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Analysis of permanence and extinction of enterprise cluster based on ecology theory

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Abstract—This paper is concerned with the permanence and extinction problem of enterprises cluster constituted by m satellite enterprises and a dominant enterprise. We present the model involving impulsive effect based on ecology theory, which effectively describe the competition and cooperation of enterprises cluster in real economic environment. Applying comparison theorem of impulsive differential equation, we establish sufficient conditions which ultimately affect the fate of enterprises: permanence, extinction, and co-existence. Finally, we present numerical examples to explain the economical significance of mathematical results.

Keywords—Enterprise cluster; Permanence; Extinction; Impulsive; Comparison theorem

I. Introduction

ENTERPRISES cluster refer to the concentration of similar or related enterprises in a specific area, which form fixed economic output and have certain economic influence on outside. Some similarities are exhibited between species population co-exist in nature and enterprises cluster in economic life such as life period, component, structure inside, openness and abundance, and so on [1].

In recent years, a few researchers presented some models about enterprises cluster based on ecology theory, which arouse growing interest in applying the methods of ecology and dynamic system theory to study enterprises cluster, for example [2-6, 15] and reference cited therein. In [2], Zhou divided enterprises cluster into two kinds of models: the concentration of subcontractors around a dominant firm and the concentration of simple competitors, called center halfback models and net models. Moreover, two kinds of models from biology were given and explained by economic view, and sufficient conditions were obtained to guarantee the co-existence and stability of enterprises cluster. In [4], the developing strategy of enterprises was analyzed based on the logistic model, the suggestions of constructing cooperative relation and choosing generalization or specialization tactics for commodity were put forward. In addition, based on the theoretical model of ecological population science, Wang [5] made a detailed analysis to the equilibrium mechanism of enterprises cluster, including the net model and the center halfback model, and drew a conclusion that: the relationship of pierce competition and beneficial cooperation among enterprises cluster was the crucial factor for them to keep stability. More related research about enterprises cluster, one can refer to literatures [7-10].

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As the birth of many species is an annual birth impulse, impulse is also an unnegligible factor of mathematical models in economy, such as annual fund regulation, staff adjustment and so on. In order to give an accurate description of the model of enterprises cluster in economy, we need consider the effect of impulse. The research on theory and application of impulsive differential equations had been made in many nice works, one can see [11-14]. Our practical interest in economy is the question of whether or not impulse can cause effect on the fate of enterprises cluster. Recently, the literature [15] considered the competition and cooperation system of two enterprises based on ecosystem:

$$\begin{cases} x_1'(t) = r_1 x_1(t)\left[1 - \dfrac{x_1(t)}{K} - \dfrac{\alpha(x_2(t)-c_2)^2}{K}\right], \\ x_2'(t) = r_2 x_2(t)\left[1 - \dfrac{x_2(t)}{K} + \dfrac{\beta(x_1(t)-c_1)^2}{K}\right], \end{cases} \tag{1}$$

where $x_1(t), x_2(t)$ represent the output of enterprises A and B, r_1, r_2 are the intrinsic growth rate, K denotes the carrying capacity of market under nature unlimited conditions, α, β are the competitive coefficient of two enterprises, c_1, c_2 are the initial production of two enterprises. If we let $a_1 = r_1/K, a_2 = r_2/K, b_1 = \alpha/K, b_2 = \beta/K$, system (1) becomes

$$\begin{cases} x_1'(t) = x_1(t)[r_1 - a_1 x_1(t) - b_1(x_2(t)-c_2)^2], \\ x_2'(t) = x_2(t)[r_2 - a_2 x_2(t) + b_1(x_1(t)-c_1)^2]. \end{cases}$$

Motivated by the above work, we consider the following impulsive competitive and cooperation model of m satellite enterprises and a dominant enterprise under center halfback model:

$$\begin{cases} x_i'(t) = x_i(t)\Big[r_i - a_i x_i(t) - \sum\limits_{j=1, j\neq i}^{m} b_{ij} x_j(t) \\ \qquad\qquad -d_i(y(t)-c)^2\Big], \\ y'(t) = y(t)\Big[r - ay(t) + d\sum\limits_{i=1}^{m}(x_i(t)-c_i)^2\Big], \\ \Delta x_i(t) = x_i(t^+) - x_i(t) = 0, \\ \Delta y(t) = y(t^+) - y(t) = -Ey(t), \end{cases} \tag{2}$$

where $x_i(t), y(t)$ represent the output of satellite enterprises A_{x_i} and core enterprise A_y, respectively; r_i, r are the intrinsic growth rates, a_i, a account for their respective self-regulations, b_{ij} account for the rates of inter-enterprises

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A_{x_i} competition, d_i represents the rate of intra-enterprise competition from A_y, d represents the rate of conversion of commodity into the reproduction of enterprise A_y, c_i and c represent the initial production of the enterprises, respectively. $r_i, r, a_i, a, b_{ij}, c_i, c, d_i, d$ are positive constants, $i = 1, 2, \ldots, m$. For the former two equations, $t \neq nT, n \in N$, and for the latter two equations, $t = nT, n \in N$, $T > 0$ is the period of impulsive, $0 < E < 1$ is the proportion of harvest at fixed moments $nT, n \in N$. Our main purpose of this paper is by using the comparison theorem of impulsive differential equation, to establish sufficient conditions for the permanence, extinction, co-existence of enterprises described by (2).

The organization of the rest of this paper is as follows. In Section 2, we introduce some preliminary results which are needed in later sections. In Section 3, we establish some sufficient conditions for the permanence, extinction, co-existence of enterprises. In Section 4, we give numerical examples to explain the economical significance of mathematical results above.

II. Preliminaries

For convenience, we shall introduce some notations, definitions and lemmas which will be useful for the proofs of our main results.

Let $R_+ = [0, +\infty), R_+^{m+1} = \{\phi = (x_1, \ldots, x_m, y) | x_i > 0, y > 0, i = 1, 2, \ldots, m\}, V_0 = \{V(t, \phi) | V : R_+ \times R_+^{m+1} \to R_+, V(t, \phi)$ is continuous on $(nT, (n+1)T] \times R_+^{m+1}$ and $\lim_{(t,w)\to(nT^+,\phi)} V(t, w) = V(nT^+, \phi), n = 0, 1, 2, \ldots\}$.

Definition 1. [16] *Let $V \in V_0$, then for $(t, \phi) \in (nT, (n+1)T] \times R_+^{m+1}$, the upper right derivative of $V(t, \phi)$ with respect to the impulsive differential system* (1) *is defined as*

$$D^+V(t, \phi) = \lim_{h\to 0^+} \sup \frac{1}{h}[V(t + h, \phi + hf(t, \phi)) - V(t, \phi)],$$

where $f = (f_1, f_2, \ldots, f_{m+1})$ is defined by the right hand of the first $m + 1$ equations of system (1).

Lemma 1. [17] *Let $V \in V_0$. Assume that*

$$\begin{cases} D^+V(t, \phi) \leq g(t, V(t, \phi)), \ t \neq nT, n \in N, \\ V(t, \phi(t^+)) \leq \varphi_n(V(t, \phi)), \ t = nT, n \in N, \end{cases} \tag{3}$$

where $g : R_+ \times R_+ \to R$ is continuous in $(nT, (n+1)T] \times R_+, n = 0, 1, 2, \ldots$ and $\lim_{(t,v)\to(nT^+,v_0)} g(t, v) = g(nT^+, v_0)$, $\varphi_n : R_+ \to R_+$ is non-decreasing. Let $h(t)$ be the maximal solution of the following scalar impulsive differential equation

$$\begin{cases} u'(t) = g(t, u), \ t \neq nT, n \in N, \\ u(t^+) = \varphi_n(u(t)), \ t = nT, n \in N, \\ u(0^+) = u_0, \end{cases} \tag{4}$$

existing on $[0, +\infty)$. Then $V(t, \phi) \leq h(t), (t \geq 0)$ when $V(0^+, \phi_0) \leq u_0$, where $\phi(t)$ is any solution of (3)*, which satisfies initial condition $\phi(0^+) = \phi_0$.*

Remark 1. (i) *In Lemma 1, assume that inequality* (3) *reversed. Let $h(t)$ be the minimal solution of* (4) *existing on $[0, +\infty)$, and $\varphi_n : R_+ \to R_+$ is non-increasing, then $V(t, \phi) \geq h(t), (t \geq 0)$ when $V(0^+, \phi_0) \geq u_0$.*

(ii) *If we have some smooth conditions of $g(t)$ to guarantee the existence and uniqueness of solution for* (4)*, then $h(t)$ is exactly the unique solution of* (4).

For system (2), the following lemma is obvious:

Lemma 2. *Suppose $\phi(t) = (x_1(t), x_2(t), \ldots, x_m(t), y(t))$ is a solution of* (2) *with $\phi(0^+) \geq 0$, then $\phi(t) \geq 0$ for all $t \geq 0$. And if $\phi(0^+) > 0$, then $\phi(t) > 0$ for all $t \geq 0$.*

Definition 2. [18] *System* (2) *is said to be permanent if there exist two positive constants m_0, M_0 such that each positive solution $(x_1(t), x_2(t), \ldots, x_m(t), y(t))$ of system* (2) *satisfies*

$$m_0 \leq \liminf_{t\to\infty} x_i(t) \leq \limsup_{t\to\infty} x_i(t) \leq M_0, \ i = 1, 2, \ldots, m,$$

$$m_0 \leq \liminf_{t\to\infty} y(t) \leq \limsup_{t\to\infty} y(t) \leq M_0.$$

Remark 2. *System* (2) *is said to be permanent if there exist two positive constants $\hat{m}, \hat{M}$ such that each positive solution $(x_1(t), x_2(t), \ldots, x_m(t), y(t))$ of system* (2) *satisfies*

$$\hat{m} \leq x_i(t) \leq \hat{M}, \ \ \hat{m} \leq y(t) \leq \hat{M}.$$

Lemma 3. [18] *Any positive solution $x(t)$ of the inequality problem*

$$x'(t) \leq x(t)[a - bx(t)]$$

satisfies $\limsup_{t\to\infty} x(t) \leq \frac{a}{b}$, if $a > 0, b > 0$.

Now, we consider the following impulsive system:

$$\begin{cases} y'(t) = y(t)[r - ay(t)], \ \ t \neq nT, n \in N, \\ \Delta y(t) = y(t^+) - y(t) = -Ey(t), \ \ t = nT, n \in N, \end{cases} \tag{5}$$

where notations in above system (5) are the same meaning as those in system (2). For system (5), we can obtain the following result:

Lemma 4. *System* (5) *has a positive periodic solution*

$$y^*(t) = \frac{r(1 - E - e^{-rT})}{a(1 - E - e^{-rT}) + aEe^{-r(t-nT)}}, \tag{6}$$

where $t \in (nT, (n+1)T], n = 0, 1, \ldots$. Moreover,

(1) *let $y(t)$ be any solution of system* (5) *satisfying initial condition $y(0^+) = c$, then $|y(t) - y^*(t)| \to 0 (t \to +\infty)$;*

(2) *$y(t) \geq y^*(t)$ if $c \geq \frac{r(1-E-e^{-rT})}{a(1-E-e^{-rT})+aE}$.*

Proof: It is not difficult to verify that $y^*(t)$ is a positive periodic solution of (5), one can refer to [16]. For any solution $y(t)$ of system (5), which satisfies the initial condition $y(0^+) = c$, we have

$$y(t) = \big(c - \frac{r(1 - E - e^{-rT})}{a(1 - E - e^{-rT}) + aE}\big)e^{-rt} + y^*(t),$$

and $|y(t) - y^*(t)| \to 0 (t \to \infty)$. It is easy to see that $y(t) \geq y^*(t)$ if $c \geq \frac{r(1-E-e^{-rT})}{a(1-E-e^{-rT})+aE}$. ■

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III. Main results

Theorem 1. *Suppose that* $E > \max\limits_{1\le i\le m}\left\{1-e^{(\frac{ac}{2}-\frac{r_i a}{2d_i c}-r)T}\right\}$ *and* $c^2-\frac{r_i}{d_i}<\frac{2rc}{a}$, *then*

$$x_i(t)\to 0(i=1,2,\dots,m),\ y(t)\to y^*(t),\ (t\to+\infty)$$

for any solution $(x_1(t),x_2(t),\dots,x_m(t),y(t))$ *of system* (2), *where* $y^*(t)$ *is defined as* (6). *That is, enterprises* A_{x_i} *will be bankruptcy, and enterprise* A_y *will keep permanence.*

Proof: Firstly, we will prove that $x_i(t)\to 0, t\to+\infty$, $i=1,2,\dots,m$.

Since $E>\max\limits_{1\le i\le m}\{1-e^{(\frac{ac}{2}-\frac{r_i a}{2d_i c}-r)T}\}$, we can choose $\varepsilon_0>0$ such that

$$\delta:=\left(2d_i c\varepsilon_0+d_i c^2-r_i-\frac{2d_i cr}{a}\right)T-\frac{2d_i c}{a}\ln(1-E)>0.$$

Noting that $y'(t)\ge y(t)[r-ay(t)]$, we consider the impulsive differential equation

$$\begin{cases}y_1'(t)=y_1(t)[r-ay_1(t)]),\ t\ne nT, n\in N,\\ \Delta y_1(t)=-Ey_1(t),\ t=nT, n\in N,\\ y_1(0^+)=y(0^+)>0.\end{cases}$$

From Lemma 1 and Lemma 4, we have $y(t)\ge y_1(t)$ and $y_1(t)\to y^*(t)(t\to+\infty)$. Thus

$$y(t)\ge y_1(t)>y^*(t)-\varepsilon_0 \tag{7}$$

holds for all large enough t. For simplicity, we may assume that (7) holds for all $t\ge 0$.

From the first equation of system (2), we have

$$\begin{aligned}x_i'(t)\ &\le\ x_i(t)\Bigg[r_i-a_ix_i(t)-\sum_{j=1,j\ne i}^{m}b_{ij}x_j(t)\\ &\quad -d_i(y^*(t)-\varepsilon_0-c)^2\Bigg]\\ &\le\ x_i(t)[r_i+2d_ic(y^*(t)-\varepsilon_0)-d_ic^2],\ i=1,2\dots,m.\end{aligned}$$

Integrating the above formula on $(nT,(n+1)T]$, we obtain

$$\begin{aligned}x_i((n+1)T)\ &\le\ x_i(nT)\exp\left(\int_{nT}^{(n+1)T}\Big[r_i+2d_ic(y^*(t)\right.\\ &\quad \left.-\varepsilon_0)-d_ic^2\Big]\mathrm{d}t\right)\\ &=\ x_i(nT)\exp\Big(r_iT-2d_ic\varepsilon_0T\\ &\quad -d_ic^2T+\frac{2d_ic}{a}[rT+\ln(1-E)]\Big)\\ &=\ x_i(nT)\exp(-\delta), i=1,2\dots,m,\end{aligned}$$

then $x_i(nT)\le x_i(0^+)\exp(-n\delta)$ and $\lim\limits_{n\to\infty}x_i(nT)=0$, $i=1,2\dots,m$.

Notice that $x_i'(t)\le x_i(t)r_i$, we have $0<x_i(t)\le x_i(nT)\times\exp[r_i(t-nT)]\le x_i(nT)\exp(r_iT), t\in(nT,(n+1)T]$, which yields $x_i(t)\to 0$ as $t\to+\infty, i=1,2,\dots,m$.

Now, we will prove that $y(t)\to y^*(t)$ when $x_i(t)\to 0(i=1,2\dots,m)$, $t\to+\infty$.

For $\varepsilon_1>0$(ε_1 is small enough), there exists a $\tilde{T}>0$ such that

$$0<x_i(t)<\varepsilon_1 \text{ for all } t>\tilde{T}, i=1,2\dots,m. \tag{8}$$

For simplicity, we may assume (8) holds for all $t\ge 0$. Thus, we have

$$y(t)[r-ay(t)]\le y'(t)\le y(t)[r+dm(\varepsilon_1)^2-ay(t)].$$

Considering the following impulsive differential systems

$$\begin{cases}y_1'(t)=y_1(t)[r-ay_1(t)]),\ t\ne nT, n\in N,\\ \Delta y_1(t)=-Ey_1(t),\ t=nT, n\in N,\\ y_1(0^+)=y(0^+)>0\end{cases}$$

and

$$\begin{cases}y_2'(t)=y_2(t)[r+dm(\varepsilon_1)^2-ay_2(t)]),\ t\ne nT, n\in N,\\ \Delta y_2(t)=-Ey_2(t),\ t=nT, n\in N,\\ y_2(0^+)=y(0^+)>0,\end{cases}$$

we have $y_1(t)\le y(t)\le y_2(t)$ and $y_1(t)\to y^*(t)(t\to+\infty), y_2(t)\to y_*(t)(t\to+\infty)$, where

$$y^*(t)=\frac{r(1-E-e^{-rT})}{a(1-E-e^{-rT})+aEe^{-r(t-nT)}},$$

$$\begin{aligned}&y_*(t)\\ &=\frac{(r+dm(\varepsilon_1)^2)(1-E-e^{-(r+dm(\varepsilon_1)^2)T})}{a(1-E-e^{-(r+dm(\varepsilon_1)^2)T})+aEe^{-(r+dm(\varepsilon_1)^2)(t-nT)}}.\end{aligned}$$

Then, for any $\epsilon>0$, there exists a $\bar{T}>0$ such that

$$y^*(t)-\epsilon<y_1(t)\le y(t)\le y_2(t)<y_*(t)+\epsilon$$

for $t>\bar{T}$. Let $\varepsilon_1\to 0$, we obtain that $y(t)\to y^*(t)$, $t\to+\infty$. The proof is completed. ■

Theorem 2. *Suppose that*

(H_1) $\frac{r_i}{d_i}=\frac{c^2T-\frac{2c}{a}(rT+\ln(1-E))}{T}$, $i=1,2,\dots,m$;

(H_2) $c\ge\frac{r(1-E-e^{-rT})}{a(1-E-e^{-rT})+aE}$

hold, then any solution $(x_1(t),x_2(t),\dots,x_m(t),y(t))$ *of system* (2) *satisfies*

$$x_i(t)\to 0(i=1,2,\dots,m),\ y(t)\to y^*(t),\ (t\to+\infty),$$

where $y^*(t)$ *is defined as* (6). *That is, enterprises* A_{x_i} *will be bankruptcy, and the enterprise* A_y *will keep permanence.*

Proof: Noting that $y'(t)\ge y(t)[r-ay(t)]$, we consider the impulsive differential system

$$\begin{cases}y_3'(t)=y_3(t)[r-ay_3(t)]),\ t\ne nT, n\in N,\\ \Delta y_3(t)=-Ey_3(t),\ t=nT, n\in N,\\ y_3(0^+)=y(0^+)>0.\end{cases}$$

By condition (H_2) and Lemma 4, we get $y(t)\ge y_3(t)\ge y^*(t)$ and $y_3(t)\to y^*(t)(t\to+\infty)$, where $y^*(t)$ is defined as (6).

From the first equation of system (2), we have

$$x_i'(t)\ \le\ x_i(t)\Bigg[r_i-a_ix_i(t)-\sum_{j=1,j\ne i}^{m}b_{ij}x_j(t)$$

$$-d_i(y^*(t)-c)^2\Big]$$
$$\le x_i(t)[r_i - a_i x_i(t) + 2d_i c y^*(t) - d_i c^2],$$

where $i = 1, 2 \ldots, m$. Integrating the above formula over $(nT, (n+1)T]$ and applying condition (H_1), we obtain that

$$\begin{aligned} x_i[(n+1)T] &\le x_i(nT)\exp\Big(\int_{nT}^{(n+1)T}[r_i - a_i x_i(t) \\ &\quad +2d_i c y^*(t) - d_i c^2]\mathrm{d}t\Big) \\ &= x_i(nT)\exp\Big(r_i T - d_i c^2 T \\ &\quad -a_i\int_{nT}^{(n+1)T} x_i(t)\mathrm{d}t \\ &\quad +\frac{2d_i c}{a}[rT + \ln(1-E)]\Big) \\ &= x_i(nT)\exp(-X_n), i = 1, 2 \ldots, m, \quad (9) \end{aligned}$$

where $X_n = a_i\int_{nT}^{(n+1)T} x_i(t)\mathrm{d}t, i = 1, 2 \ldots, m$. We conclude that $\lim\limits_{n\to\infty} X_n = 0$. If it is not true, we assume that $\limsup\limits_{n\to\infty} X_n = \omega > 0$, there exists subsequence $\{X_{n_k}\}$ of $\{X_n\}$ such that $\limsup\limits_{k\to\infty} X_{n_k} = \omega$. Then, there exists a $K \in N$ such that $X_{n_k} > \frac{\omega}{2}$ for $k > K$, which yields

$$\sum_{n=1}^{\infty} X_n > \sum_{k=1}^{\infty} X_{n_k} \to +\infty.$$

From $x_i[(n+1)T] \le x_i(nT)\exp(-X_n)$, $i = 1, 2, \ldots, m$, we have

$$x_i(nT) \le x_i(0^+)\exp(-\sum_{j=0}^{n} X_j) \to 0 \ (n \to \infty),$$

where $i = 1, 2 \ldots, m$.

Notice that $x_i'(t) \le x_i(t)r_i$, we have $0 < x_i(t) \le x_i(nT)\exp(r_i T)$, $t \in (nT, (n+1)T]$, then $x_i(t) \to 0$ as $t \to +\infty$, $i = 1, 2 \ldots, m$. Moreover, $X_n \to 0$ as $n \to \infty$, which is a contradiction. Therefore, $\lim\limits_{n\to\infty} X_n = 0$. From (9), it is easy to see that $x_i(t) \to 0 (i = 1, 2 \ldots, m), t \to +\infty$.

Next, similar to the proof of the second part of Theorem3.1, combined with the above proof of this theorem, we can finally obtain that $y(t) \to y^*(t) (t \to +\infty)$, where $y^*(t)$ is defined as (6). This completes the proof. ■

One of the most important questions in economy is to find the permanence conditions for enterprises cluster, which has received a great deal of attention of many mathematicians and economists. Now, we will present the result of permanence for enterprises cluster.

Theorem 3. *Suppose that*
(H_3) $0 < E < 1 - e^{-rT}$;
(H_4) $\min\limits_{1\le i\le m}\{\frac{r_i}{d_i}\} > M^2$, *where*

$$M = \max\Big\{M_1, \frac{(r + dm(M_1)^2)(1 - E - e^{-(r+dm(M_1)^2)T})}{a(1 - E - e^{-(r+dm(M_1)^2)T})}\Big\},$$

$$M_1 = \max_{1\le i\le m}\{\frac{r_i}{a_i}\}$$

hold, then system (2) *is permanent. That is, the enterprises A_{x_i} and A_y will be co-existence.*

Proof: By condition (H_4), there exists a positive constant ϵ_0 such that

$$\delta := r_i - d_i M^2 - \Big(a_i + \sum_{j=1, j\ne i}^{m} b_{ij}\Big)\epsilon_0 > 0.$$

Since $x_i'(t) \le x_i(t)[r_i - a_i x_i(t)], i = 1, 2, \ldots, m$, by Lemma 3 and Lemma 4, we have

$$\limsup_{t\to\infty} x_i(t) \le \frac{r_i}{a_i} \le \max_{1\le i\le m}\{\frac{r_i}{a_i}\} = M_1, \ \ i = 1, 2, \ldots, m.$$

Noting that $y(t)[r - ay(t)] \le y'(t) \le y(t)[r + dm(M_1)^2 - ay(t)]$, similar to the proof of Theorem 1, we obtain that

$$y_1(t) \le y(t) \le y_2(t)$$

and

$$y_1(t) \to \tilde{y}^*(t), \ y_2(t) \to \tilde{y}_*(t) (t \to +\infty),$$

where y_1, y_2 are the solutions of the following impulsive differential equations

$$\begin{cases} y_1'(t) = y_1(t)[r - ay_1(t)]), \ t \ne nT, n \in N, \\ \Delta y_1(t) = -Ey_1(t), \ t = nT, n \in N, \\ y_1(0^+) = y(0^+) > 0 \end{cases}$$

and

$$\begin{cases} y_2'(t) = y_2(t)[r + dm(M_1)^2 - ay_2(t)]), \ t \ne nT, n \in N, \\ \Delta y_2(t) = -Ey_2(t), \ t = nT, n \in N, \\ y_2(0^+) = y(0^+) > 0, \end{cases}$$

respectively. Moreover, we have

$$\tilde{y}^*(t) = \frac{r(1 - E - e^{-rT})}{a(1 - E - e^{-rT}) + aEe^{-r(t-nT)}},$$

$$\begin{aligned} &\tilde{y}_*(t) \\ &= \frac{[r + dm(M_1)^2][1 - E - e^{-(r+dm(M_1)^2)T}]}{a(1 - E - e^{-(r+dm(M_1)^2)T}) + aEe^{-(r+dm(M_1)^2)(t-nT)}}. \end{aligned}$$

It follows that for any $\epsilon > 0$, there exists a $T' > 0$ such that

$$\tilde{y}^*(t) - \epsilon < y_1(t) \le y(t) \le y_2(t) < \tilde{y}_*(t) + \epsilon, \ \ t > T'.$$

Since the arbitrariness of ϵ, we have

$$\tilde{y}^*(t) \le y(t) \le \tilde{y}_*(t).$$

Let

$$\begin{aligned} M_2 &= \sup\{\tilde{y}_*(t), t \ge 0\} \\ &= \frac{(r + dm(M_1)^2)(1 - E - e^{-(r+dm(M_1)^2)T})}{a(1 - E - e^{-(r+dm(M_1)^2)T})}, \\ m_2 &= \inf\{\tilde{y}^*(t), t \ge 0\} = \frac{r(1 - E - e^{-rT})}{a(1 - E - e^{-rT}) + aEe^{rnT}}, \end{aligned}$$

then $m_2 < y(t) < M_2, 0 < m_2, M_2 < +\infty, t \ge 0$.

Therefore,

$$x_i(t) < M, \ \ y(t) < M, \ t \ge 0, \ i = 1, 2, \ldots, m. \quad (10)$$

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Now, we will prove that there exists a constant $m_1 > 0$ such that $m_1 \le \liminf_{t\to\infty} x_i(t)$.

Assume that there exists positive constant $\hat{T}$ such that $x_i(t) \le \epsilon_0$ for all $t \ge \hat{T}$, then when n is large enough such that $nT > \hat{T}$, for $t \in (nT, (n+1)T]$, we have

$$\begin{aligned} x_i((n+1)T) &> x_i(nT)\exp\Bigg(\int_{nT}^{(n+1)T}\Bigg[r_i - a_i\epsilon_0 \\ &\quad -\epsilon_0\sum_{j=1,j\neq i}^{m} b_{ij} - d_i(M_2)^2\Bigg]\mathrm{d}t\Bigg) \\ &> x_i(nT)\exp\Bigg(\int_{nT}^{(n+1)T}\Bigg[r_i - d_iM^2 \\ &\quad -\Bigg(a_i + \sum_{j=1,j\neq i}^{m} b_{ij}\Bigg)\epsilon_0\Bigg]\mathrm{d}t\Bigg) \\ &= x_i(nT)\exp(\delta T),\ i = 1, 2, \ldots, m, \end{aligned}$$

hence $x_i(nT) > x_i(0^+)\exp(n\delta T) \to +\infty, n \to \infty$. Thus, $x_i(t) \to +\infty (i = 1, 2, \ldots, m)$ as $n \to \infty$, which is a contradiction.

For $i = 1, 2, \ldots, m$, suppose that $x_i(t)$ are oscillatory about ϵ_0. Choose two sequences $\{u_n\}$ and $\{\bar{u}_n\}$ satisfying

$$0 < u_1 < \bar{u}_1 < u_2 < \bar{u}_2 < \ldots < u_n < \bar{u}_n < \ldots$$

and $\lim_{n\to\infty} u_n = \lim_{n\to\infty} \bar{u}_n = +\infty$, $\bar{u}_n - u_n \ge T$, and

$$x_i(u_n) \ge \epsilon_0,\ x_i(u_n^+) \le \epsilon_0,\ x_i(\bar{u}_n) \le \epsilon_0,\ x_i(\bar{u}_n^+) \ge \epsilon_0,$$

where $n = 1, 2, \ldots$, then

$$\begin{aligned} x_i(t) &\le \epsilon_0,\ t \in (u_n, \bar{u}_n],\ n = 1, 2, \ldots, \\ x_i(t) &\ge \epsilon_0,\ t \in (\bar{u}_n, u_{n+1}],\ n = 1, 2, \ldots. \end{aligned}$$

For $t \in (u_n, \bar{u}_n]$, we have

$$\begin{aligned} x_i'(t) &> x_i(t)\Bigg[r_i - a_i\epsilon_0 - \epsilon_0\sum_{j=1,j\neq i}^{m} b_{ij} - d_i(M_2)^2\Bigg], \\ x_i(t) &> x_i(u_n)\exp\Bigg(\int_{u_n}^{t}\Bigg[r_i - a_i\epsilon_0 \\ &\quad -\epsilon_0\sum_{j=1,j\neq i}^{m} b_{ij} - d_i(M_2)^2\Bigg]\mathrm{d}s\Bigg) \\ &> \epsilon_0\exp(-\xi T),\ \ i = 1, 2, \ldots, m, \end{aligned}$$

where $\xi := \max_{1\le i\le m}\Bigg\{r_i + a_i\epsilon_0 + \epsilon_0\sum_{j=1,j\neq i}^{m} b_{ij} + d_i(M_2)^2\Bigg\}$. Thus,

$$x_i(t) > \epsilon_0\exp(-\xi T),\ t \in (u_n, \bar{u}_n].$$

On the other hand, for $t \in (\bar{u}_n, u_{n+1}]$, we have $x_i(t) \ge \epsilon_0 > \epsilon_0\exp(-\xi T)$. Therefore, $x_i(t) > \epsilon_0\exp(-\xi T)$ for all $t \ge 0$. Let $m_1 := \epsilon_0\exp(-\xi T)$, we have $x_i(t) > m_1$.

Choose $m := \min\{m_1, m_2\}$, we have

$$x_i(t) > m,\ \ y(t) > m,\ \ t \ge 0,\ \ i = 1, 2, \ldots, m. \qquad (11)$$

From (10) and (11), we conclude that system (2) is permanent. The proof is completed. ■

IV. Examples

In this section, we present three examples to illustrate the feasibility and effectiveness of our results obtained in Section 3.

Example 1. *In system* (2)*, we take* $T = 1, m = 2, E = 0.8, r_1 = 1, r_2 = 1.5, a_1 = 0.8, a_2 = 0.9, b_{12} = 0.5, b_{21} = 0.8, d_1 = 0.9, d_2 = 0.8, r = 1.8, a = 0.8, d = 0.6, c_1 = 1.5, c_2 = 1.8, c = 2$*. We consider the following system:*

$$\begin{cases} x_1'(t) = x_1(t)[1 - 0.8x_1(t) - 0.5x_2(t) - 0.9(y(t) - 2)^2], \\ x_2'(t) = x_2(t)[1.5 - 0.9x_2(t) - 0.8x_1(t) - 0.8(y(t) - 2)^2], \\ y'(t) = y(t)[1.8 - 0.8y(t) + 0.6(x_1(t) - 1.5)^2 \\ \qquad +0.6(x_2(t) - 1.8)^2], \\ \Delta x_i(t) = x_i(t^+) - x_i(t) = 0, \\ \Delta y(t) = y(t^+) - y(t) = -0.8y(t). \end{cases}$$

By calculating, it is easy to check that all conditions in Theorem 1 are fulfilled. Hence, by Theorem 1, we have the enterprises A_{x_i} *do not develop and may be bankruptcy, and the enterprise* A_y *keep certain power to be permanent.*

Example 2. *In system* (2)*, we take* $T = 1$*,* $m = 2$*,* $E = 0.8$*,* $r_1 = 1.64, r_2 = 1.8$*,* $a_1 = 0.7, a_2 = 0.5$*,* $b_{12} = 0.75, b_{21} = 0.8$*,* $d_1 = 0.8, d_2 = 0.88$*,* $r = 2, a = 0.45, d = 0.3$*,* $c_1 = 1$*,* $c_2 = 1.5$*,* $c = 2$*. We consider the following system:*

$$\begin{cases} x_1'(t) = x_1(t)[1.64 - 0.7x_1(t) - 0.75x_2(t) - 0.8(y(t) - 2)^2], \\ x_2'(t) = x_2(t)[1.8 - 0.5x_2(t) - 0.8x_1(t) - 0.88(y(t) - 2)^2], \\ y'(t) = y(t)[2 - 0.45y(t) + 0.3(x_1(t) - 1)^2 \\ \qquad +0.3(x_2(t) - 1.5)^2], \\ \Delta x_i(t) = x_i(t^+) - x_i(t) = 0, \\ \Delta y(t) = y(t^+) - y(t) = -0.8y(t). \end{cases}$$

By calculating, we can check that all conditions in Theorem 2 are fulfilled. Therefore, by Theorem 2, we have the enterprises A_{x_i} *do not develop and may be bankruptcy, and the enterprise* A_y *keep certain power to be permanent.*

Example 3. *In system* (2)*, we take* $T = 1$*,* $m = 2$*,* $E = 0.6$*,* $r_1 = 2.8, r_2 = 3$*,* $a_1 = 1, a_2 = 1$*,* $b_{12} = 0.3, b_{21} = 0.22$*,* $d_1 = 0.1, d_2 = 0.12$*,* $r = 2.5, a = 0.9, d = 0.1$*,* $c_1 = 1$*,* $c_2 = 1.5$*,* $c = 2$*. We consider the following system:*

$$\begin{cases} x_1'(t) = x_1(t)[2.8 - 0.8x_1(t) - 0.3x_2(t) - 0.125(y(t) - 2)^2], \\ x_2'(t) = x_2(t)[3 - 0.75x_2(t) - 0.22x_1(t) - 0.12(y(t) - 2)^2], \\ y'(t) = y(t)[2.5 - 0.6y(t) + 0.3(x_1(t) - 1)^2 \\ \qquad +0.3(x_2(t) - 1.5)^2], \\ \Delta x_i(t) = x_i(t^+) - x_i(t) = 0, \\ \Delta y(t) = y(t^+) - y(t) = -0.6y(t). \end{cases}$$

We check that all conditions in Theorem 3 are fulfilled. Hence, by Theorem 3, we have system (2) *is permanent, which indicates that the enterprises* A_{x_i} *and enterprise* A_y *will be co-existence.*

From examples above, we can see that core enterprise has certain capability and competitive power, which make it occupy a certain superiority in center halfback model.

For satellite enterprise, except for self-development, it has to face the competition from other satellite enterprises and dominant enterprise. When intrinsic growth rate decreases and the competitive power of competitors increase, the fate of decline even extinction is unavoidable, which is according to economic rules. In addition, as we can see from theoretical results, impulsive effect is also an important factor to affect the fate of enterprises cluster. Therefore, self-development, market-saturation degree, competition power and impulse perturbations are important factors to affect the fate of enterprises cluster.

The study has some theoretical and practical meanings and value in a certain extent. However, in real and complex economic situation, considering into more factors which affect enterprises cluster, the model will be more complex. The model we establish is autonomous system and we ignore some factors such as the effect of time delay and control variables, and so on, which will or not affect the fate of enterprises cluster? It is our future work.

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