World Academy of Science, Engineering and Technology
International Journal of Mechanical and Mechatronics Engineering
Vol:4, No:2, 2010

Dynamic Stability of Beams with Piezoelectric Layers Located on a Continuous Elastic Foundation

A. R. Nezamabadi, and M. Karami Khorramabadi

Open Science Index, Mechanical and Mechatronics Engineering Vol:4, No:2, 2010 publications.waset.org/2350/pdf

***Abstract*—**This paper studies dynamic stability of homogeneous beams with piezoelectric layers subjected to periodic axial compressive load that is simply supported at both ends lies on a continuous elastic foundation. The displacement field of beam is assumed based on Bernoulli-Euler beam theory. Applying the Hamilton's principle, the governing dynamic equation is established. The influences of applied voltage, foundation coefficient and piezoelectric thickness on the unstable regions are presented. To investigate the accuracy of the present analysis, a compression study is carried out with a known data.

***Keywords*—**Dynamic stability, Homogeneous graded beam-Piezoelectric layer, Harmonic balance method.

I. Introduction

THE dynamic stability of structures is a subject of considerable engineering importance, and many investigations have been carried out in this subject. In 1985, Bailey and Hubbard [1] investigated the active vibration control of a cantilever beam using distributed piezoelectric polymer as an actuator. Crawley and de Luis [2] developed analytical models for the dynamic response of a cantilever beam with segmented piezoelectric actuators that are either bonded to an elastic substructure or embedded in a laminated composite. Shen [3] used the finite element method to study the free vibration problems of beams containing piezoelectric sensors and actuators.

Pierre and Dowell [4] reported the dynamic instability of plates using an extended incremental harmonic balance method. Liu et al. [5] used a finite element model to analyze the shape control and active vibration suppression of laminated composite plates with integrated piezoelectric sensors and actuators. By a feedback control loop, Tzou and Tseng [6] and Ha et al. [7] formulated three-dimensional incompatible finite elements for vibration control of structures containing piezoelectric actuators and sensors. The dynamic instability of a structure subjected to periodic axial compressive forces has attracted a lot of attention.. Bolotin [8] summarized the results achieved in comprehensive studies for the dynamic stability of machine components and structural members. Briseghella et al. [9] used beam elements without axial deformability to solve the dynamic stability problem of beam structures. The load bending contribution was taken into account by means of a second-order approach. Takahashi et al. [10] investigated dynamically unstable regions of cantilever rectangular plates. They presented the numerical results obtained for various loading conditions that are applied along the edge. Recently, Zhu et al. [11] presented a three-dimensional theoretical analysis of the dynamic instability region of functionally graded piezoelectric circular cylindrical shells subjected to a combined loading of periodic axial compression and electric field in the radial direction.

To the author's knowledge, there is no analytical solution available in the open literatures for dynamic stability of homogeneous beams with piezoelectric actuators located on a continuous elastic foundation. In the present work, the dynamic stability of a homogeneous beam with piezoelectric actuators subjected to periodic axial compressive loads located on a continuous elastic foundation is studied. Appling the Hamilton's principle, the dynamic equation of beam is derived and solved using the harmonic balance method. The effect of the applied voltages, piezoelectric thicknesses and foundation coefficient on the unstable regions of beam are also discussed.

II. Formulation

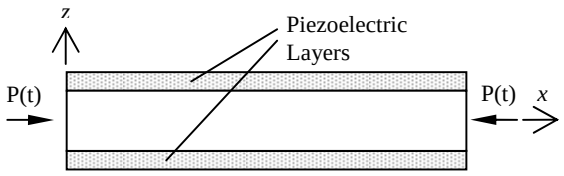


Fig. 1 Schematic of the problem studied.

Consider a homogeneous beam with piezoelectric actuators and rectangular cross-section as shown in Fig. 1. The thickness, length, and width of the beam are denoted, respectively, by $h, L,$ and $b.$ Also, h_T and h_B are the thickness of top and bottom of piezoelectric actuators, respectively. The $x-y$ plane coincides with the midplane of the beam and the $z-$axis located along the thickness direction. The Young's modulus E and the Poisson's ratio ν are assumed to be constant. The beam is assumed to be slender, thus, the Euler-Bernoulli beam theory is adopted. The piezoelectric layers are also assumed to be polarized along the thickness direction. The axial stress and electrical displacement can be written as:

A. R. Nezamabadi, is with the Department of Mechanical Engineering, Islamic Azad University, Arak Branch, Arak, Iran (e-mail: alireza.nezamabadi@gmail.com).

M. Karami Khorramabadi is with the Department of Mechanical Engineering, Islamic Azad University, Khorramabad Branch, Khorramabad, Iran (corresponding author to provide phone: +98 661 2204463; fax: +98 661 2214281; e-mail: mehdi_karami2001@yahoo.com).

$$\sigma_{xx} = \frac{E}{1-\nu^2}\varepsilon_{xx} - e_{31}E_z \qquad (1)$$
$$D_z = e_{31}\varepsilon_{xx} + \eta_{33}E_z$$

where

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} - z\frac{\partial^2 w}{\partial x^2} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 \qquad (1)$$

and

$$E_z = \frac{V}{h} \qquad (3)$$

where $\sigma_{xx}, D_z, e_{31},$ and η_{33} are the normal stress, electrical displacement, piezoelectric elastic stiffness, and permittivity coefficient, respectively, and u and w are the displacement components in the $x-$ and $z-$directions, respectively.

The potential energy can be expressed as:

$$U = \frac{1}{2}\int_v (\sigma_{xx}\varepsilon_{xx} - D_z E_z)\,\mathrm{d}v \qquad (4)$$

Substituting Eqs. (2)-(4) into Eq. (5) and neglecting the higher-order terms, we obtain:

$$\begin{aligned} U = {} & \frac{1}{2}\int_v \frac{E}{1-\nu^2}\left(\frac{\partial u}{\partial x} - z\frac{\partial^2 w}{\partial x^2}\right)^2 \mathrm{d}v \\ & + \int_v \left[\frac{E}{1-\nu^2}\left(\frac{\partial u}{\partial x} - z\frac{\partial^2 w}{\partial x^2}\right) - e_{31}\frac{V}{h}\right]\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 \mathrm{d}v \\ & - \int_v \left(\frac{\partial u}{\partial x} - z\frac{\partial^2 w}{\partial x^2}\right)e_{31}\frac{V}{h}\mathrm{d}v - \frac{1}{2}\int_v \left(\frac{V}{h}\right)^2 \eta_{33}\mathrm{d}v \end{aligned} \qquad (5)$$

The width of beam is assumed to be constant, which is obtained by integrating along y over v. Then Eq. (5) becomes

$$\begin{aligned} U = {} & \frac{b}{2}\int_0^L \left[A_{11}\left(\frac{\partial u}{\partial x}\right)^2 - 2B_{11}\frac{\partial u}{\partial x}\frac{\partial^2 w}{\partial x^2} + D_{11}\left(\frac{\partial^2 w}{\partial x^2}\right)^2\right]\mathrm{d}x \\ & + \frac{b}{2}\int_0^L P'\left(\frac{\partial w}{\partial x}\right)^2 \mathrm{d}x - \frac{b}{2}\int_0^L \eta_{33}\left(\frac{V_B^2}{h_B} + \frac{V_T^2}{h_T}\right)\mathrm{d}x \\ & - b\int_0^L \Bigg(\int_{-h_B-\frac{h}{2}}^{-\frac{h}{2}} \left(\frac{\partial u}{\partial x} - z\frac{\partial^2 w}{\partial x^2}\right)e_{31}\frac{V_B}{h_B}\mathrm{d}z + \\ & \int_{\frac{h}{2}}^{h_T+\frac{h}{2}} \left(\frac{\partial u}{\partial x} - z\frac{\partial^2 w}{\partial x^2}\right)e_{31}\frac{V_T}{h_T}\mathrm{d}z\Bigg)\mathrm{d}x \end{aligned} \qquad (6)$$

where

$$(A_{11}, B_{11}, D_{11}) = \frac{1}{1-\nu^2}\int_{-h_B-\frac{h}{2}}^{h_T+\frac{h}{2}} (1, z, z^2)E\mathrm{d}z \qquad (7)$$

and

$$\begin{aligned} P' = {} & \int_{-h_B-\frac{h}{2}}^{h_T+\frac{h}{2}} \left[\frac{E}{1-\nu^2}\left(\frac{\partial u}{\partial x} - z\frac{\partial^2 w}{\partial x^2}\right) - e_{31}E_z\right]\mathrm{d}z \\ = {} & A_{11}\frac{\partial u}{\partial x} - B_{11}\frac{\partial^2 w}{\partial x^2} - \int_{-h_B-\frac{h}{2}}^{h_T+\frac{h}{2}} e_{31}\frac{V}{h}\mathrm{d}z \end{aligned} \qquad (8)$$

where $A_{11}, B_{11}, D_{11}, V_T, V_B$ and P' are the extensional stiffness, coupling stiffness, bending stiffness, applied voltages on the top and bottom actuators and piezoelectric force, respectively. When the applied voltage is negative, the piezoelectric force is tensile. Note that, no residual stresses due to the piezoelectric actuator are considered in the present study and the extensional displacement is neglected. Thus, the potential energy can be written as:

$$\begin{aligned} U = {} & \frac{b}{2}\int_0^L D_{11}\left(\frac{\partial^2 w}{\partial x^2}\right)^2 \mathrm{d}x + \frac{P'b}{2}\int_0^L \left(\frac{\partial w}{\partial x}\right)^2 \mathrm{d}x - \frac{b}{2}\int_0^L \eta_{33}\left(\frac{V_B^2}{h_B} + \frac{V_T^2}{h_T}\right)\mathrm{d}x \\ & + b\int_0^L \left[\int_{-h_B-\frac{h}{2}}^{-\frac{h}{2}} z\left(\frac{\partial^2 w}{\partial x^2}\right)e_{31}\frac{V_B}{h_B}\mathrm{d}z + \int_{\frac{h}{2}}^{h_T+\frac{h}{2}} z\left(\frac{\partial^2 w}{\partial x^2}\right)e_{31}\frac{V_T}{h_T}\mathrm{d}z\right]\mathrm{d}x \end{aligned} \qquad (9)$$

The beam is subjected to the periodic axial compressive loads, $P(t)$ as shown in Fig. 2.

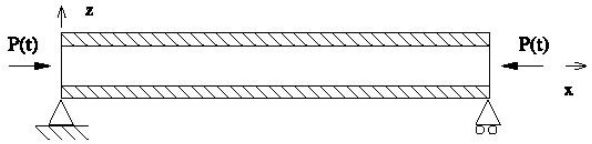


Fig. 2 Simply supported beam under periodic loads

$$P(t) = P_0 + P_t\cos\theta t = \alpha P^* + \beta P^*\cos\theta t \qquad (10)$$

Here, α and β are the static and dynamic load factors. The work done by the periodic axial compressive load can be expressed as:

$$W = \frac{1}{2}\int_0^L P(t)\left(\frac{\partial w}{\partial x}\right)^2 \mathrm{d}x \qquad (11)$$

Open Science Index, Mechanical and Mechatronics Engineering Vol:4, No:2, 2010 publications.waset.org/2350/pdf

Open Science Index, Mechanical and Mechatronics Engineering Vol:4, No:2, 2010 publications.waset.org/2350/pdf

The kinetic energy can be expressed as:

$$T = \frac{1}{2}\int_{\circ}^{L} m\left(\frac{\partial w}{\partial t}\right)^2 dx \tag{12}$$

where m is the mass per unit length of the beam. We apply the Hamilton's principle to derive the dynamic equation of beam, that is:

$$\delta\int_{0}^{t} (T - U + W)\,\mathrm{d}t = 0 \tag{13}$$

Substitution from Eqs. (9), (11), and (12) into Eq. (13) leads to the following dynamic equation:

$$m\frac{\partial^2 w}{\partial t^2} + bD_{11}\frac{\partial^4 w}{\partial x^4} + (P(t) - bP')\frac{\partial^2 w}{\partial x^2} = 0 \tag{14}$$

Assume that a homogeneous beam with piezoelectric actuators that is simply supported at both ends lies on a continuous elastic foundation, whose reaction at every point is proportional to the deflection (Winkler foundation). The dynamic equation of the homogeneous beams with piezoelectric layers located on a continuous elastic foundation subjected to a periodic axial compressive load is obtained from Eq. (14) by the addition of ηw for the foundation reaction as:

$$m\frac{\partial^2 w}{\partial t^2} + bD_{11}\frac{\partial^4 w}{\partial x^4} + (P(t) - bP')\frac{\partial^2 w}{\partial x^2} + \eta w = 0 \tag{15}$$

where η is the foundation coefficient.

III. Stability Analysis

For the simply supported boundary condition, the solution of the dynamic equation is assumed to be in the following form:

$$w(x,t) = f(t)\sin\frac{n\pi x}{L}, \qquad n = 1,2,3,... \tag{16}$$

where $f_k(t)$ are as yet undetermined function of time, satisfies this equation. Substituting expression Eq. (16) into Eq. (15) leads to the following equation:

$$f_k'' + \omega_k^2\left(1 - \frac{p(t) - bp'}{p_{*k}}\right) f_k = 0 \tag{17}$$

where

$$\omega_k^2 = \frac{1}{m}[bD_{11}(\frac{k\pi}{l})^4 + \eta] \tag{18}$$

and

$$p_{*k} = (\frac{k\pi}{l})^2 bD_{11} + \frac{l^2\eta}{k^2\pi^2} \tag{19}$$

where ω_k is the kth free vibration frequency of homogeneous beam with piezoelectric actuators loaded by a constant axial force P and p_{*k} is the critical buckling load. Analogous equations are obtained by considering the case of an infinitely long beam. In this case, Eq.(16) will be satisfied by assuming that:

$$w(x,t) = f\ (t,\lambda)\sin\frac{\pi x}{\lambda} \tag{20}$$

where the length of the half-wave λ can take on arbitrary values from zero to infinity. Substitution leads to Eq.(18), where the parameter λ plays the part of the index k; the coefficient of the equations depend on this parameter in the following manner:

$$\omega^2(\lambda) = \frac{1}{m}[\frac{bD_{11}\pi^4}{\lambda^4} + \eta] \tag{21}$$

and

$$p_*(\lambda) = \frac{\pi^2 bD_{11}}{\lambda^2} + \frac{\eta\lambda^2}{\pi^2} \tag{22}$$

Thus, for a given length of the half-wave, the boundaries of the principal regions of dynamic instability can be determined by the harmonic balance method [8]. Therefore, the boundary frequency of the instability region obtained as follow:

$$\theta_*^2(\lambda) = \frac{4}{m}[\frac{\pi^2 bD_{11}}{\lambda^4} - \frac{\pi^2(p_0 \pm \frac{p_t}{2} - bp')}{\lambda^2} + \eta] \tag{23}$$

By neglecting the piezoelectric effect and foundation coefficient, Eq. (15) is reduced to the parametric resonance of homogeneous beams:

$$m\frac{\partial^2 w}{\partial t^2} + b\overline{D}_{11}\frac{\partial^4 w}{\partial x^4} + P(t)\frac{\partial^2 w}{\partial x^2} = 0 \tag{24}$$

where

$$\overline{D}_{11} = \frac{Eh^3}{12} \tag{25}$$

Eq. (25) has been reported by Bolotin [8].

Open Science Index, Mechanical and Mechatronics Engineering Vol:4, No:2, 2010 publications.waset.org/2350/pdf

A. Numerical Results

The unstable and stable regions of homogeneous beams with piezoelectric actuators subjected to periodic axial compressive loads are studied in this paper.

It is assumed that both the top and bottom piezoelectric layers have the same thickness; $h_T = h_B$ and the same voltages are applied to both actuators. The material properties of the beam are listed in Table I.

TABLE I
MATERIAL PROPERTIES

Property	Piezoelectric layer	Homogeneous layer
Young's modulus E (GPa)	63	223.95
Poisson's ratio ν	0.3	0.3
Length L (m)	0.3	0.3
Thickness h (m)	0.00005	0.01
Density ρ (Kgm^{-3})	7600	8900
Piezoelectric constant e_{31}, e_{32} (Cm^{-2})	17.6	-

The effect of the applied voltages on the dynamic stability of homogeneous beam with piezoelectric actuators is shown in Fig. 3. As can be seen the parametric resonance frequency becomes smaller when the applied voltages are positive. The unstable region enlarges and shifts to the left when the applied voltages are positive and decrease and shift to the right when the applied voltages are negative. Fig. 4 illustrates the effect of the static load factor α on the unstable regions. The applied voltage is -50 V in the two cases.

As the static load factor increases, the unstable regions enlarge and the parametric resonance frequencies are lower. The effect of the piezoelectric force P on the dynamic stability of beam with different actuator thicknesses h_a and $2h_a$ is presented in Fig. 5.

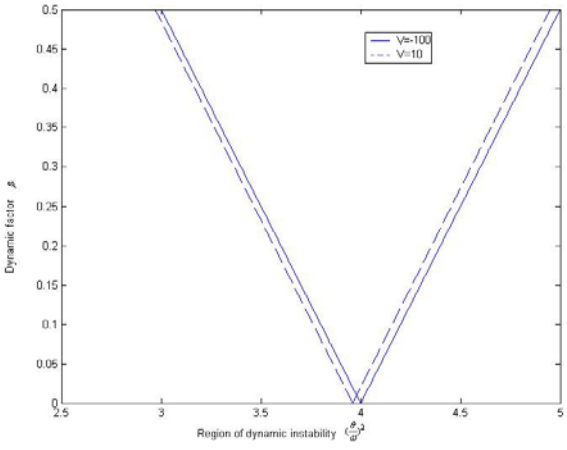


Fig. 3 Effect of applied voltage on the unstable region

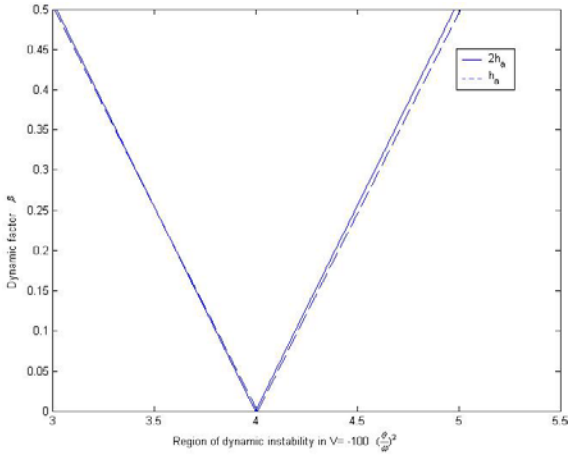


Fig. 4 Effect of piezoelectric actuators thickness on the unstable region

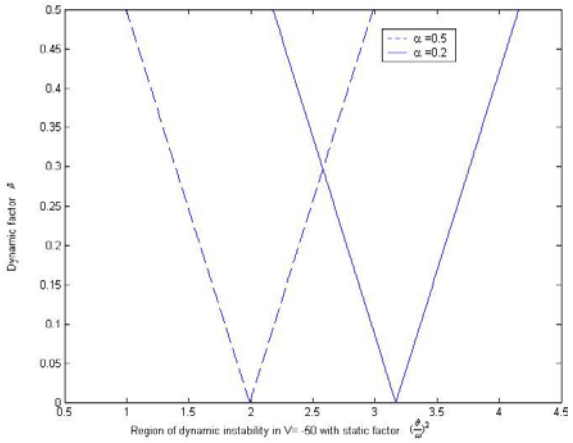


Fig. 5 Effect of static and dynamic factors on the unstable region

IV. CONCLUSION

The dynamic instability analysis of a homogeneous beam with piezoelectric actuators has been presented. It was shown that the piezoelectric actuators induce tensile piezoelectric force produced by applying negative voltages that significantly affect the dynamic instability of the homogeneous beam with piezoelectric actuators. The width of the unstable region decreases when the applied voltage is negative. The homogeneous beam with a thinner actuator thickness is more efficient in reducing the width of the dynamic stability region.

REFERENCES

[1] Bailey T. and Hubbard Jr.J.E., Distributed Piezoelectric-polymer Active Vibration Control of A cantilever Beam. Journal of Guidance Control and Dynamic, Vol. 8, 1985, pp. 605–611.
[2] Crawley E.F. and de Luis J., Use Piezoelectric Actuators As Elements of Intelligent Structures, AIAA Journal, Vol. 8, 1987, pp. 1373–1385.
[3] Shen M.H., Analysis of Beams Containing Piezoelectric Sensors and Actuators, Smart Materials and Structures, Vol. 3, 1994, pp. 439–347.
[4] Pierre C. and Dowell E.H., A Study of Dynamic Instability of Plates By An Extended Incremental Harmonic Balance Method, Journal of Applied Mechanics, Vol. 52, 1985, pp. 693–697.
[5] Liu G.R., Peng X.Q., and Lam K.Y., Vibration Control Simulation of Laminated Composite Plates with Integrated Piezoelectrics, Journal of Sound and Vibration, Vol . 220, No. 5, 1999, 827–846.

[6] Tzou H.S. and Tseng C.I., Distributed Piezoelectric Sensor/Actuator Design For Dynamic Measurement/Control of Distributed Parameter System: A Piezoelectric Finite Element Approach, Journal of Sound and Vibration, Vol. 138, 1990, pp. 17–34.
[7] Ha S.K., Keilers C., and Chang, F.K., Finite Element Analysis of Composite Structures Containing Distributed Piezoceramic Sensor and Actuators, AIAA Journal, Vol. 30, 1992, pp. 772–780.
[8] Bolotin V.V., The dynamic Stability of Elastic Systems, San Francisco: Holden Day, 1964.
[9] Briseghella L., Majorana C.E., Pellegrino C., Dynamic Stability of Elastic Structures: A Finite Element Approach, Computers and. Structures, Vol. 69, 1998, pp. 11–25.
[10] Takahashi K., Wu M., and Nakazawa N., Vibration Buckling and Dynamic Stability of A Cantilever Rectangular Plate Subjected to Inplane Force, Engineering Mechanics, Vol. 6, 1998, pp. 939-953.
[11] Zhu J., Chen C., Shen Y., and Wang S., Dynamic Stability of Functionally Graded Piezoelectric Circular Cylindrical Shells, Materials Letters, Vol. 59, 2005, pp. 477-485.
[12] Reddy J.N. and Praveen G.N., Nonlinear Transient Thermoelastic Analysis of Functionally Graded Ceramic-metal Plates, International Journal of. Solids and Structures, Vol. 35, 1998, pp. 4467-4476.

Open Science Index, Mechanical and Mechatronics Engineering Vol:4, No:2, 2010 publications.waset.org/2350/pdf