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Determining Optimal Demand Rate and Production Decisions: A Geometric Programming Approach

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***Abstract*—**In this paper a nonlinear model is presented to demonstrate the relation between production and marketing departments. By introducing some functions such as pricing cost and market share loss functions it will be tried to show some aspects of market modelling which has not been regarded before. The proposed model will be a constrained signomial geometric programming model. For model solving, after variables' modifications an iterative technique based on the concept of geometric mean will be introduced to solve the resulting non-standard posynomial model which can be applied to a wide variety of models in non-standard posynomial geometric programming form. At the end a numerical analysis will be presented to accredit the validity of the mentioned model.

***Keywords*—**Geometric programming**,** marketing, nonlinear optimization, production.

I. Introduction

IN traditional production and lot sizing models, demand and variable productions cost are assumed to be constant, while it is not the same in real world. In the past few years, models attempting to link production and marketing decisions, have been proposed to determine an item's price, lot size and marketing expenditure per unit simultaneously.

Production and marketing departments are two highly interdependent units of all the firms. Decisions of one department frequently influence the performance and decision of the other. For instance the information presented by marketing department can be used as a basis for production department to set its production planning. On the other hand, information such as capacity constraint or degree of flexibility from production department can influence the marketing department performance [1].

Lee [2] considered the price dependent demand to maximize the profit. He finally determined the price and lot size as decision variables of the model. Lee and Kim [3] presented partial and full models to maximize the profit of a firm facing constant but price and marketing dependent demand over a planning horizon to determine price, marketing cost and lot size simultaneously. In 1998 Kim and Lee [4] considered the fixed and variable capacity problems of jointly determining an item's price and lot size for a profit-maximizing firm regarding nonlinear demand function. In their paper, demand is regarded as a nonlinear function of selling price. Chen introduced an inventory model under return on-inventory-investment maximization for an intermediate organization to determine the selling quantity and purchase cost of a product via geometric programming approach [5]. Sadjadi *et al*. studies the effects of integrated production and marketing decisions in a profit maximizing firm. Their model can determine price, marketing expenditure, and lot size for a single product simultaneously [6]. Fathian *et al.* proposed a model assuming demand as a nonlinear function of price, marketing cost and service cost. They determine decision variables via geometric programming [7].

Geometric programming (GP) has been very popular in engineering design research since its inception in the early 1960s. Geometric programming is an efficient method when decision variables interact in a nonlinear, specifically in an exponential form. If the primal problem is in posynomial form, then a global optimum to that problem is guaranteed and can be obtained by solving the dual program. The dual constraints are linear, and linearly constrained programs are generally easier to solve than ones with nonlinear constraints [8].

The main obstacle in using Geometric Programming is the fact that most of the real-world- problems are in signomial or nonstandard posynomial form. Literally speaking, it was observed that when the model is of signomial form with more than one posynomial term or nonstandard posynomial form, the heuristic methods like Genetic Algorithm were used to solve the model [9]. Another obstacle is that even when the problems are of posynomial form, with increasing the degree of difficulty, the resolving procedure is still hard and time-consuming. In this paper, after model construction the main focus will be on applying a transformation technique which is easier to use than other methods in the literature for solving signomial and nonstandard geometric programming models.

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II. Problem Statement

In this paper, a producer of a single product is considered who wants to sell its product in a competitive market with one major competitor. It is also assumed that product demand is a nonlinear function of a unit selling price and marketing cost per unit. Marketing, production, set-up, holding and pricing costs besides other costs associated with market share loss are considered in the proposed model. So, the goal of the present model will be maximizing the sales revenue profit, considering above costs under following assumptions:

1-Production rate is instantaneous. 2-shortage is not allowed. 3- Lead time is zero. 4-deman and variable production costs are not known before.

A. Notations and assumptions

$$I)D = \alpha\, p^{-\gamma}.M^{\theta} \qquad II)C = \beta.Q^{-\lambda}$$
$$III)\,pricing\ \cos t\ function = .5g.\alpha.p^{2} \qquad (1)$$
$$IV)h = i.c$$

D: Demand rate
P: Unit selling price (Decision variable)
M: Marketing cost per unit (Decision variable)
Q: Economic lot size per production cycle (Decision variable)
α: Scaling constant for demand function α>0
γ: Elasticity of demand for product γ>1
θ: Elasticity of marketing cost for product demand 0 < θ <1
C: Production cost per unit
β : Scaling constant for production cost per unit $\beta > 0$
λ : Lot size elasticity of production unit cost $0 < \lambda < 1$
h: Inventory holding cost per item per unit time
i: Holding cost rate
A: Set up cost per production run
g: Scaling constant in pricing cost function
π: Market share loss cost
p': Competitor's selling price

The equation "I" is to show the relationship between demand, price and marketing costs of each unit. This type of relationship is widely used in the literature. As for the parameter α, it is obvious that it is a positive parameter due to the fact that demand rate must be of positive value. Parameter γ is representing that price is the key factor in demand rate. Therefore, γ<1 shows that price is of less importance in demand rate, which is not desirable in the current problem. Parameter γ, represents the relative change in the demand with respect to the corresponding relative change in the price. Parameter 0<θ<1, represents the relative change in the demand with respect to the corresponding relative change in the marketing cost.

The equation II represents the unit production cost, considering the learning concept. For parameter β, the $\beta > 0$ condition sounds obvious, since the value of cost must be positive. λ is set to show the rate of changes in production costs when the lot size increases by one unit. Also, the $0 < \lambda < 1$ condition for the parameter λ is set which has a great use in the related literature. (γ>1 sounds unrealistic) and mostly contains tiny value.

Equation III presents the pricing cost function. This function shows the price direct effect on a firm's profitability if it chooses to increase its price. For example, when the firm decides to charge higher prices, it may have to increase its advertising expenditure and service level for customers. These direct pricing costs are presented by an increasing function of P. The below relation implies that a unit increase in price will cause αg(P+.5) $ increase in marginal costs [10].

$$\Delta(P) = pricing\ \cos t(P+1) - pricing\ \cos t(r) = \alpha.g(P+\frac{1}{2}) \quad (2)$$

The forth equation shows the holding cost per unit of product per unit time, considering i as the holding rate cost.

In this model, the cost associated with market share loss is also considered. Since the firm is about to sell the product in a single-rival-market, the unit selling price is of a great impact in market penetration which is symmetric in selling price. Letting φ as the market penetration rate factor and P and P' as unit selling price and the competitor's selling price respectively, the following equation will be achieved:

$$\varphi = \frac{P'}{P+P'} \qquad (3)$$

Considering φ as market share factor, it can be said that the amount of related market loss which equally means rival's market share acquisition, is equal to:

$$1-\varphi = \frac{P}{P+P'} \qquad (4)$$

On the other hand, the marketing department can use market research methods to become aware of the rival's selling price (P') and consequently sets its optimized price with regards to rival's price and restricting its own selling price.

B. Model Development

Regarding mentioned assumptions, the problem formulation which is maximizing the profit would be as follows:

$$Max \quad PD - MD - CD - \frac{AD}{Q} - h\frac{Q}{2} - \frac{1}{2}g.\alpha.P^{2} - \pi.\alpha(\frac{P}{P+P'})$$
$$s.t.\begin{cases} P \le (P'+1) \\ P, M, Q > 0 \end{cases} \qquad (5)$$

The above constraint shows that the unit selling price should not exceed the rival's selling price more than one unit. Rival's selling price is acquired by marketing department through market research studies and rival's market status. Now the denominator of market share term could be estimated with the positive variable L. Therefore, the following revised equation is obtained:

$$Max \quad PD - MD - CD - \frac{AD}{Q} - h\frac{Q}{2} - \frac{1}{2}g.\alpha.P^{2} - \pi.\alpha.P.L^{-1}$$

$$s.t.\begin{cases} P+P' \geq L \Rightarrow PL^{-1}+P'L^{-1} \geq 1 \\ P.(P'+1)^{-1} \leq 1 \\ P,M,Q,L>0 \end{cases} \quad (6)$$

Using the following substitutions will ease the problem solving process:

$x_1 \leftarrow P_1$, $x_2 \leftarrow M$
$x_3 \leftarrow Q$, $x_4 \leftarrow L$

So program (6) will be changes as below:

$$Max \;\; \alpha.x_1^{1-\gamma}.x_2^{\theta} - (\alpha.x_1^{-\gamma}.x_2^{\theta+1} + \alpha\beta x_1^{-\gamma}.x_2^{\theta}.x_3^{-\lambda} + A\alpha.x_1^{-\gamma}.x_2^{\theta}.x_3^{-1} + \frac{i\beta}{2}x_3^{1-\lambda} + \frac{g\alpha}{2}x_1^2 + \pi\alpha x_1 x_4^{-1})$$

$$s.t.\begin{cases} x_1 x_4^{-1} + p' x_4^{-1} \geq 1 \\ x_1.(p'+1)^{-1} \leq 1 \\ x_1, x_2, x_3, x_4 > 0 \end{cases} \quad (7)$$

This problem is a signomial geometric programming problem with 5 degrees of difficulty. It is needed to make some necessary modifications in order to change the objective function into a posynomial problem. But one problem still exists: the first constraint is in non- standard (greater than) posynomial form which must be changed to a standard one.

For transformation, it is assumed that there is a lower bound for the objective function such that maximizing the model will be equivalent to maximizing the lower bound of objective function. Therefore, the signomial GP is transformed into a posynomial one with an additional constraint and variable as follows:

$$Max\; x_0 = Min\; x_0^{-1}$$
$$s.t:$$

$$\alpha.x_1^{1-\gamma}.x_2^{\theta} - (\alpha.x_1^{-\gamma}.x_2^{\theta+1} + \alpha\beta x_1^{-\gamma}.x_2^{\theta}.x_3^{-\lambda} + A\alpha.x_1^{-\gamma}.x_2^{\theta}.x_3^{-1} + \frac{i\beta}{2}x_3^{1-\lambda} + \pi\alpha x_1 x_4^{-1} + \frac{g\alpha}{2}x_1^2) \geq x_0$$

$$\Rightarrow x_0 + \alpha.x_1^{-\gamma}.x_2^{\theta+1} + \alpha\beta x_1^{-\gamma}.x_2^{\theta}.x_3^{-\lambda} + A\alpha.x_1^{-\gamma}.x_2^{\theta}.x_3^{-1} + \frac{i\beta}{2}x_3^{1-\lambda} + \pi\alpha x_1 x_4^{-1} + \frac{g\alpha}{2}x_1^2 \leq \alpha.x_1^{1-\gamma}.x_2^{\theta}$$

$$\Rightarrow \alpha^{-1}.x_0.x_1^{\gamma-1}.x_2^{-\theta} + x_1^{-1}.x_2 + \beta.x_1^{-1}x_3^{-\lambda} + A.x_1^{-1}.x_3^{-1} + \frac{i.\beta}{2.\alpha}x_1^{\gamma-1}.x_2^{-\theta}.x_3^{1-\lambda} + \pi..x_1^{\gamma}.x_2^{-\theta}x_4^{-1} + \frac{g}{2}x_1^{1+\gamma}.x_2^{-\theta} \leq 1 \quad (8)$$

Consequently:

$$Max\;\; x_0 = Min\; x_0^{-1}$$
$$s.t:$$

$$\alpha^{-1}.x_0.x_1^{\gamma-1}.x_2^{-\theta} + x_1^{-1}.x_2 + \beta.x_1^{-1}x_3^{-\lambda} + A.x_1^{-1}.x_3^{-1} + \frac{g}{2}x_1^{1+\gamma}.x_2^{-\theta} + \frac{i.\beta}{2.\alpha}x_1^{\gamma-1}.x_2^{-\theta}.x_3^{1-\lambda} + \pi..x_1^{\gamma}.x_2^{-\theta}x_4^{-1} \leq 1$$

$$x_1 x_4^{-1} + p' x_4^{-1} \geq 1$$
$$x_1.(p'+1)^{-1} \leq 1$$
$$x_1, x_2, x_3, x_4, x_0 > 0 \quad (9)$$

C. Solution Approach

The above model is a non standard posynomial problem containing 5 degrees of difficulty which can be converted to standard GP form. For means of transformation one can use the following inequality relationship between arithmetic and geometric mean to convert a non-standard posynomial form to a standard one, containing only prototype $y \leq 1$ [8]. In this relation v_i are positive quantities and t_i are nonnegative weights which must sum to one. ($\sum_{i=1}^{N} t_i = 1$)

$$\prod_{i=1}^{N}(v_i)^{t_i} \leq \sum_{i=1}^{N} t_i.v_i \quad (10)$$

Taking $t_i.v_i \equiv u_i$ in mind, will give:

$$\prod_{i=1}^{N}(\frac{u_i}{t_i})^{t_i} \leq \sum_{i=1}^{N} u_i \quad (11)$$

The variables t_i are positive quantities and these inequalities hold at equalities if and only if:

$$t_i = \frac{u_i}{\sum_{i=1}^{N} u_i} \quad (12)$$

So, it will be apparent determining the related weighs as the above, will made both sides of the relation (11) to be equivalent:

$$y_m(x) = \sum_{i=1}^{N} u_i \geq 1 \quad \Rightarrow \quad y_m^{-1}(x) = (\sum_{i=1}^{N} u_i)^{-1} \leq 1$$
$$\Rightarrow \quad \prod_{i=1}^{N}(\frac{u_i}{t_i})^{-t_i} \leq 1 \quad (13)$$

Therefore, in the proposed model, considering the above relations will give t_i as below:

$$t_1 = \frac{x_1 x_4^{-1}}{x_1 x_4^{-1} + p' x_4^{-1}} \qquad t_2 = \frac{p' x_4^{-1}}{x_1 x_4^{-1} + p' x_4^{-1}} \quad (14)$$
$$\rightarrow \; t_1 + t_2 = 1$$

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The values of t_i are known once a (x) vector is specified. After all transformations, the proposed model will be converted to following formulation:

$$Max\ \ x_0 = Min\ x_0^{-1}$$
$$s.t:$$
$$\alpha^{-1}.x_0.x_1^{\gamma-1}.x_2^{-\theta} + x_1^{-1}.x_2 + \beta.x_1^{-1}x_3^{-\lambda} + A.x_1^{-1}.x_3^{-1} + \frac{i.\beta}{2.\alpha}x_1^{\gamma-1}.x_2^{-\theta}.x_3^{1-\lambda} + \frac{g}{2}x_1^{1+\gamma}.x_2^{-\theta} + \pi.x_1^{\gamma}.x_2^{-\theta}x_4^{-1} \le 1$$
$$(\frac{x_1x_4^{-1}}{t_1})^{-t_1}.(\frac{p'x_4^{-1}}{t_2})^{-t_2} \le 1$$
$$x_1.(p'+1)^{-1} \le 1$$
$$x_1, x_2, x_3, x_4, x_0 > 0 \qquad (15)$$

As can be seen, by means of this transformation method, proposed model is reduced to a standard posynomial GP with 4 degrees of difficulty, which means one degree reduction.

D. Algorithmic Procedure

Iterative procedure for solving the model is presented as follow:

1. Determine a primal feasible initial solution for x_1.
2. Using the amount of step1, calculate the weighs t_i.
3. Solve the standard posynomial model (15).
4. Evaluate the original constraint set in model (7) at the new solution vector (x) from step3. If the original constraints are satisfied (note that the first constraint in model (7) which is conducted from objective function transformation must be tight at optimality) terminate. If not, return to step2 to calculate new t_i.

The advantageous of this algorithm is its ease of use and reduction in degree of difficulty. Also the iterations of this algorithm do not depend on the value of initial feasible solution. For solving the model (15) we use GGPLAB [11] which is based on primal – dual interior point method [12] which does not have much restriction on number of degree of difficulty. By means of this toolbox, global optimum can be guaranteed for every convex posynomial model.

III. Computational Results

For the purpose of this example, we consider a manufacturing company which wants to sell its product regarding different cost categories and the assumptions given in section II-1. In other words, this company wants to examine whether the investment in the market is economically justifiable or not. The required parameters to build the model are set after the interview with expert people and market research studies. These parameters are as follows:

$\alpha = 3 \times 10^{6}$, $\beta = 6$, $\gamma = 2$, $\lambda = 0.01$, $\theta = 0.03$, $i = .1$, $A = 100$, $P' = 9$, $\pi = 0.06$, $g = 0.0000003$

For the initial feasible solution one can consider x_1=8.5 and the results obtained are t_1=0.4857 and t_2=0.5143, then GGPLAB is applied for solving the Model 15 to calculate the amount of decision variables. Table I shows the convergence procedure to optimum solution through 6 iterations considering x_1=8.5 as initial solution.

Numerical experiments imply that the number of iterations does not depend much on the value of initial solution. This independency is one of the advantageous of the proposed algorithm. For instance starting the algorithm with x_1=6.1$, will lead to convergence to optimum solution after 6 iterations as it is shown in table II. Finally, resulting values for the vector of (x_1,x_2,x_3,x_0) which equals to (unit price, marketing cost per unit, lot size, total profit) are 9.7141$,0.1227$,7496.6867, 26187.1756$, respectively.

IV. Sensitivity Analysis

A. Unit Selling Price and Associated Parameter

As Fig. 1 shows, the more γ increases, the more decrease in product price will be achieved. This is obvious from practical perspective as well, because when a market is highly dependent on price changes (large γ), the reduction in price will lead to increased demand rate.

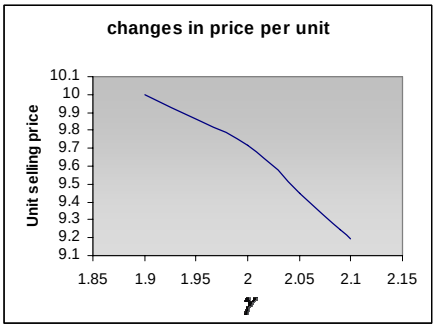


Fig. 1 Changes in Unit Price Regarding Changes in γ

B. Unit Selling Price and Associated Parameter

As Fig. 2 shows, the more θ increases, the more unit marketing expenditure will increase. This is obvious from practical perspective as well, because when customers are highly sensitive to the marketing costs, increasing the marketing expenditure can be followed by increased demand rate. As can be seen in Fig. 3, increasing the parameter θ along with increasing marketing cost per unit will lead to increase in the unit selling price, which is necessary for maintaining profit level.

TABLE I
ITERATIVE PROCEDURE FOR (x_1=8.5) AS INITIAL SOLUTION

	Initial solution	Iteration1	Iteration2	Iteration3	Iteration4	Iteration5	Iteration6
x1	8.5	9.6332	9.7092	9.7138	9.7141	9.7141	9.7141
x2	*	0.1204	0.1226	0.1227	0.1227	0.1227	0.1227
x3	*	7598.9483	7502.9495	7497.064	7496.7098	7496.6867	7496.6867
x4	*	18.5968	18.7089	18.7138	18.7141	18.7141	18.7141
x0	*	25992.3384	26186.4097	26187.1728	26187.1756	26187.1756	26187.1756

TABLE II
ITERATIVE PROCEDURE FOR (x_1=6.1) AS INITIAL SOLUTION

	Initial solution	Iteration1	Iteration2	Iteration3	Iteration4	Iteration5	Iteration6
x1	6.1	9.4244	9.6959	9.713	9.7141	9.7141	9.7141
x2	*	0.1144	0.1222	0.1227	0.1227	0.1277	0.1227
x3	*	7874.0704	7519.4498	7498.0895	7496.7714	7496.6867	7496.6867
x4	*	18.0011	18.6941	18.713	18.7141	18.7141	18.7141
x0	*	23854.4213	26177.1427	26187.137	26187.1755	26187.1756	26187.1756

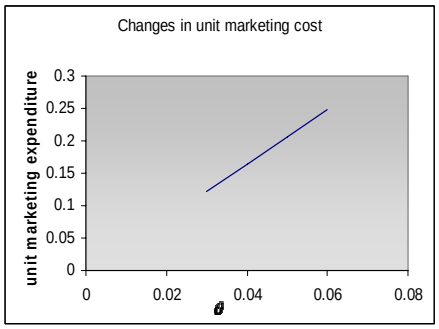


Fig. 2 Changes in Unit Marketing Cost Regarding Changes In θ

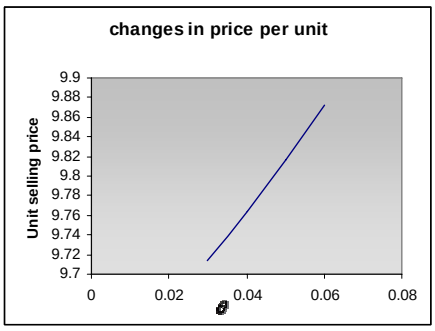


Fig. 3 Changes in Unit Price Regarding Changes In θ

C. Unit Production Variable Cost and Associated Parameter

According to Fig. 4, the more λ increases, the more lot size will increase. This is obvious from practical perspective as well, because increasing this parameter means decreasing costs in the greater production volume for each run. Also as it is shown in Fig. 5, increasing the parameter λ along with increasing the volume will lead to price reduction, which is obvious with regards to production cost reduction.

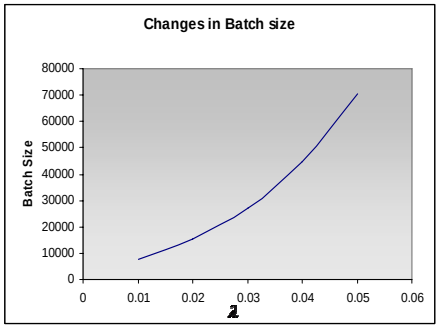


Fig. 4 Changes in Batch Size Regarding Changes in λ

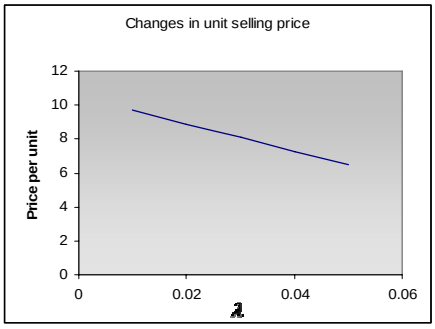


Fig. 5 Changes in Unit Price Regarding Changes in λ

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V. Conclusion

This paper formulated a comprehensive previously unexplored model regarding market share loss and pricing cost functions to demonstrate the relation between two important managerial departments of production and marketing. As the model was in signomial form, it was needed to change that to standard posynomial form. In order to do this change, the concepts behind the relations between geometric and arithmetic means have been applied. After transformation, as the dual program was a parametric equation system, an efficient iterative algorithm is used to solve the resulting model. Considering the given numerical example the model is analyzed and investigated for the effect of changes in parameters on decision variables. The analysis of the model could approve the validity of the proposed model. This study provides a rather comprehensive framework for joint pricing and lot sizing decisions, and can be easily expanded to investigating multi- product firms using the mentioned techniques.

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