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Group Similarity Transformation of a Time Dependent Chemical Convective Process

M. M. Kassem, and A. S. Rashed

***Abstract*—**The time dependent progress of a chemical reaction over a flat horizontal plate is here considered. The problem is solved through the group similarity transformation method which reduces the number of independent by one and leads to a set of nonlinear ordinary differential equation. The problem shows a singularity at the chemical reaction order n=1 and is analytically solved through the perturbation method. The behavior of the process is then numerically investigated for $n \neq 1$ and different Schmidt numbers. Graphical results for the velocity and concentration of chemicals based on the analytical and numerical solutions are presented and discussed.

***Keywords*—**Time dependent, chemical convection, group transformation method, perturbation method.

Nomenclature

Latin characters

a_1, a_2= unity elements
c = species concentration
C = non-dimensional species concentration
c_0 = concentration next to the plate
c_∞ = ambiant concentration
D = chemical molecular diffusivity.
F' = horizontal velocity after transformation
g = gravitational acceleration
G = group
k = chemical rate constant
n = chemical reaction order
Q, T = real valued coefficients
S = subgroup
Sc = Schmidt number ν/ D
u = velocity in x direction
v = velocity in y direction

Greek characters

β= volumetric coefficient of expansion with concentration
ν = μ/ρ = kinematic viscosity of fluid
ρ = fluid density
ψ = stream function
η =similarity variable

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I. Introduction

RECENTLY Rashed *et al* [12] analyzed the problem of steady stated chemical coatings past a vertical plate. Here, the study is concerned with time dependant natural convection. This problem reported by Levich [1] and Gebhart *et al* [5] concerns the immersion of a plate is in a fluid solution having a concentration $c_0(x,t)>0$. When the plate touches the solution a chemical reaction takes place inducing a change of concentration and implying density gradients in the presence of gravitational field [3].

Ganesan *et al* [4] analyzed the diffusion of chemically reactive species for a convective unsteady flow along a vertical cylinder using an implicit finite difference method. Makinde [10] investigated a convective flow with thermal radiation and mass transfer past a moving vertical porous plate and assumed a time-dependency for the vertical velocity. The resultant similarity equations were solved numerically using a superposition method. Ibrahim *et al* [6] analytically derived the heat and mass transfer of a chemical convective process assuming an exponentially decreasing suction velocity at the surface of a porous plate and a two terms harmonic function for the rest of the variables. The mathematical technique used in the present analysis is a two parameter group transformation of the variables. This method developed by Morgan [11] reduces the number of variables by one and generates a set of ordinary differential equations. This method adopted by Kassem [7-8] and Abd-el Malek *et al* [1] proved to be efficient for an analysis of various flow problems.

In the present work we reduce the field equations and related boundary conditions through a two parameters group. This transformation results in a system of nonlinear differential equations with appropriate boundary conditions. The obtained system of equations shows a singularity at the chemical reaction order n=1. At this value the concentration profile of chemicals in the boundary layer is analytical derived using the perturbation method. The behavior of the flow is then numerically investigated for $n \neq 1$ and different Schmidt numbers using the shooting method and the results are plotted. For n=1 the concentration is analytically evaluated and plotted for different chemical molecular diffusivity rates.

II. Mathematical Formulation

The following study is concerned with time dependant convection and diffusion within a thin boundary layer adjacent to an horizontal plate immersed in a fluid, having a chemical reaction of order n. For this, it is convenient to consider an idealized system illustrated in Fig. 1 and

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composed of a semi infinite plate set in a fluid of infinite extent. The natural convection is described by the following equations;

$$\frac{\partial u}{\partial x}+\frac{\partial \mathrm{v}}{\partial y}=0 \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+\mathrm{v}\frac{\partial u}{\partial y}-g\beta c-\nu\frac{\partial^2 u}{\partial y^2}=0 \tag{2}$$

$$\frac{\partial c}{\partial t}+u\frac{\partial c}{\partial x}+\mathrm{v}\frac{\partial \mathrm{c}}{\partial \mathrm{y}}+kc^n-D\frac{\partial^2 c}{\partial y^2}=0 \tag{3}$$

subjected to the following initial and boundary conditions

$$u(x,y,0)=u_0(x,y)\ ,\ \mathrm{v}(x,y,0)=\mathrm{v}_0(x,y)\ ,\ c(x,y,0)=0 \tag{4}$$

$$u(x,0,t)=\mathrm{v}(x,0,t)=0\quad ,\quad c(x,0,t)=c_0(x,t) \tag{5}$$

$$\begin{aligned}&\lim_{y\to\infty} u(x,y,t)=\lim_{y\to\infty}\mathrm{v}(x,y,t)=0\\&\lim_{y\to\infty} c(x,y,t)=0\end{aligned} \tag{6}$$

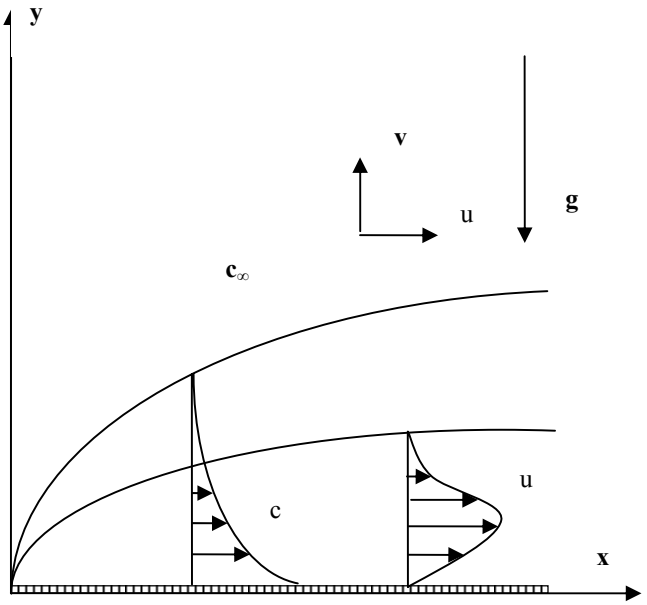


Fig. 1 Illustration of the flow over a semi-infinite horizontal plate

Equation (1) is eliminated and concentration at the wall (5) is normalized through the transformations

$$u=\frac{\partial \psi}{\partial y},\qquad \mathrm{v}=-\frac{\partial \psi}{\partial x} \tag{7}$$

$$C(x,y,t)=\frac{c(x,y,t)}{c_0(x,t)} \tag{8}$$

In this case the flow equations (1) - (6) reduce to

$$\frac{\partial^2\psi}{\partial y\partial t}+\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial x\partial y}-\frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y^2}-g\beta c_0 C-\nu\frac{\partial^3\psi}{\partial y^3}=0 \tag{9}$$

$$\begin{aligned}&\frac{\partial C}{\partial t}+\frac{C}{c_0}\frac{\partial c_0}{\partial t}+\frac{\partial\psi}{\partial y}\frac{\partial C}{\partial x}+\frac{C}{c_0}\frac{\partial\psi}{\partial y}\frac{\partial c_0}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial C}{\partial y}\\&+kc_0^{n-1}C^n-D\frac{\partial^2 C}{\partial y^2}=0\end{aligned} \tag{10}$$

subjected to the boundary conditions

$$\begin{aligned}&\frac{\partial\psi(x,y,0)}{\partial y}=u_0(x,y),\\&\frac{\partial\psi(x,y,0)}{\partial x}=-\mathrm{v}_0(x,y),\\&C(x,y,0)=0\end{aligned} \tag{11}$$

$$\begin{aligned}&\frac{\partial\psi(x,0,t)}{\partial y}=\frac{\partial\psi(x,0,t)}{\partial x}=0,\\&C(x,0,t)=1\end{aligned} \tag{12}$$

$$\left.\frac{\partial\psi}{\partial y}\right|_{y\to\infty}=\left.\frac{\partial\psi}{\partial x}\right|_{y\to\infty}=0\ ,\ \lim_{y\to\infty}C(x,y,t)=0 \tag{13}$$

III. Group Formulation of the Problem

From the group definition [2]

$$G:\bar{S}=Q^s(a_1,a_2)S+T^s(a_1,a_2) \tag{14}$$

where G is a two parameter group , $\bar{S}$ and S stand for the system variables (t, x; ψ, C and c_0) before and after transformation, Q^s, T^s are real valued coefficients at least differentiable in the group parameters (a_1,a_2). First and second partial derivatives are defined as;

$$\left.\begin{aligned}&\bar{S}_{\bar{i}}=\left(\frac{Q^s}{Q^i}\right)S_i\\&\bar{S}_{\bar{i}\,\bar{j}}=\left(\frac{Q^s}{Q^iQ^j}\right)S_{ij}\end{aligned}\right\}\ i,j=x,y \tag{15}$$

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where S stands for the dependent variables $(\psi, C \text{ and } c_0)$. From the above definitions (9) is transformed to

$$\frac{\partial^2 \overline{\psi}}{\partial \overline{y} \partial \overline{t}} + \frac{\partial \overline{\psi}}{\partial \overline{y}} \frac{\partial^2 \overline{\psi}}{\partial \overline{x} \partial \overline{y}} - \frac{\partial \overline{\psi}}{\partial \overline{x}} \frac{\partial^2 \overline{\psi}}{\partial \overline{y}^2} - g\beta \overline{c}_0 \overline{C} - \nu \frac{\partial^3 \overline{\psi}}{\partial \overline{y}^3}$$
$$= H_1(a_1, a_2)\left(\frac{\partial \psi}{\partial y} \frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial \psi}{\partial x} \frac{\partial^2 \psi}{\partial y^2} - g\beta c_0 C - \nu \frac{\partial^3 \psi}{\partial y^3} \right) \tag{16}$$

$$\left(\frac{Q^{\psi}}{Q^y Q^t} \right) \frac{\partial^2 \psi}{\partial y \partial t} + \left(\frac{Q^{\psi}}{Q^y} \frac{Q^{\psi}}{Q^x Q^y} \right) \frac{\partial \psi}{\partial y} \frac{\partial^2 \psi}{\partial x \partial y}$$
$$- \left(\frac{Q^{\psi}}{Q^x} \frac{Q^{\psi}}{(Q^y)^2} \right) \frac{\partial \psi}{\partial x} \frac{\partial^2 \psi}{\partial y^2} - g\beta Q^{c_0} Q^C c_0 C$$
$$- \nu \left(\frac{Q^{\psi}}{(Q^y)^3} \right) \frac{\partial^3 \psi}{\partial y^3} + R_1$$
$$= H_1(a_1, a_2)\left(\frac{\partial \psi}{\partial y} \frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial \psi}{\partial x} \frac{\partial^2 \psi}{\partial y^2} - g\beta c_0 C - \nu \frac{\partial^3 \psi}{\partial y^3} \right) \tag{17}$$

Where;

$$R_1 = -g\beta(Q^{c_0} T^C c_0 + T^{c_0} Q^C C + T^{c_0} T^C) \tag{18}$$

The invariant transformation of (17) implies that $H_1(a_1,a_2)=1$ and $R_1 = 0$ giving ;

$$T^{c_0} = T^C = 0 \tag{19}$$

$$\frac{Q^{\psi}}{Q^y Q^t} = \frac{(Q^{\psi})^2}{Q^x (Q^y)^2} = Q^C Q^{c_0} = \frac{Q^{\psi}}{(Q^y)^3} = 1 \tag{20}$$

similarly (10)- (14) are transformed

$$\frac{Q^C}{Q^t} = \frac{Q^{\psi} Q^C}{Q^y Q^x} = (Q^{c_0})^{n-1} (Q^C)^n = \frac{Q^C}{(Q^y)^2} = 1 \tag{21}$$

$$Q^C = 1 \quad , \quad T^y = T^t = 0 \tag{22}$$

and the ratios (20) to (22) reduce to

$$Q^{c_0} = 1 \, , \; Q^t = (Q^y)^2 \, , \; Q^{\psi} = (Q^y)^3 \, , \; Q^x = (Q^y)^4 \tag{23}$$

giving a group structure of the form;

$$G \begin{cases} G_1 \begin{cases} \overline{x} = (Q^y)^4 x + T^x \\ \overline{y} = Q^y y \\ \overline{t} = (Q^y)^2 t \end{cases} \\ G_2 \begin{cases} \overline{\psi} = (Q^y)^3 \psi + \mathrm{T}^{\psi} \\ \overline{C} = C \\ \overline{c_0} = c_0 \end{cases} \end{cases} \tag{24}$$

where G_1 and G_2 are subgroups describes the independent and dependant variables and dashes stand for their transformation.

A. Group Transformation of the System Variables

The flow equations order is reduced by one if it satisfies Morgan's theorem [11]

$$\left. \begin{aligned} \sum_{i=1}^{6} [\alpha_i u(i) + \alpha_{i+1}] \frac{\partial \overline{u}(i)}{\partial u(i)} = 0 \\ \sum_{i=1}^{6} [\beta_i u(i) + \beta_{i+1}] \frac{\partial \overline{u}(i)}{\partial u(i)} = 0 \end{aligned} \right\} \tag{25}$$

where $u(i), \overline{u}(i), i = 1, 2, \ldots 6$ stand for the six system variables (x,y,t; ψ, C, c_0) before and after transformation

$$\alpha_i = \frac{\partial Q^{s_i}(a_1, a_2)}{\partial a_1} \quad , \quad \alpha_{i+1} = \frac{\partial T^{s_i}(a_1, a_2)}{\partial a_1} \tag{26}$$

$$\beta_i = \frac{\partial Q^{s_i}(a_1, a_2)}{\partial a_2} \, , \quad \beta_{i+1} = \frac{\partial T^{s_i}(a_1, a_2)}{\partial a_2} \tag{27}$$

B. Transformation of the Independent Variables

The similarity variable η(x,y,t) is derived through the application of Morgan's theorem (25)

$$(\alpha_1 x + \alpha_2) \frac{\partial \eta}{\partial x} + \alpha_3 y \frac{\partial \eta}{\partial y} + \alpha_5 t \frac{\partial \eta}{\partial t} = 0 \tag{28}$$

$$(\beta_1 x + \beta_2) \frac{\partial \eta}{\partial x} + \beta_3 y \frac{\partial \eta}{\partial y} + \beta_5 t \frac{\partial \eta}{\partial t} = 0 \tag{29}$$

Eliminating $\frac{\partial \eta}{\partial y}$ and $\frac{\partial \eta}{\partial x}$ respectively from (28) and (29);

$$(\lambda_{13} x + \lambda_{23}) \frac{\partial \eta}{\partial x} + \lambda_{53} t \frac{\partial \eta}{\partial t} = 0$$
$$\lambda_{ij} = \alpha_i \beta_j - \alpha_j \beta_i \tag{30}$$

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$$(\lambda_{31}x+\lambda_{32})y\frac{\partial\eta}{\partial y}+(\lambda_{51}x+\lambda_{52})t\frac{\partial\eta}{\partial t}=0 \tag{31}$$

then invoking (24) group structure we obtain;

$$\alpha_5=2\alpha_3 \tag{32}$$

$$\beta_5=2\beta_3 \tag{33}$$

i.e $\lambda_{53}=\alpha_5\beta_3-\alpha_3\beta_5=0$ and (30) reduces to

$$(\lambda_{13}x+\lambda_{23})\frac{\partial\eta}{\partial x}=0 \tag{34}$$

$$\frac{\partial\eta}{\partial x}=0 \tag{35}$$

$$\eta=\eta(y,t) \tag{36}$$

from (36) in (31) we obtain the similarity variable

$$\eta=y\pi(t) \tag{37}$$

where:

$$\pi(t)=at^b \tag{38}$$

without loss of generality let a=1 while b will be determined later.

C. Transformation of the Dependant Variables

Transformation of C and c_0 is obtained directly from (24) group structure

$$\overline{C}\left(\bar{x},\bar{y}\right)=C(\eta) \tag{39}$$

$$\overline{c_0}(\bar{x},\bar{t})=\Gamma(x,t) \tag{40}$$

Γ(x,t) will be determined later. Similarly ψ is transformed applying (25)

$$\begin{aligned}&(\alpha_1x+\alpha_2)\frac{\partial g_3(\bar{x},\bar{y},\bar{t};\overline{\psi})}{\partial x}+\alpha_3y\frac{\partial g_3(\bar{x},\bar{y},\bar{t};\overline{\psi})}{\partial y}\\&+(\alpha_5\psi+\alpha_6)\frac{\partial g_3(\bar{x},\bar{y},\bar{t};\overline{\psi})}{\partial\psi}=0\\&(\beta_1x+\beta_2)\frac{\partial g_3(\bar{x},\bar{y},\bar{t};\overline{\psi})}{\partial x}+\beta_3y\frac{\partial g_3(\bar{x},\bar{y},\bar{t};\overline{\psi})}{\partial y}\\&+(\beta_5\psi+\beta_6)\frac{\partial g_3(\bar{x},\bar{y},\bar{t};\overline{\psi})}{\partial\psi}=0\end{aligned} \tag{41}$$

the solution of this differential equation results in

$$g_3\left(\bar{x},\underline{y},\bar{t};\overline{\psi}\right)=\phi_1\left(\psi/\omega(x)\right)=F(\eta) \tag{42}$$

and for $\phi_1=1$ (42) reduces to

$$\overline{\psi}(\bar{x},\bar{y},\bar{t})=\omega(x,t)F(\eta) \tag{43}$$

then replacing for the variables C, c_0 and ψ described in (39), (40) and (43) and their derivatives in the flow equations (9) and (10) we obtain

$$\begin{aligned}&\frac{d^3F}{d\eta^3}-A_1\eta\frac{d^2F}{d\eta^2}+A_2F\frac{d^2F}{d\eta^2}-\\&\left(A_1+A_3\right)\frac{dF}{d\eta}-A_2\left(\frac{dF}{d\eta}\right)^2+A_4C=0\end{aligned} \tag{44}$$

$$\begin{aligned}&\frac{d^2C}{d\eta^2}-\frac{\nu}{D}\eta\frac{dC}{d\eta}+\frac{\nu}{D}F\frac{dC}{d\eta}-A_5C\\&-A_6\frac{dF}{d\eta}C-A_7C^n=0\end{aligned} \tag{45}$$

where

$$A_1=\frac{d\pi/dt}{\nu\pi^3},\quad A_2=\frac{\partial\omega/\partial x}{\nu\pi} \tag{46}$$

$$A_3=\frac{\partial\omega/\partial t}{\nu\omega\pi^2},\quad A_4=\frac{g\beta\Gamma}{\nu\omega\pi^3} \tag{47}$$

$$A_5=\frac{\partial\Gamma/\partial t}{D\Gamma\pi^2},\quad A_6=\frac{\omega}{D\pi\Gamma}\frac{\partial\Gamma}{\partial x} \tag{48}$$

$$A_7=\frac{k\Gamma^{n-1}}{D\pi^2} \tag{49}$$

In order to reduce (44), (45) to a system of ordinary equations, the coefficients A_i must be function of η or constants. For this we set $A_7=\frac{k}{D}$ in (49). This leads to;

$$\Gamma(x,t)=\left(\pi(t)\right)^{\frac{2}{n-1}} \tag{50}$$

from (38) in (50)

$$\Gamma(t)=\left(t^b\right)^{\frac{2}{n-1}} \tag{51}$$

where Γ(t) is the concentration of chemicals next to the plate. Then from (51) in (48) we obtain;

$$A_6 = 0 \tag{52}$$

$$A_5 = \frac{2b}{(n-1)D} t^{-(2b+1)} \tag{53}$$

A_5 will be constant if b = -1/2 . i.e $A_5 = \frac{-1}{D(n-1)}$ and the similarity variable (37) reduces to;

$$\eta = \frac{y}{\sqrt{t}} \tag{54}$$

then setting $A_4 = \frac{g\beta}{\nu}$ results in

$$\omega(x,t) = \omega(t) = t^{\frac{3n-5}{2(n-1)}} \qquad , n \neq 1 \tag{55}$$

where ω(t) is part of the stream function described in (43). Replacing for ω, π, Γ in (46) and (47) we obtain the remaining constants

$$A_3 = \frac{3n-5}{2(n-1)\nu} \qquad , n \neq 1 \tag{56}$$

$$A_2 = 0 \tag{57}$$

$$A_1 = \frac{-1}{2\nu} \tag{58}$$

Substituting for the A's in (44), (45) yields a system of differential equations

$$\frac{d^3F}{d\eta^3} + \left(\frac{\eta}{2\nu}\right)\frac{d^2F}{d\eta^2} - \left(\frac{2n-4}{2(n-1)\nu}\right)\frac{dF}{d\eta} + \frac{g\beta}{\nu}C = 0 \tag{59}$$

$$\frac{d^2C}{d\eta^2} + \left(\frac{\eta}{2D}\right)\frac{dC}{d\eta} + \frac{1}{D(n-1)}C - \frac{k}{D}C^n = 0 \tag{60}$$

subjected to the boundary conditions

$$\text{for} \quad \eta = 0; \quad F((0) = F'(0) = 0 \, , C(0) = 1 \tag{61}$$

$$\lim \eta \to \infty \, ; \quad F'(\infty) = C(\infty) = 0 \tag{62}$$

IV. Perturbation Analysis

Eqs. (59) and (60) are singular at n=1. An analytical solution of (60) at n=1 is derived assuming a perturbation factor ε .

$$C = C_0 + \varepsilon C_1 + \varepsilon^2 C_2 + \ldots \tag{63}$$

The elements of (63) are obtained through the following steps:

i- From the boundary condition (61)

$$C_0(0) + \varepsilon C_1(0) + \varepsilon^2 C_2(0) + \varepsilon^3 C_3(0) + ... = 1 \tag{64}$$

and equating the powers of $\varepsilon^0, \varepsilon^1$ leads to

$$C_0(0) = 1 \tag{65}$$

$$C_1(0) = C_2(0) = C_3(0) = ... = 0 \tag{66}$$

ii- eq. (60) is then multiplied by (n-1)

$$(n-1)\left[\frac{d^2C}{d\eta^2} + \left(\frac{\eta}{2D}\right)\frac{dC}{d\eta} - \left(\frac{k}{D}\right)C^n\right] + \frac{C}{D} = 0 \tag{67}$$

at $n \to 1$ we assume that $\varepsilon = n - 1$

$$\varepsilon\left[\frac{d^2C}{d\eta^2} + \left(\frac{1}{2D}\right)\eta\frac{dC}{d\eta} - \left(\frac{k}{D}\right)C^n\right] + \frac{C}{D} = 0 \tag{68}$$

substituting for (63) in (68) results in

$$\begin{aligned} &\varepsilon\left(\frac{d^2C_0}{d\eta^2} + \varepsilon\frac{d^2C_1}{d\eta^2} + \varepsilon^2\frac{d^2C_2}{d\eta^2} + \ldots\right) \\ &+ \frac{\eta}{2D}\left(\frac{d\ C_0}{d\eta} + \varepsilon\frac{d\ C_1}{d\eta} + \varepsilon^2\frac{dC_2}{d\eta} + \ldots\right) \\ &\qquad - \frac{k}{D}\left(C_0 + \varepsilon C_1 + \varepsilon^2 C_2 + \ldots\right)^n \\ &\qquad + \frac{1}{D}\left(C_0 + \varepsilon C_1 + \varepsilon^2 C_2 + \ldots\right) = 0 \end{aligned} \tag{69}$$

equating the powers of ε^0 on both sides of this equation leads to

$$C_0(\eta) = 0 \tag{70}$$

this solution is rejected as it doesn't satisfy the boundary conditions (61). Then equating the powers of ε^1 we obtain

$$\frac{d^2C_0}{d\eta^2} + \frac{\eta}{2D}\frac{dC_0}{d\eta} - \frac{k}{D}C_0^n + \frac{1}{D}C_1 = 0 \, . \tag{71}$$

This equation is subjected to the boundary conditions

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$$C_0(0)=1,\ \lim_{\eta->\infty} C_0(\eta)=0 \tag{72}$$

$$C_1(0)=0,\ \lim_{\eta->\infty} C_1(\eta)=0 \tag{73}$$

from (73) we deduce that

$$C_1(\eta)=0 \tag{74}$$

Thus (71) reduces to

$$\frac{d^2C_0}{d\eta^2}+\frac{\eta}{2D}\frac{dC_0}{d\eta}-\frac{k}{\mathrm{D}}C_0^n=0 \tag{75}$$

for n=1 it simplifies

$$\frac{d^2C_0}{d\eta^2}+\frac{\eta}{2D}\frac{dC_0}{d\eta}-\frac{k}{\mathrm{D}}C_0=0 \tag{76}$$

This equation subjected to the boundary condition (72) has a solution of the form

$$C(\eta)=\frac{1}{2}\,e^{-\frac{\eta^2}{4D}}\left[\begin{array}{l} {}_1F_1\left(\frac{(1+2k)}{2},\frac{1}{2};\frac{\eta^2}{4D}\right) \\ +\ H\left(-1-2k;\frac{\eta}{2\sqrt{D}}\right)\end{array}\right] \tag{77}$$

where ${}_1F_1\left[\frac{1+2k}{2},\frac{1}{2};\frac{\eta^2}{4D}\right]$ and $H\left[-1-2k;\frac{\eta}{2\sqrt{D}}\right]$ are respectively hypergeometric and Hermite functions.

V. Results and Discussion

A. Numerical Solution at Values Different from n=1

The numerical solution of the differential equations (59) and (60) describing the momentum and concentration of fluid is investigated for different chemical orders *n* using the Runge Kutta method. The boundary conditions (61) and (62) are completed by guessing two additional conditions at η =0; F''(0), C'(0) and the solution is iterated on through the shooting method so that the boundary conditions at η = ∞ are satisfied. The parameters values adopted here are; gβ=1, k=10^{-3} /sec, ν= 10 in^2/sec and the velocity F'(η) and the concentration of chemicals C(η) are evaluated for different reaction orders *n* > 1-3. The decomposition of nitrogen pentoxide and the radioactive disintegration of unstable nuclei is a first order reaction while the gas phase thermal decomposition of nitrogen dioxide and hydrogen iodide are both examples of second order reactions. Third order reactions are extremely rare in engineering practice.

Fig. 2 displays the velocity profile for n=1.2. In this figure the momentum profile shows pronounced fluctuations and an over hump that attains a maximum of 800 for Sc=0. The fluctuations rapidly disappear and the over hump diminishes for a gradually increasing of the Schmidt number.

Figs. 3-5 display the effect of Schmidt number on the momentum profile for n ranging from 2-3. No orders higher than the third order in any reactant are known. As the Schmidt number increases the viscosity reduces the buoyancy effect, yielding a reduction in the fluid velocity, a decrease in the maximum velocity as well as a reduction in the convection layers widths and heights. These behaviors appear clearly in Figs 2-5.

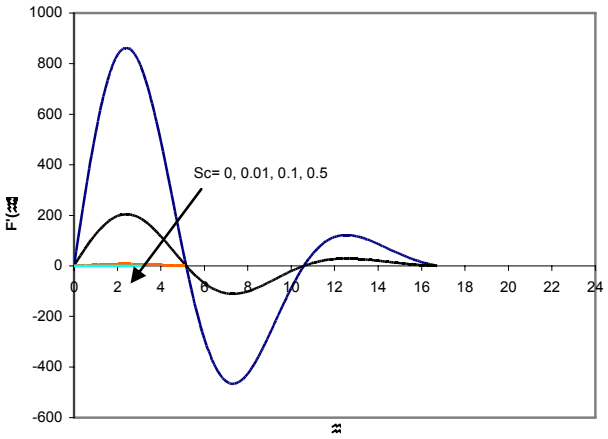


Fig. 2 Momentum profile F'(η) for n=1.2 and various Schmidt numbers

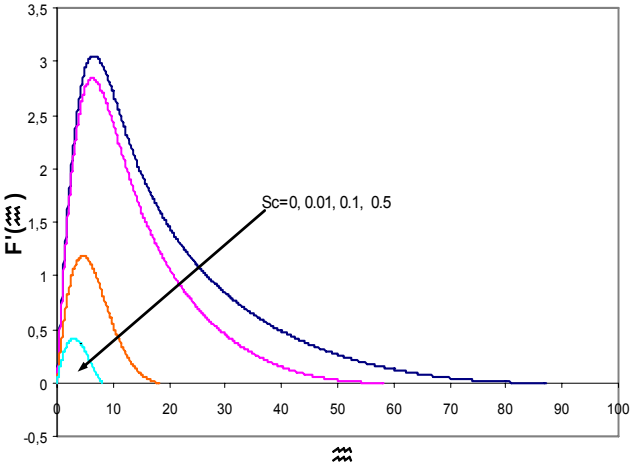


Fig. 3 Velocity profiles for various Schmidt numbers and chemical reaction order n=2

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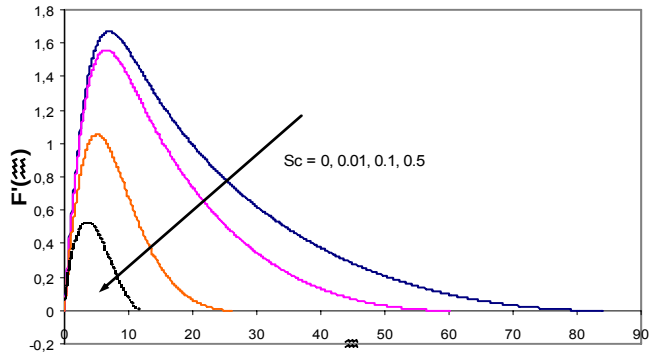


Fig. 4 Momentum profiles for various Schmidt numbers and n = 2.5

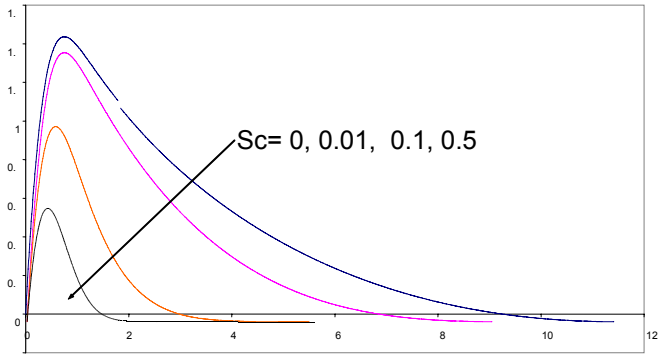


Fig. 5 Velocity profiles for various Schmidt numbers and n=3

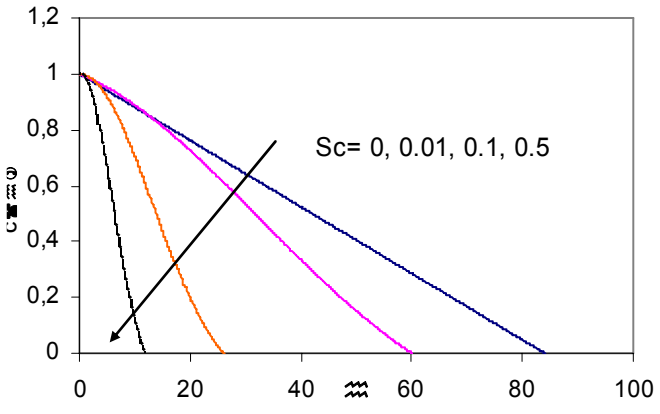


Fig. 6 Concentration profiles for different Schmidt numbers and n = 2.5

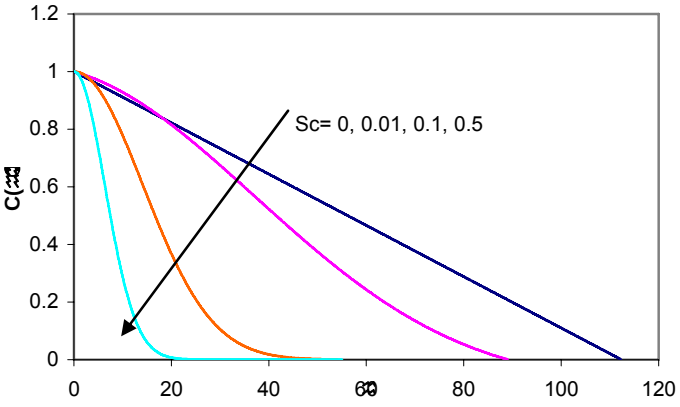


Fig. 7 Concentration profiles for different Schmidt numbers and n = 3

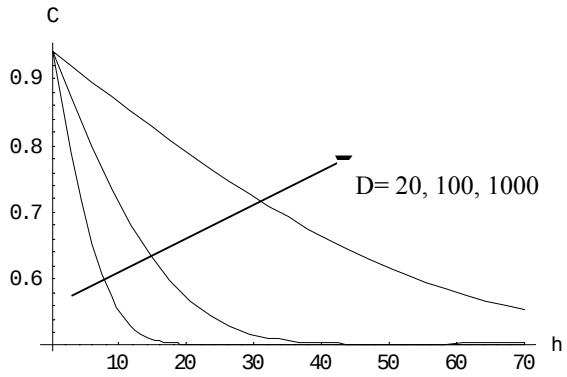


Fig. 8 Concentration for n=1, k= 10^{-3} and D = 20 to 10^{3}

Fig. 6 displays for n=2.5 different concentration profiles. For Sc=0 a linear distribution of reaction species is obtained. With the increase of the Schmidt number (Sc=0.01 → 0.5) the width of the concentration layer tends to decrease. This is due to an increase in the fluid viscosity. Further investigations for n= 3 (Fig.7) show a decrease in the concentration layer widths as the Schmidt number increases.

B. Analytical Results

The concentration of chemicals C(η) at n=1 is investigated for different values of chemical molecular viscosities and k =10^{-3}. Fig. 8 displays the concentration layer profile which tends to be linear with the increase in D.

A further analysis of the case n=1 is investigated by omitting the last term in (76) $\frac{k}{D}C_0$. This term is neglected as k= 10^{-3} and D value varies between 20 and 10^{3} . The equation thus reduces to;

$$\frac{d^2C_0}{d\eta^2}+\frac{\eta}{2D}\frac{dC_0}{d\eta}=0 \qquad (78)$$

Subjected to the boundary conditions (72) its solution is;

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$$C_0(\eta) = erfc\left(\frac{\eta}{2\sqrt{D}}\right) \tag{79}$$

which corresponds to the form of curves depicted in Fig. 8.

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