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ZMP Based Reference Generation for Biped Walking Robots

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***Abstract*—** Recent fifteen years witnessed fast improvements in the field of humanoid robotics. The human-like robot structure is more suitable to human environment with its supreme obstacle avoidance properties when compared with wheeled service robots. However, the walking control for bipedal robots is a challenging task due to their complex dynamics.

Stable reference generation plays a very important role in control. Linear Inverted Pendulum Model (LIPM) and the Zero Moment Point (ZMP) criterion are applied in a number of studies for stable walking reference generation of biped walking robots. This paper follows this main approach too.

We propose a natural and continuous ZMP reference trajectory for a stable and human-like walk. The ZMP reference trajectories move forward under the sole of the support foot when the robot body is supported by a single leg. Robot center of mass trajectory is obtained from predefined ZMP reference trajectories by a Fourier series approximation method. The Gibbs phenomenon problem common with Fourier approximations of discontinuous functions is avoided by employing continuous ZMP references. Also, these ZMP reference trajectories possess pre-assigned single and double support phases, which are very useful in experimental tuning work.

The ZMP based reference generation strategy is tested via three-dimensional full-dynamics simulations of a 12-degrees-of-freedom biped robot model. Simulation results indicate that the proposed reference trajectory generation technique is successful.

***Keywords*—**Biped robot, Linear Inverted Pendulum Model, Zero Moment Point, Fourier series approximation.

I. Introduction

The human-like robot structure is suitable for our homes and offices due to its supreme obstacle avoidance properties when compared with wheeled service robots. Robots in the human shape can be accepted as a social being by human beings. However, there are many problems which should be solved before realizing the daily life human-robot coexistence. The bipedal free-fall manipulator is inherently difficult to stabilize [1,2]. This makes the walking control a challenging task. In biped robot systems, control and gait planning go hand in hand. A stable walking reference generation is essential.

The ZMP criterion is the most widely accepted and used stability measure for the legged locomotion. The criterion states that, during the walk, the ZMP should lie within the supporting area - often called the support polygon - of the feet in contact with the ground [1].

The ZMP coordinates are functions of the positions and accelerations of the numerous links and the body of the humanoid robot. Though they can be computed – even online, – it is quite difficult to use these expressions of many variables in the design of reference generation and control algorithms. The same is true for the complicated dynamics equations of the biped robot. This is where an approximate model, rather than a detailed one, would be of better use.

The LIPM [3-5] is such an approximate model of the legged robot. It consists of a point mass of constant height and a massless rod connecting the point mass with the ground. By virtue of this model, a quite simple relation between the ZMP and the robot Center of Mass (CoM) coordinates is obtained [6-9]. This relation is exploited for ZMP based stable walking reference generation in a number of studies. In such works, robot CoM trajectory is obtained from predefined stable ZMP reference trajectories. The reference trajectories for the leg joints are obtained then via inverse kinematics from the robot CoM coordinates.

There is a freedom in choosing the ZMP reference trajectory as long as the criterion mentioned above is satisfied. A natural choice is to keep it fixed at the center of the foot sole when only one foot is supporting the body (single support phase) and interpolating between the foot centers when two feet support it (double support phase). However, studies have shown that a natural, human-like walk can be obtained by ZMP trajectories which move forward when the robot body is supported by a single leg [10-12].

In [13], Erbatur and Kurt introduce a forward moving discontinuous ZMP reference trajectory for a stable and human-like walk and employ Fourier series approximation to obtain CoM reference trajectory from this ZMP trajectory. The ZMP reference trajectory in the double support phases in [13] is obtained indirectly with a smoothing process, which also provides smoothing of the Gibbs phenomenon peaks due to Fourier approximation. Although the walk period is defined by the user, the partition of the period into the single and double support phases is due to the smoothing process, and not predefined.

This paper follows the same mechanism as in [13] in using Fourier series approximation for the computation of the CoM trajectory from a given ZMP reference curve. However, it

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defines a continuous ZMP reference and the durations of the single and double support phases are fully pre-assigned. This is quite useful since these parameters play an important role in the final parameter tuning in experimental work. The naturalness of the walk is preserved, in that the single support ZMP reference is forward moving. Further, the continuity of the introduced ZMP reference makes the after-Fourier-approximation smoothing process unnecessary. The proposed reference generation method is tested via simulations on the full dynamics three-dimensional model of a 12-dof biped robot.

The rest of the paper is organized as follows. The ZMP based reference generation for bipedal walk and created CoM reference trajectories are presented in the next section. The control structure used in the simulations is briefed in Section III. Section IV is devoted to simulation results with the generated reference trajectories. A conclusion is presented lastly.

II. ZMP Based Reference Generation

The sketch in Fig. 1 shows the typical biped robot with 6-DOF legs for which the reference generation and control algorithms presented below can be applied.

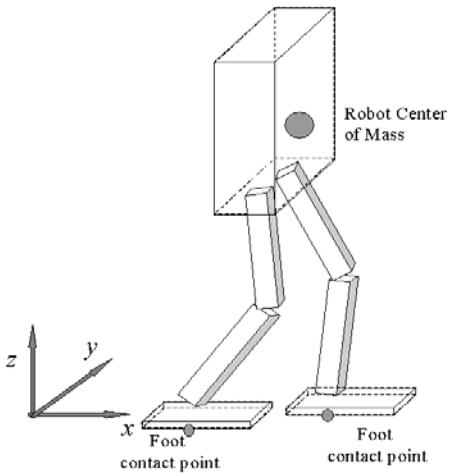


Fig. 1 Typical biped robot kinematic arrangement. In single support phases, it behaves as an inverted pendulum.

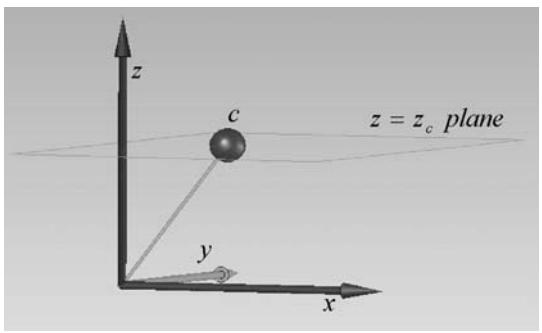


Fig. 2 The linear inverted pendulum model

Instead of using this complex full dynamics models, the simple linear inverted pendulum model is more suitable for controller synthesis. A point mass is assigned to the CoM of the robot and it represents the body (trunk) of the robot. The point mass is linked to a stable (not sliding) contact point on the ground via a massless rod, which is idealized model of a supporting leg. In the same manner, the swing leg is assumed to be massless too. With the assumption of a fixed height for the robot CoM a linear system which is uncoupled in the x and y directions is obtained. The system described above is shown in Fig. 2 $c = (c_x \quad c_y \quad c_z)^T$ is the coordinates of the point mass in this figure.

Stability of the walk is the most desired feature of a reference trajectory. The ZMP criterion is the most widely accepted and used stability criterion in biped robotics. The ZMP is defined as the point on the $x-y$ plane which no horizontal torque components exist on it for the model in Fig. 2. For the point mass structure shown in this figure, the expressions for the ZMP coordinates p_x and p_y are [14]:

$$p_x = c_x - \frac{z_c}{g}\ddot{c}_x \tag{1}$$

$$p_y = c_y - \frac{z_c}{g}\ddot{c}_y \tag{2}$$

z_c is the height of the plane where the motion of the point mass is constrained and g is the gravity constant.

The ZMP and the CoM can be related to each other with the equations (1) and (2). A suitable ZMP trajectory can be generated without difficulty for reference generation purposes. As the only stability constraint, the ZMP should always lie in the supporting polygon defined by the foot or feet touching the ground. The ZMP location is generally chosen as the middle point of the supporting foot sole. In [14], the reference ZMP trajectory shown in Fig. 3 is created with this idea. A is the distance between the foot centers in the y direction, B is the step size and T is the half of the walking period in this figure. It can be observed that from the same figure, firstly, step locations are determined. The step location selections can be based on the size of the robot and the task performed by the robot. This selection of support foot locations and the half period T defines the staircase-like p_x and the square-wave structured p_y curves.

However, in [14], the naturalness of the walk is not considered. As mentioned above, in that work ZMP stays at a fixed point under the foot sole, although investigations in [10-12] show that the human ZMP moves forward under the foot sole. Fig. 3 also shows that the transition from left single support phase to the right single support phase is instantaneous, there exists no double support phase.

In order to address the naturalness issue, the p_x reference curve shown in Fig. 4 is employed in [13]. In this figure, forward moving ZMP can be seen at the top. b defines the range of the ZMP motion under the sole in Fig. 4 and a symmetric trajectory centered at the foot frame center is assumed.

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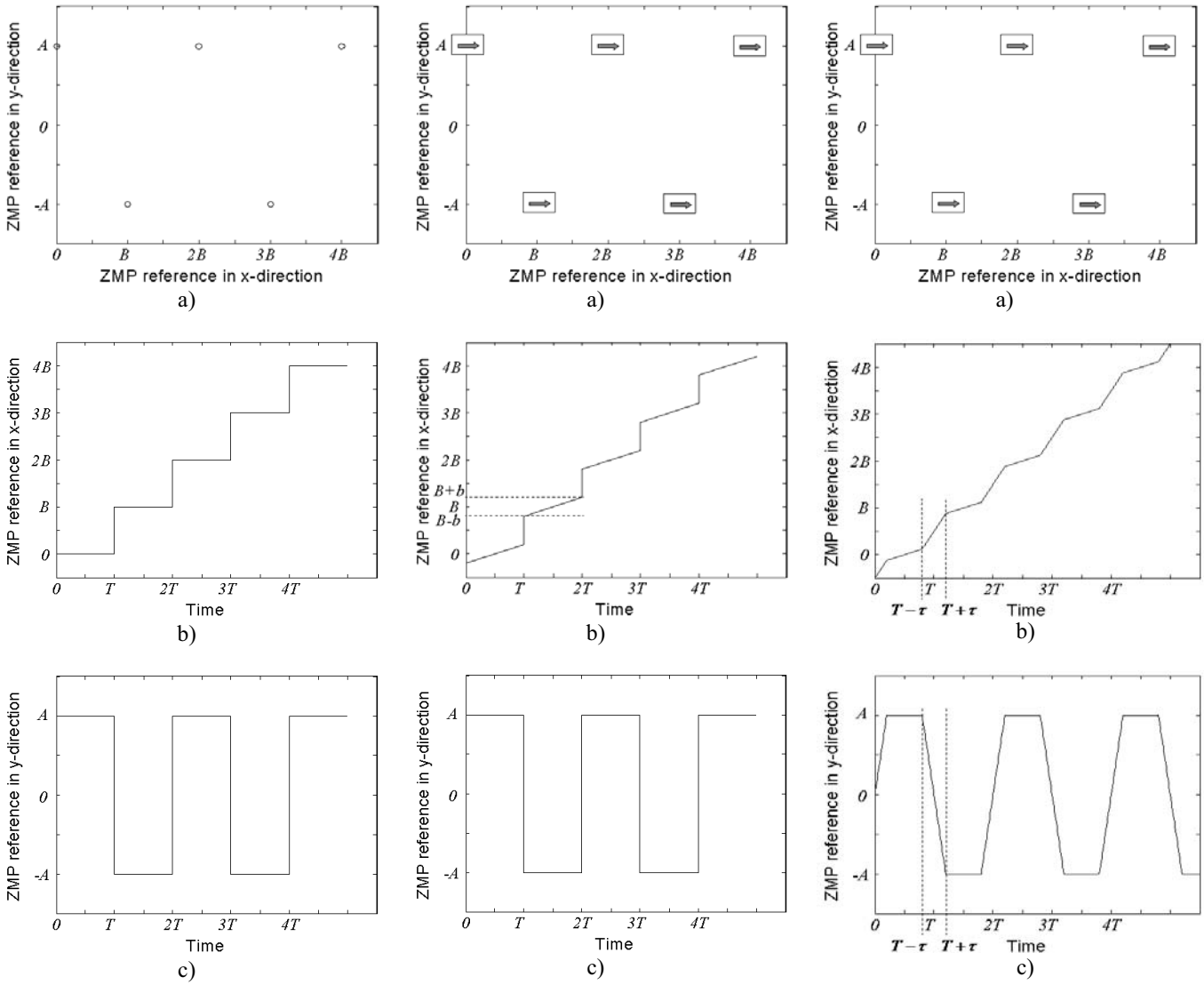


Fig. 3 Fixed ZMP references.
a) $p_x^{ref} - p_y^{ref}$ Relation on the $x-y$ plane
b) p_x^{ref}, the x-axis ZMP reference
c) p_y^{ref}, the y-axis ZMP reference

Fig. 4 Forward moving ZMP reference
a) $p_x^{ref} - p_y^{ref}$ Relation on the $x-y$ plane
b) p_x^{ref}, a natural x-axis ZMP reference. Note the difference of the x-reference with the one shown in Fig. 3.
c) p_y^{ref}, the y-axis ZMP reference.

Fig. 5 Forward moving ZMP references with pre-assigned double support phases.
a) $p_x^{ref} - p_y^{ref}$ Relation on the $x-y$ plane
b) p_x^{ref}, the x-axis ZMP reference
c) p_y^{ref}, the y-axis ZMP reference

Having defined the curves, and hence the mathematical functions for $p_x^{ref}(t)$ and $p_y^{ref}(t)$, and the next step is obtaining CoM reference trajectories from $p_x^{ref}(t)$ and $p_y^{ref}(t)$. Position control schemes for the robot joints with joint references obtained by inverse kinematics from the CoM locations can be obtained once the CoM trajectory is computed. The computation of CoM trajectory from the given ZMP trajectory can be carried out in a number of ways [6, 14].

[14], for the reference ZMP trajectories in Fig. 3, propose an approximate solution with the use of Fourier series representation to obtain CoM references for reference generation.

Taking an approach similar to the one in [14], [13] develops an approximate solution for the c_x and c_y references corresponding to the moving ZMP references in Fig.4. In this process Fourier series approximations of the ZMP references $p_x^{ref}(t)$ and $p_y^{ref}(t)$ and of the CoM reference are obtained. Note that, although the ZMP reference in the x-direction in Fig. 4 is forward moving and hence natural as desired, it is not continuous. So is the ZMP reference of Fig. 4 in the y-direction. The y-direction reference is in the form of

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as square wave as in Fig. 3. This discontinuous function corresponds to an instantaneous switching of the support foot, from right to left and from left to right foot, without an intermediate double support phase. Apart from the difficulties in the realization of such an instantaneous switching there is one more reason why such reference is undesirable: Natural, human-like motion will be lost with such a ZMP reference trajectory. Humans walk with a double support phase of nonzero duration. In [13], smooth transition between single support phases with a double support phase is achieved by an additional smoothing action based on Lanczos sigma factors smoothing. Lanczos sigma factor smoothing is a technique in which the so called Gibbs Phenomenon – peaks of Fourier series approximations at the discontinuities of the original function is cured by introducing weighting coefficients multiplying Fourier series coefficients. There weighting coefficients are called Lanczos sigma factors. This method of smoothing achieves two objectives at once: i) Suppressing the Gibbs phenomenon, ii) Introducing a double support phase. Different levels of smoothing can be achieved by modifying the Lanczos sigma factors. Also at the same time different double support periods can be obtained. This mechanism, however, introduces some shortcomings too: Gibbs suppression and double support period determination are coupled processes. Walk pattern design may impose a short double support phase (as in the case of humans), whereas this, due to the coupling between this period and Gibbs suppression level, would lead to ZMP reference curves with pronounced oscillations at the foot switching times. Furthermore, our simulation studies and experimental work with a number of reference generation techniques [15-17] suggest that having the single and double support periods a freely and directly adjustable parameters plays a vital role in final tuning of walking pattern. With this motivation, in this paper, a new ZMP reference trajectory is introduced. This trajectory, as in [33], has forward moving x-direction components for the naturalness of the walk. However, it is continuous and includes double support phases in its original description. The solution for the CoM trajectory from the given ZMP trajectory follows the same lines as in [13]. However, thanks to continuity of the ZMP reference signal, the Gibbs phenomenon is not observed and there is no need for smoothing. Also, the double and single support phase durations are freely selectable parameters of the reference generation algorithm. The newly introduced ZMP reference trajectory is presented in Fig. 5. It is a modified version of the trajectory in Fig. 4. The double support phase is introduced by using the parameter τ in this figure. A linear interpolation interval is inserted around multiples of the half walking period T. The durations of the intervals are equal to 2τ and they correspond to double support periods. Hence the double support period is freely adjustable with the parameter τ.

The mathematical description of the $p_x^{ref}(t)$ in Fig. 5 is given by

$$p_x^{ref} = \frac{B}{T}(t-\frac{T}{2}) + p_x'^{ref} \tag{3}$$

where $p_x'^{ref}$ is periodic with period T. $p_x'^{ref}$ can be expressed as a combination of three line segments on $[0, T]$.

$$p_x'^{ref} = \begin{cases} \Omega_1 + \sigma_1 t & \text{if} \quad 0 \le t \le \tau \\ \Omega_2 + \sigma_2 t & \text{if} \quad \tau < t \le T-\tau \\ \Omega_3 + \sigma_3 t & \text{if} \quad T-\tau < t \le T \end{cases} \tag{4}$$

Here,

$$\Omega_1 = 0, \;\; \sigma_1 = \frac{\delta}{\tau},$$
$$\Omega_2 = \delta - \tau\sigma_2, \;\; \sigma_2 = \frac{-2\delta}{T-2\tau}, \tag{5}$$
$$\Omega_3 = -\delta - (T-\tau)\sigma_3, \;\; \sigma_3 = \sigma_1.$$

with

$$\delta = \frac{T-2\tau}{T}(\frac{B}{2} - b). \tag{6}$$

Note that δ is the magnitude of peak difference between p_x^{ref} and the non-periodic component $\frac{B}{T}(t-\frac{T}{2})$ of p_x^{ref}. δ can be computed form Fig. 6 geometrically.

$p_y^{ref}(t)$ is Fig. 5 is expressed as

$$p_y^{ref} = \sum_{k=1}^{\infty} \mathrm{A}(-1)^k \Big[\frac{2}{2\tau}(t-kT)\big[u(t-(kT-\tau)) - u(t-(kT+\tau))\big] + \big[u(t-(kT+\tau)) - u(t-(kT+T-\tau))\big] \Big] \tag{7}$$

where $u(\cdot)$ represents the unit step function.

Defining $\omega_n \equiv \sqrt{g/z_c}$, we can rewrite (1) and (2) for the reference variables as follows.

$$\ddot{c}_x^{ref} = \omega_n^2 c_x^{ref} - \omega_n^2 p_x^{ref} \tag{8}$$
$$\ddot{c}_y^{ref} = \omega_n^2 c_y^{ref} - \omega_n^2 p_y^{ref} \tag{9}$$

Note that the y-direction ZMP reference $p_y^{ref}(t)$ is a periodic function with the period $2T$. It is reasonable to assume that $c_y^{ref}(t)$ is a periodic function too and that it has the same period. Hence, it can be approximated by a Fourier series

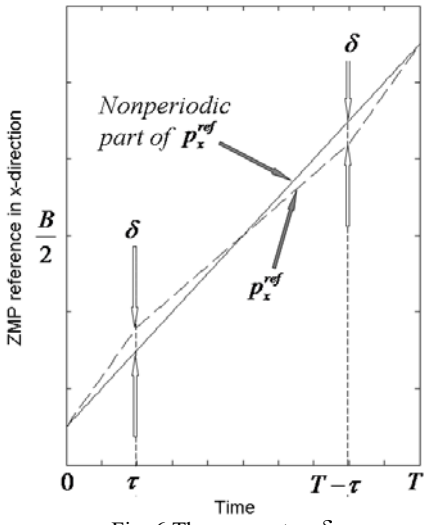


Fig. 6 The parameter δ

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$$c_y^{ref}(t) = \frac{a_0}{2} + \sum_{k=1}^{\infty} a_k \cos(\frac{2\pi kt}{2T}) + b_k \sin(\frac{2\pi kt}{2T}) \quad (10)$$

By (9) and (10), p_y^{ref} can be expressed as

$$\begin{aligned} p_y^{ref}(t) &= c_y^{ref} - \frac{1}{\omega_n^2}\ddot{c}_y^{ref} \\ &= \frac{a_0}{2} + \sum_{k=1}^{\infty} a_k (1 + \frac{\pi^2 k^2}{\omega_n^2 T^2}) \cos(\frac{2\pi kt}{2T}) \\ &\quad + b_k (1 + \frac{\pi^2 k^2}{\omega_n^2 T^2}) \sin(\frac{2\pi kt}{2T}) \end{aligned} \quad (11)$$

Noting that this expression in the form of a Fourier series for $p_y^{ref}(t)$, and since $p_y^{ref}(t)$ is an odd function, we can conclude that the coefficients $a_0/2$ and $a_k(1+(\pi^2k^2)/(\omega_n^2T^2))$ for $k = 1,2,3,\cdots$ are zero. In order to compute the coefficients $b_k(1+(\pi^2k^2)/(\omega_n^2T^2))$ we can employ the Fourier integral:

$$b_k(1 + \frac{\pi^2 k^2}{\omega_n^2 T^2}) = \frac{2}{2T}\int_0^{2T} p_y^{ref} \sin(\frac{2\pi kt}{2T}) dt \quad (12)$$

As a result, after some arithmetical steps (omitted here due to space considerations), the Fourier coefficients b_k of $c_y^{ref}(t)$ in (10) can be obtained as

$$b_k = \begin{cases} \frac{\omega_n^2 T^2}{\omega_n^2 T^2 + \pi^2 k^2} \frac{2A}{\pi k} \left\{ \left[\frac{2}{\tau} \left\langle \frac{T}{\pi k} \sin(\frac{\pi k \tau}{T}) - \tau \cos(\frac{\pi k \tau}{T}) \right\rangle \right] \right. & \text{if } k \text{ is odd} \\ \left. + \left[\cos(\frac{\pi k \tau}{T}) - \cos(\frac{\pi k (T-\tau)}{T}) \right] \right\} & \\ 0 & \text{if } k \text{ even} \end{cases}$$

for $k = 1,2,3,\cdots$. (13)

The second step is finding the Fourier series coefficients for c_x^{ref} in Fig. 5 is not a periodic function. It cannot be expressed as a Fourier series. However, as expressed above, this function is composed of the periodic function $p_x'^{ref}$ and the non-periodic function $(\frac{B}{T}(t - \frac{T}{2}))$. The periodic part of $p_x^{ref}(t)$ are shown in Fig. 7. It is again a reasonable assumption that c_x^{ref} has a periodic part and a non-periodic part too. Further, if we suppose that the two non-periodic parts (of $p_x^{ref}(t)$ and c_x^{ref}) are non-equal, then the difference $p_x^{ref}(t) - c_x^{ref}$ will be non-periodic. This is not expected in a continuous walk as the one described in Fig. 5.

Therefore we conclude that the non-periodic parts of the two functions are equal. Note that, as shown in Fig. 5, the period of the periodic part of $p_x^{ref}(t)$ is T and we can make the same statement for the period of the periodic part of c_x^{ref}. Finally, c_x^{ref} can be expressed as

$$c_x^{ref} = \frac{B}{T}(t - \frac{T}{2}) + \frac{\alpha_0}{2} + \sum_{n=1}^{\infty} \alpha_k \cos(\frac{2\pi nt}{T}) + \beta_k \sin(\frac{2\pi nt}{T}) \quad (14)$$

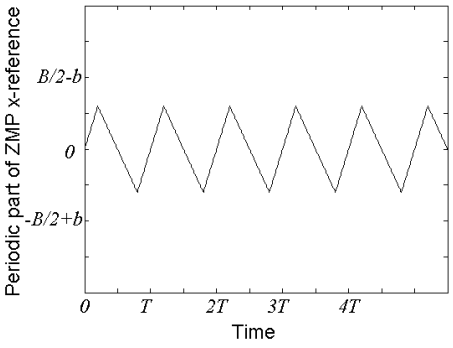


Fig. 7 $p_x'^{ref}(t)$, the periodic part of the x-direction ZMP reference $p_x^{ref}(t)$

Recalling (8), with (14) the expression for $p_x^{ref}(t)$ with a Fourier series is

$$\begin{aligned} p_x^{ref}(t) &= c_x^{ref} - \frac{1}{\omega_n^2}\ddot{c}_x^{ref} \\ &= \frac{B}{T}(t - \frac{T}{2}) + \frac{\alpha_0}{2} + \sum_{n=1}^{\infty} \alpha_k (1 + \frac{\pi^2 k^2}{\omega_n^2 T^2}) \cos(\frac{2\pi kt}{T}) \\ &\quad + \beta_k (1 + \frac{\pi^2 k^2}{\omega_n^2 T^2}) \sin(\frac{2\pi kt}{T}) \end{aligned} \quad (15)$$

Therefore the Fourier coefficients of $p_x'^{ref}(t)$, the periodic part of $p_x^{ref}(t)$, are $\alpha_0/2$, $\alpha_k(1+\pi^2k^2/\omega_n^2T^2)$ and $\beta_k(1+\pi^2k^2/\omega_n^2T^2)$ for $k = 1, 2, 3, \cdots$. The Fourier coefficients $\alpha_0/2$, $\alpha_k(1+\pi^2k^2/\omega_n^2T^2$ of $p_x'^{ref}(t)$ shown in Fig. 7 are zero because this is an odd function. The coefficients for $\beta_k(1+(\pi^2k^2)/(\omega_n^2T^2))$ can be found by

$$\beta_k(1 + \pi^2 k^2/\omega_n^2 T^2) = \frac{2}{T}\int_0^T p_x'^{ref}(t) \sin(\frac{2\pi kt}{T}) dt \quad (16)$$

This yields the result

$$\begin{aligned} \beta_k = \frac{\omega_n^2 T^2}{\pi^2 k^2 + \omega_n^2 T^2} \frac{2}{\pi k} \Bigg\{ & \sigma_1 \left[-\tau \cos(\frac{2\pi k \tau}{T}) + \frac{T}{2\pi k} \sin(\frac{2\pi k \tau}{T}) \right] \\ & + \sigma_2 \left[\tau \cos(\frac{2\pi k \tau}{T}) - \frac{T}{2} \left(\cos\left(\frac{2\pi k \tau}{T} \right) \right) \right. \\ & \left. - \frac{T}{2\pi k} \sin(\frac{2\pi k \tau}{T}) \right] \Bigg\} \end{aligned}$$

for $k = 1, 2, 3, \cdots$. (17)

The curves obtained for c_x^{ref} and c_y^{ref} are shown in Fig. 8 together with the corresponding original ZMP references (as defined in Fig. 5). The infinite sums in (10) and (14) are approximated by finite sums of N terms ($N = 24$).In Fig 8, the following parameter values are used: $A = 0.1$ m, $B = 0.1$ m, $b = 0.04$, $T = 1$ s and $\tau = 0.2$ s.

In addition to the CoM references, foot position reference trajectories have to be designed too: Inverse kinematics then can be employed to find the reference positions of the leg joints which bridge the CoM and the feet. The x and z-

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direction components of the foot trajectories used in this paper are shown in Fig. 9. These curves are smooth combinations of sinusoidal and constant segments. h_s is the step height parameter. T_d and T_s represent the double and single support times, respectively. B is the step size from Fig.5. The y direction trajectories are constant at $-A$ and A for the right and left feet, respectively, where A is half of the foot to foot y direction distance also shown in Fig. 5. The foot orientation references used in inverse kinematics are fixed and they are computed for feet parallel to the robot body.

III. Control Algorithm

The control algorithm is a simple one based on independent joint PID position controllers. The joint position references are generated through inverse kinematics from CoM and swing foot references defined in world frame coordinates. The PID controller gains are obtained via trial and error.

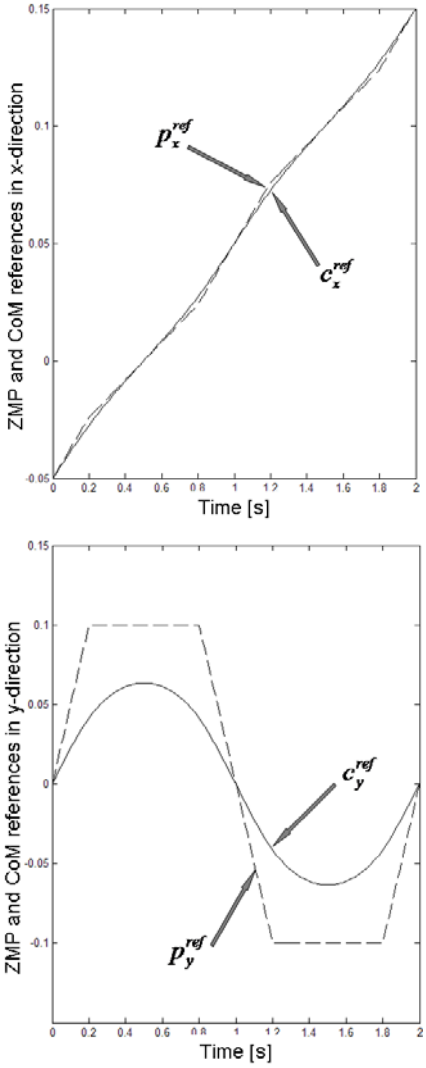


Fig. 8 x and y -direction CoM references together with the corresponding original ZMP references

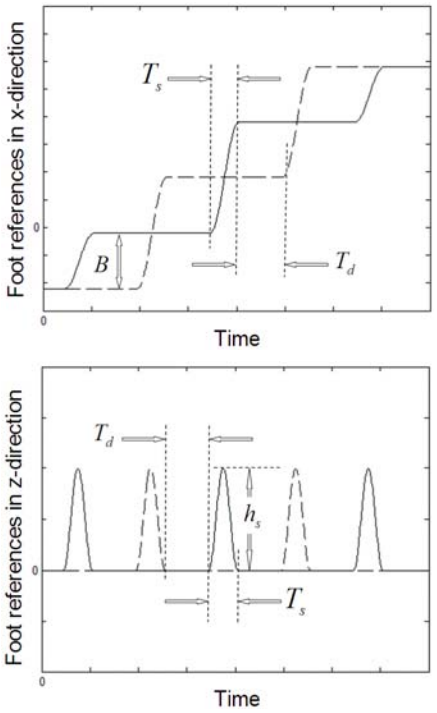


Fig.9 x and z -direction foot references in as expressed in the world frame. Solid curves belong to the right foot, dashed curves indicate left foot trajectories.

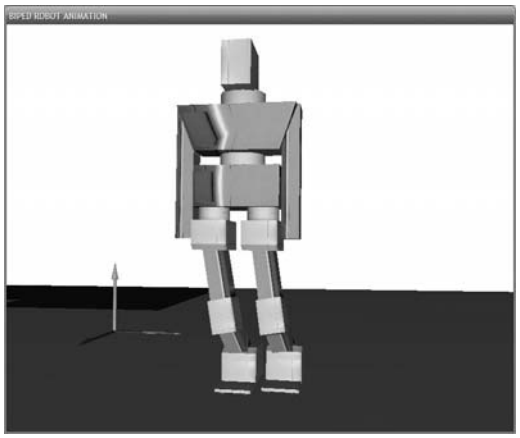


Fig. 10 A snapshot from the animation window.

The controller structured this way, except for the servo control loops, is an open-loop one. However, it achieves walking when stable reference trajectories (like the ones obtained in the previous section) are employed.

IV. Simulation Results

The biped model used in this paper consists of two 6-DOF legs and a trunk connecting them (Fig. 1). Three joint axes are positioned at the hip. Two joints are at the ankle and one at the knee. Link sizes and the masses of the biped are given in Table I.

Simulation studies are carried out with this robot model, with references generated in Section II and control mechanism discussed in Section III. A view of the animation window is

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shown in Fig. 10. The full-dyamics 3D simulation scheme is similar to the one in [18]. The ground contact is modeled by an adaptive penalty based method. The details of the simulation algorithm and contact modeling can be found in [19]. Parameters used for reference generation are presented in Table II.

TABLE I

MASSES AND DIMENSIONS OF THE ROBOT LINKS

A. Link	**Dimensions (LxWxH) [m]**					**Mass [kg]**
Trunk	0.2	x	0.4	x	0.5	50
Thigh	0.27	x	0.1	x	0.1	12
Calf	0.22	x	0.05	x	0.1	0.5
Foot	0.25	x	0.12	x	0.1	5.5

TABLE II

REFERENCE GENERATION PARAMETERS

Parameter	***Value***
Step height h_s	0.01 m
Step period $2T$	3 s
Single support period T_s	0.6 s
Double support period T_d	0.9
Foot to foot y-direction distance $2A$	0.24 m
Step size B	0.1 m
ZMP motion under the support foot $2b$	0.08 m

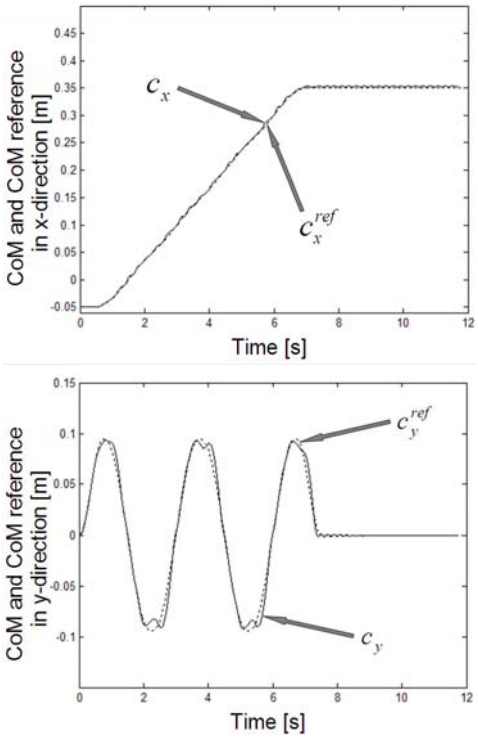


Fig. 11 CoM (solid) and reference CoM (dashed) trajectories in the moving single support ZMP case.

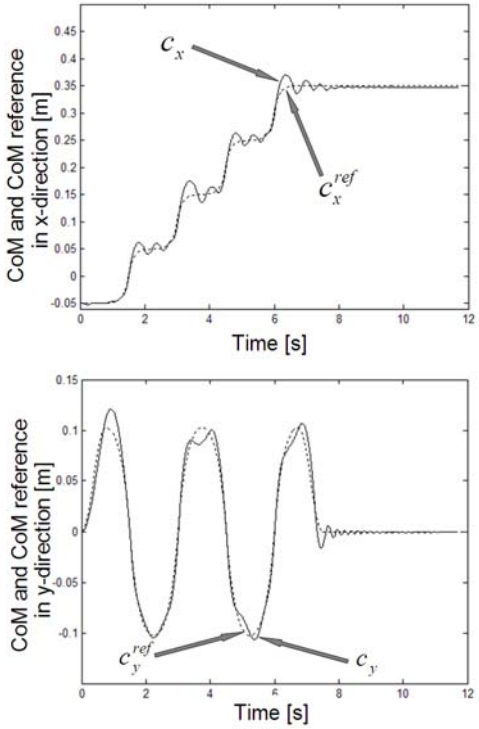


Fig. 12 CoM (solid) and reference CoM (dashed) trajectories in the fixed single support ZMP case.

For comparison purposes, simulations with fixed support foot ZMP references are carried out too.

Fig. 11 shows the experimental result obtained with forward moving ZMP references generated in this paper. As can be seen the x-direction reference CoM reference is tracked accurately. y-direction CoM reference tracking errors at the peaks are more pronounced. This is mainly due to the difference of the LIPM and the robot parameter in Table I. The legs are 15 kg, far from being "massless". Also no closed loop trajectory compensation technique is applied in this paper since our objective is to evaluate the reference generation method. Fig. 11 shows a successful work of 5 steps.

The CoM tracking with fixed x-direction single support ZMP references are shown in Fig. 12. This figure is obtained with the same reference generation and control routines. The differences from the previous simulation are the following. The parameter b of Table II is modified to zero. The double support period is taken as 0.0001 seconds (We did not make it equal to zero since this would cause a division by zero in equation 5.). The walk period $2T$ is kept however equal by increasing the single support duration to 1.4999 seconds. Note that these choices define a walk with the kind of ZMP references shown in Fig. 4. Actually, we can state that the reference trajectories in figures 4 and 5 are special cases of the newly proposed trajectory in Fig. 6. A trajectory like in Fig. 5 can be obtained by changing the double support period to a small value, keeping $2T$ constant in Table II with no modification of the parameter b.

The CoM trajectory performance in Fig. 12 in the y-direction is similar to that of Fig. 11. Only a slightly better tracking performance is observed in Fig. 11 in this direction.

More significant differences, however, are observed in the x - direction plots in the two figures. The CoM behavior with the fixed single support ZMP references is oscillatory. This is because of the repeated acceleration and deceleration patterns in the CoM reference trajectory in Fig. 12. This is a difficult to follow reference when compared with the CoM reference in Fig. 11. this also justifies that a more natural walk is obtained by moving ZMP references. The observations are in parallel with [10-13].

V. Conclusion

In this paper, a forward moving continuous Zero Moment Point based reference trajectory generation is presented for a stable and human-like walk of the bipedal humanoid robots. The relation between the Zero Moment Point and the robot Center of Mass coordinates is obtained via the Linear Inverted Pendulum Model. In order to obtain the Center of Mass reference trajectory, ZMP reference trajectories are approximated with Fourier series approximation. Continuous nature of the ZMP reference trajectories provides non-oscillatory references, so that the smoothing with Lanczos sigma factors are not necessary unlike it is the case in [33]. Another contribution in this paper is that the single support and double support phase durations are introduced as predefined separate parameters to be able to adjust them in the final parameter tuning of the generated references. Simulation results show that the generated stable human-like ZMP reference trajectories successfully enables a stable bipedal walk without a fall with a step size of 10 cm.

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