could find the thermal effect increasing with a larger k0.

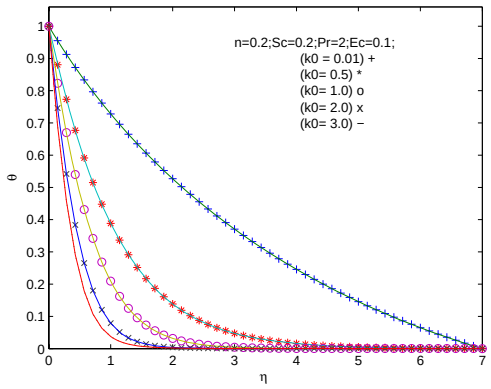


Fig. 4 θ vs. η for varies parameters

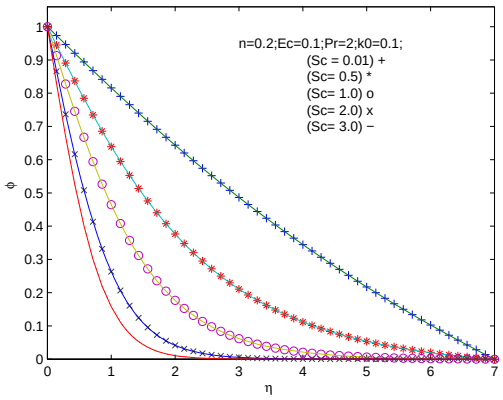


Fig. 5 ϕ vs. η for varies parameters

Fig. 5 depicts dimensionless concentration profiles ϕ vs. η as Pr=2.0, k0=0.1, Ec=0.1 and Sc = 0.01, 0.5, 1.0, 2.0, 3.0. The effect of Schmidt number Sc on mass transfer process may be analysis from Figure 5 for the case of prescribed concentration and prescribed mass flux, respectively. Fig. 5 also shows that the increase of value of Schmidt number Sc results in the decrease of concentration distribution as a result of decrease of the concentration boundary layer thickness with the increased values of Sc. From the result could find the mass transfer effect increasing with a larger Sc.

V. Conclusion

There are some important conclusions as:

(1) It seemed that the increase of Prandtl number Pr results in the decrease of temperature distribution at a particular point of the flow region.
(2) It was found that when parameter n increased, the fluid velocity decreased. However it is observed that the effect of momentum in the boundary layer, which causes the temperature to decrease, while the presence of heat

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absorption effects caused reductions in the fluid temperature, which results in decreasing the fluid velocity.

(3) It was become that the increase of Ec results in the increase of temperature distribution at a particular point of the flow region.

(4) It is also observed that increase in thermal radiation parameter k0 produces a significant decrease in the thickness of the thermal boundary layer of the fluid and so as the temperature decreases in presence/absence of thermal conductivity parameter.

(5) The effect of Schmidt number Sc on mass transfer process may show that the increase of value of Schmidt number Sc results in the decrease of concentration distribution as a result of decrease of the concentration boundary layer thickness with the increased values of Sc.

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REFERENCES

[1] J.P. Hartnett, Viscoelastic fluids: a new challenge in heat transfer, Transactions of ASME, 296-303, (1992).

[2] K.R. Rajagopal, A.S. Gupta and T.Y. Na, A note on the Falkner-Skan flows of a non-Newtonian fluid, Int. J. Non-linear Mech., 313-320, (1983).

[3] M. Massoudi and M. Ramezan, Effect of injection or suction on the Falkner-Skan flows of second grade fluids, Int. J. Non-linear Mech., 221-227, (1989).

[4] K. R. Rajagopal, Boundary Layers in Non-linear Fluids, Pitman Monographs and Surveys in Pure and Applied Mathematics, 209-218, (1995).

[5] K. Vajravelu and E. Soewono, Fourth order non-Linear systems arising in combined free and forced convection flow of a second order fluid, Int. J. Non-Linear Mech. Vol. 29, No. 6, 861-869 (1994).

[6] A.A. Raptis, Heat Transfer From Flow of an Elastico Viscous Fluid, Int. Comm. Heat Mass Transfer Vol. 16, 193-197, (1989).

[7] Sujit Kumar Khan, Heat transfer in a viscoelastic fluid flow over a stretching surface with heat source/sink, suction/blowing and radiation, International Journal of Heat and Mass Transfer, Volume 49, Issues 3-4, February 2006, Pages 628-639.

[8] Rafael Cortell, Effects of viscous dissipation and radiation on the thermal boundary layer over a nonlinearly stretching sheet Physics Letters A, Volume 372, Issue 5, 28 January 2008, Pages 631-636.

[9] S. Awang Kechil and I. Hashim, Series solution of flow over nonlinearly stretching sheet with chemical reaction and magnetic field Physics Letters A, In Press, Corrected Proof, Available online 19 November 2007.

[10] Rafael Cortell Bataller, Similarity solutions for flow and heat transfer of a quiescent fluid over a nonlinearly stretching surface Journal of Materials Processing Technology, In Press, Corrected Proof, Available online 22 October 2007.

[11] Rafael Cortell, Viscous flow and heat transfer over a nonlinearly stretching sheet Applied Mathematics and Computation, Volume 184, Issue 2, 15 January 2007, Pages 864-873.

[12] K. Vajravelu, Viscous flow over a nonlinearly stretching sheet Applied Mathematics and Computation, Volume 124, Issue 3, 15 December 2001, Pages 281-288.

[13] Sanjayanand, Emmanuel; Khan, Sujit Kumar, On heat and mass transfer in a viscoelastic boundary layer flow over an exponentially stretching sheet, International Journal of Thermal Sciences Volume: 45, Issue: 8, August, 2006, pp. 819-828.

[14] Cortell, Rafael, MHD flow and mass transfer of an electrically conducting fluid of second grade in a porous medium over a stretching sheet with chemically reactive species, Chemical Engineering & Processing Volume: 46, Issue: 8, August, 2007, pp. 721-728.

[15] Seddeek, M.A., Heat and mass transfer on a stretching sheet with a magnetic field in a visco-elastic fluid flow through a porous medium with heat source or sink, Computational Materials Science Volume: 38, Issue: 4, February, 2007, pp. 781-787.

[16] R. S. Rivlin and J. L. Ericksen, Stress deformation relation for isotropic materials, J. Rat. Mech. Analysis, 323-425, (1955).

[17] T. Cebeci and P. Bradshaw, Physical and Computational Aspects of Convective Heat Transfer, Springer-Verlag, (1984).

[18] Kai-Long Hsiao and Guan-Bang Chen, Conjugate Heat Transfer of Mixed Convection for Viscoelastic Fluid Past A Stretching Sheet, Mathematical Problems in Engineering, Volume 2007, Article ID 17058, 21 pages doi:10.1155/2007/17058.

[19] Kai-Long Hsiao, Conjugate Heat Transfer of Magnetic Mixed Convection with Radiative and Viscous Dissipation Effects for Second Grade Viscoelastic Fluid past a Stretching Sheet, Applied Thermal Engineering, 27/11-12 pp. 1895-1903 (2007).

[20] Kai-Long Hsiao, Heat and Mass Transfer for Electrical Conducting Mixed Convection with Radiation Effect for Viscoelastic Fluid past a Stretching Sheet, Journal of Mechanics, Vol.24, No.2, June 2008, (pp.N21-N27).

[21] Kai-Long Hsiao, MHD Mixed Convection of Viscoelastic Fluid over a Stretching Sheet with Ohmic Dissipation, Journal of Mechanics, Vol.24, No.3, September 2008, (pp.N29-N34).

[22] Kai-Long Hsiao and C.H. Hsu, Conjugate heat transfer of mixed convection for viscoelastic fluid past a horizontal flat-plate fin, Applied Thermal Engineering, 29/1, pp 28-36, January 2009.

[23] Kai-Long Hsiao and C.H. Hsu, Conjugate heat transfer of mixed convection for visco-elastic fluid past a triangular fin, Nonlinear Analysis (Series B: Real World Applications), Vol.10/1, pp 130-143, February 2009.

[24] K. Vajravelu, Convection Heat Transfer at a Stretching Sheet with Suction and Blowing, J. of Mathematical Analysis and Application, 188, 1002-1011(1994).

[25] Chapra and Canale, Numerical Methods for Engineers, McGRAW-HILL, 2ed, 1990.



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