

# Iterative Joint Power Control and Partial Crosstalk Cancellation in Upstream VDSL

H. Bagheri, H. Emami, M. R. Pakravan

**Abstract**—Crosstalk is the major limiting issue in very high bit-rate digital subscriber line (VDSL) systems in terms of bit-rate or service coverage. At the central office side, joint signal processing accompanied by appropriate power allocation enables complex multiuser processors to provide near capacity rates. Unfortunately complexity grows with the square of the number of lines within a binder, so by taking into account that there are only a few dominant crosstalkers who contribute to main part of crosstalk power, the canceller structure can be simplified which resulted in a much lower run-time complexity.

In this paper, a multiuser power control scheme, namely iterative waterfilling, is combined with previously proposed partial crosstalk cancellation approaches to demonstrate the best ever achieved performance which is verified by simulation results.

**Keywords**—iterative waterfilling, partial crosstalk cancellation, run-time complexity, VDSL.

## I. INTRODUCTION

VERY high bit-rate digital subscriber line (VDSL) offers multi-ten-Mbps services by using up to 20 MHz region in the ordinary telephone copper twisted pairs. High frequency application imposes some distortions such as attenuation, crosstalk, impulsive and radio noises which limit capacity of the access channel. In these systems, crosstalk due to electromagnetic induction of neighborhood lines is the major concern. It arises in near-end (NEXT) and far-end crosstalk (FEXT) types, wherein NEXT refers to the crosstalk created at the same side of the cable while FEXT is generated at the other side. NEXT is usually much stronger than FEXT since FEXT is attenuated as it travels along the loop length. Using time or frequency division duplexing (TDD and FDD as here assumed) circumvents generation of self NEXT (other VDSL users' NEXT). Other systems' NEXT usually don't impinge on VDSL systems substantially, as they occupy much narrower bandwidth, consequently self FEXT cancellation becomes the target of VDSL crosstalk cancellation schemes, especially as a result of short length VDSL lines. FEXT can be very

destructive in near-far scenarios in which strong signal of near-end transmitter destroys attenuated signal of far-end transmitter in the way to their target points. This might occur only in upstream transmission to central office (CO) for VDSL systems; however Asymmetric DSL (ADSL) systems may suffer from it in the downstream direction [1].

Impairments due to high frequency usage make telephone channels severely frequency selective, as a result, intersymbol interference (ISI) occurs which leads to imperfect detection. ISI effect can be mitigated by dividing the spectrum into  $N$  approximately flat subchannels, which is called discrete multitone (DMT) modulation. By applying well known water-filling (WF) algorithm, the transmitter can distribute its power among subchannels according to their signal to noise ratios (SNRs) in order to maximize its bit-rate.

Multiuser power control and multiuser detection (MUD) based schemes are the two general approaches to reduce crosstalk destructive effects. While the former tries to vary power spectral density (PSD) of the users in order to lessen generation of crosstalk, the latter attempts to cancel the existing one. Although MUD based approaches outperform the avoidance ones, they suffer from their run-time complexity ( $\sim$  multi billions multiplications/second) which is not currently realizable and may remain infeasible economically for several years [2].

The main part of crosstalk power seen by each user comes from a few numbers of its neighboring lines in the binder. In addition, crosstalk cancellation may lead to large performance gains only in a small sub-set of tones. Taking these points into account, large run-time complexity reduction can be achieved via partial crosstalk cancellation (canceling only dominant crosstalkers in space and frequency domains).

In this paper, a multiuser power control procedure, known as iterative waterfilling (IWF), is joined with existing partial crosstalk cancellation (PCC) algorithms to achieve near capacity rates, which is not considered before. It also, leads to larger run-time complexity reduction at the expense of slightly higher initialization complexity in comparison to previous algorithms wherein power control schemes have not been applied. Various simulation results show that the iterative algorithm almost always converges after 1~2 iterations, so the initialization complexity increases marginally. It must be noted that the initialization complexity is not taken into account, in crosstalk cancellation complexity calculations due to almost static nature of DSL channels.

The rest of the paper is organized as follows. Section II

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formulates the problem and considers full crosstalk cancellation algorithm, known as vectored DMT (VDMT). Section III, describes various PCC schemes. IWF and its combination with PCC algorithms are addressed in Section IV. Section V shows the simulation results and conclusions are drawn in section VI.

## II. FULL CROSSTALK CANCELLATION

### A. System Model

In VDSL systems, synchronized symbol transmission and reception at CO/ONU enables us to model crosstalk independently on each tone [3]. Assuming  $N$  users in a binder each with  $K$  tones, upstream transmission of a single DMT block can be modeled as

$$\begin{bmatrix} y_k^1 \\ \vdots \\ y_k^N \end{bmatrix} = \begin{bmatrix} h_k^{(1,1)} & h_k^{(1,N)} \\ \vdots & \ddots \\ h_k^{(N,1)} & h_k^{(N,N)} \end{bmatrix} \begin{bmatrix} x_k^1 \\ \vdots \\ x_k^{N+1} \end{bmatrix} + \begin{bmatrix} z_k^1 \\ \vdots \\ z_k^N \end{bmatrix}$$

$$y_k = H_k x_k + z_k \quad (1)$$

Where  $x_k^n$  and  $y_k^n$  are the transmitted and received symbols by user  $n$  on tone  $k$  respectively. The tone  $k$  is within  $1, \dots, K$  where  $K$  is the number of DMT upstream tones.  $h_k^{(n,n)}$  is the direct channel of user  $n$  at tone  $k$ , and  $h_k^{(n,m)}$  is the crosstalk channel from user  $m$  into user  $n$ .  $z_k^n$  is the additive white Gaussian noise experienced by user  $n$  on tone  $k$  with  $E\{z_k z_k^H\} = \sigma_k^2 I_N$  auto-correlation matrix. The transmit autocorrelation on tone  $k$  is  $E\{x_k x_k^H\} = S_k$  with  $S_k^m = [S_k]_{m,m}$ . Note that  $S_k$  is a diagonal matrix since co-ordination is not possible between CP transmitters. It should be mentioned that our notation here is like the one used in [2].

### B. Vectored DMT

When both the transmitters and receivers are coordinated, channel capacity can be achieved by means of appropriate pre and post signal processing modules which can be found via channel decomposition schemes. Since only receivers are coordinated in our case, these rates are not achievable. In such a case, [3] using channel QR decomposition, has proposed a nonlinear MUD based approach which performs successive interference cancellation (SIC). The received signal at tone  $k$  is processed by the unitary matrix  $Q_k^H$ , resulting in (2).

$$y_k = R_k X_k + Q_k^H Z_k \quad (2)$$

The vector  $X_k$  can be recovered by SIC using upper triangular matrix  $R_k$ .

## III. PARTIAL CROSSTALK CANCELLATION

Run-time complexity is a limiting factor in applying mentioned cancellation schemes since  $KN(N-1)$

multiplications per DMT block (with a block rate of 4000 blocks per second) is required. Partial cancellation schemes have been proposed to provide a practical balance between bit-rate and run-time complexity by canceling dominant crosstalkers in each frequency tone.

Space and frequency are the two possible dimensions for selection of dominant crosstalkers. Accordingly, three partial cancellation approaches have been proposed previously, namely line selection (LS), tone selection (TS), and joint line and tone selection (JLTS).

While LS cancels  $C(N-1)$  dominant crosstalkers in each tone, TS, removes all crosstalkers existing in only  $CK$  important tones. These are one-dimensional selective algorithms; however JLTS which operates on both space and frequency dimensions and cancels  $CK(N-1)$  important (crosstalk, tone) pairs, outperforms LS and TS. Note that in each algorithm the possible  $C$  values are the fractions that make complexity limit integer.

The algorithms differ in the way of spending their complexity, hence one may suppose complexity distribution problem as a resource allocation problem, in which complexity resources should be allocated to dimensions properly.

After applying PCC schemes the signal to interference plus noise ratio (SINR) at the input of decision device is obtained by (3).

$$SINR_k^n \approx \frac{|h_k^{(n,n)}|^2 s_k^n}{\sum |h_k^{(n,m)}|^2 s_k^m + \sigma_k^2} \quad (3)$$

Note that in derivation of (3), it is assumed that interference plus noise is Gaussian. This is a realistic assumption when sufficient numbers of crosstalkers (more than 2) exist.

### A. Line Selection Algorithm

For user  $n$  assume,  $N-1$  crosstalk powers are sorted in descending order as in (4).

$$\{q_{k,n}(1), \dots, q_{k,n}(N-1)\} = \text{sort}(|h_k^{(n,i)}|^2 s_k^i) \quad \forall i \ni q_{k,n}(i) \neq n \quad (4)$$

Where  $q_{k,n}(i)$  is the  $i^{\text{th}}$  sorted crosstalk for the user  $n$  at tone  $k$ . LS simply cancels crosstalkers with  $i=1, \dots, C(N-1)$ .

### B. Tone Selection Algorithm

Here, the  $CK$  important tones are selected according to the amount of their offered incremental bit-rates assuming all of their crosstalkers are cancelled. It is expected that the most of the selected tones will be in the intermediate frequency region, since the attenuated direct channel supports minimal bit-loading in high frequency region and also crosstalk has marginal effects on the lower frequency tones.

### C. Optimal Joint Line and Tone Selection Algorithm

The two previous algorithms are not optimum since they solve the problem for one dimension only. However, optimum JLTS (OJLTS) distributes its complexity resources among the two dimensions. The algorithm works for each user as follows:

In each tone, crosstalk powers are sorted according to (4). Using (5) a table can be formed with entries representing the

average gain per complexity for user  $n$  if up to  $p$  crosstalkers are cancelled where  $p$  varies from 1 to  $N-1$ .

$$v_{k,n}(p) = \frac{r_{k,n}(p) - r_{k,n}(0)}{p} \quad (5)$$

While the values are calculated, the algorithm finds the maximum entry in the table. Suppose this occurs at tone  $K_s(1)$  with cancellation value of  $P_s(1)$ . Then the total complexity resource is reduced to  $CK(N-1) - P_s(1)$ , the entries of column  $K_s(1)$  are updated in such a way that the first  $P_s(1)$  entries are set to zero, the remainder entries are calculated by ranging  $p$  from 1 to into  $N-1-P_s(1)$ . Next, the maximum entry is selected (for example this occurs at tone  $K_s(2)$  with  $P_s(2)$ ) and this procedure is repeated until all complexity resources are allocated. Note that it may possible that at the final stage, the remaining resource is less than  $P_s(\text{last stage})$ , in this way, the nearest entry with  $P_s'(\text{last stage})$ , should be selected.

#### D. Simplified Joint Line and Tone Selection Algorithm

In [2], a simplified JLTS (SJLTS) has been introduced and stated that it is quite close to optimum. However we will show that this is not the case for some simulation scenarios. This algorithm works as follows:

A table with entries  $\bar{g}_{k,n}(m)$  is created according to (6) and the  $CK(N-1)$  largest entries are selected for cancellation. Note that this algorithm ignores the effect of other crosstalkers in gain calculation.

$$\bar{g}_{k,n}(m) \triangleq \log_2 \left( 1 + \frac{|h_k^{(n,n)}|^2 s_k^n}{\Gamma \sigma_k^2} \right) - \log_2 \left( 1 + \frac{1}{\Gamma} \frac{|h_k^{(n,m)}|^2 s_k^m + \sigma_k^2}{|h_k^{(n,m)}|^2 s_k^m + \sigma_k^2} \right) \quad (6)$$

### IV. MULTIUSER POWER CONTROL

#### A. Iterative Water-Filling

In a multiuser channel, since the PSD of each user influences the crosstalk it induces on the other lines, it must be assigned in an appropriate way. This is known as *spectrum management* (SM) which has static and dynamic types.

Static SM is the traditional approach which considers worst case scenarios to impose fixed spectral masks which leads to poor performance [4]. Dynamic SM overcomes this problem by joint transmit spectra and signals optimization based on the direct and crosstalk channels seen by each user.

IWF [5] is a DSM algorithm in which each user obtains its most favorable PSD by doing single user WF iteratively until its PSD becomes stable.

WF is the optimum single user bit-loading algorithm used in DMT systems where in, the inverse channel SNR  $(\frac{\Gamma S_{noise}(f)}{|H(f)|^2})$  is filled by water/power until available power has been used [6]. The transmit spectrum must satisfy (7).

$$\lambda = S_x(f) + \frac{\Gamma S_{noise}(f)}{|H(f)|^2} \quad (7)$$

Where  $f$  represents frequency and  $\Gamma$  denotes the SNR gap which depends on the coding, modulation and the probability of error. Note that DSL channels are approximately time invariant, hence assumption of having channel information in obtaining channel SNR is reasonable. The resulting bit-rate is:

$$R = \frac{1}{T} \sum_{n=1}^K \log_2 \left( 1 + \frac{SNR_n}{\Gamma} \right) \quad (8)$$

Where  $1/T$  represents VDSL symbol rate.

By applying IWF, each user allocates its PSD in frequency regions wherein less crosstalk exists.

#### B. Iterative Joint Power Control and PCC

Here OJLTS and IWF are jointly employed to make crosstalk cancellation as feasible and efficient as possible. The algorithm consists of two loops. In the inner loop users sequentially form (4) and virtually cancel all their  $CK(N-1)$  (pair, tone) crosstalkers using OJLTS. Then apply conventional single user WF using SINRs obtained by (3) and consequently update their spectra. In the outer loop this procedure is repeated until each user achieves a stable PSD. We have done simulations on various scenarios and the results show that in all cases the algorithm converges after 1~2 iterations. Fig. 1 shows a simplified illustration of the algorithm.

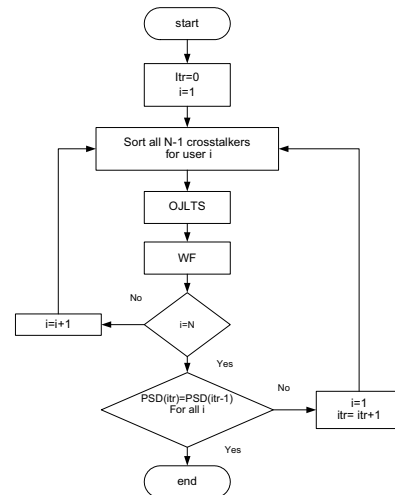


Fig.1. A simplified illustration of the algorithm

### V. SIMULATION RESULTS

The upstream VDSL scenario with 8 users ranging from 500 ft to 4000 ft in 500 ft increments is simulated. The general parameters are as Table III in [2] except we use also the first optional transmission band in FDD 998 and assume the specified coding gain, noise margin, and probability of error target lead to a transmission gap of 12.8 dB. Infinite granularity for the number of bits on each tone is assumed, and error propagation effects are ignored. We now evaluate the performance of different selection algorithms. Each scheme has been considered with (our algorithm) and without (previously ones with flat PSD) IWF. Figures 2~4 show the

bit-rates obtained by each of LS, TS and OJLTS algorithms, versus run-time complexity budget for the nearest, farthest and middle-ranged users respectively. Comparing these figures, the following observations can be made.

1. Our algorithm (OJLTS + WF) always outperforms the previous ones especially in low complexity budgets and in some cases it can increase the bit-rate more than 250%.
2. The difference between the achievable bit-rates using our algorithm and flat OJLTS becomes smaller as complexity budget increases and users' length become shorter. This is largely due to approximately equal channel SINRs in short loops which results in near flat PSDs using IWF.
3. Although LS and TS performance vary considerably with the scenario as stated in [2] but it can be seen that TS performs roughly linear in short length users as all subchannels' conditions are almost the same. However, in far-end users with much greater lengths, it acts far more rapidly compared to LS.
4. Using IWF with LS and TS may be especially beneficial at far end-users and in low complexity limits compared to LS and TS with flat PSDs.

The performance of SJLTS and OJLTS for these users are considered in figures 5~7 respectively. It can be seen that SJLTS is lagging behind our algorithm by up to 33 percent in some cases. This is particularly obvious for middle-ranged users in mid complexity limits.

## VI. CONCLUSION

In this paper we presented a novel near optimal partial crosstalk cancellation scheme for upstream VDSL transmission which employs IWF algorithm to achieve greater run-time complexity reductions compared to previous ones while maintaining similar performance. It can also lead to even larger bit-rates in far-end users. These are made possible by accepting a slight increase in initialization complexity which is not taken into account in complexity calculations due to almost static nature of DSL channels.

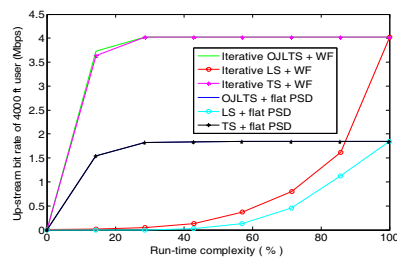


Fig.2. far-end user data rate vs. run-time complexity

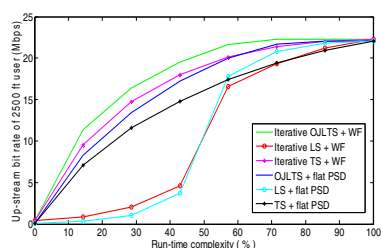


Fig.3. middle-ranged user data rate vs. run-time complexity

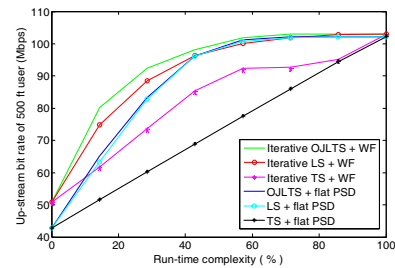


Fig.4. near-end data rate vs. run-time complexity

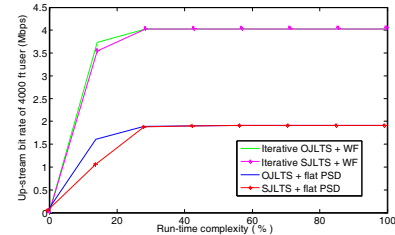


Fig.5. OJLTS and SLTS comparison for far-end user

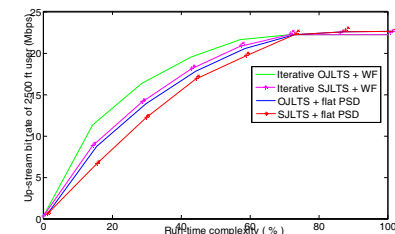


Fig.6. OJLTS and SLTS comparison for middle-ranged user

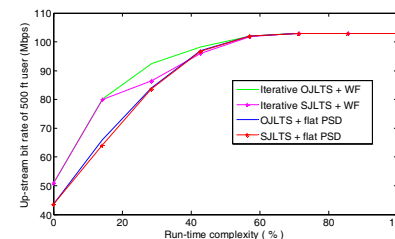


Fig.7. OJLTS and SLTS comparison for near-end user

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