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Modeling and Identification of Hammerstein System by using Triangular Basis Functions

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***Abstract*—**This paper deals with modeling and parameter identification of nonlinear systems described by Hammerstein model having Piecewise nonlinear characteristics such as Dead-zone nonlinearity characteristic. The simultaneous use of both an easy decomposition technique and the triangular basis functions leads to a particular form of Hammerstein model. The approximation by using Triangular basis functions for the description of the static nonlinear block conducts to a linear regressor model, so that least squares techniques can be used for the parameter estimation. Singular Values Decomposition (SVD) technique has been applied to separate the coupled parameters. The proposed approach has been efficiently tested on academic examples of simulation..

***Keywords*—**Identification, Hammerstein model, Piecewise nonlinear characteristic, Dead-zone nonlinearity, Triangular basis functions, Singular Values Decomposition

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I. Introduction

MODELING and identification of nonlinear dynamic systems constitute an essential stage in practical control design. Indeed, several researchers published interesting works about this theme and many classes of nonlinear systems have been studied in the literature. One of the studied classes is the block-oriented nonlinear systems having piecewise static nonlinearities [1], [2], [3], [4], [5], [6]. Among these models, we can cite particularly Hammerstein model, Wiener model and Hammerstein-Wiener model. Hammerstein models with piecewise nonlinearities are frequently used in nonlinear systems control. We find such nonlinearities in some actuator families [7]. Many methods of identifications operating on-line have been proposed for this kind of nonlinear models [8], [5].

The recursive techniques of identification are extensively developed. They are well adapted particularly to a great number of applications, in real time. In addition, they can be easily combined with the control strategies operating on-line such as the adaptive control algorithms [4]. They can also be applied to systems with time-varying parameters or Hammerstein nonlinear systems with different forms of discontinuities [1], [3], [4], [10]. In these approaches, a parameter redundancy was considered in the chosen form of the appropriate Hammerstein model description, leading to a significant increase of the number of estimated parameters. Other forms of Hammerstein models, which can be provided by a decomposition based on the "Key term" principle, are proposed in the literature. In this case, the static nonlinearity is represented by the piecewise nonlinearities [11], [12], [13], [14]. The considered decomposition leads to the form of a model where the parameters of the linear and nonlinear blocks are separated. It results that the obtained model is linear in parameters and the RLS algorithm can be well applied [12]. Other shapes of Hammerstein models using various static nonlinearities (dead zone, saturation nonlinearity, preload nonlinearity,…) and the key term principle have been proposed. In this situation, the output of the model is considered as an output of Multi-Inputs Single Output (MISO) system [11]. This Hammerstein model contains an internal variable which is not available to the measure, what does not allow the parameters estimation directly by the ordinary method of least squares. For this reason, it is necessary to estimate the internal variable to build the observations vector. This estimation requires of manipulating a great number of iterations to ensure the convergence of the least squares algorithm [1], [11], [10], [6].

In this paper, we propose an approach which consists in building Hammerstein model having Piecewise nonlinear characteristics such as Dead-zone nonlinearity characteristic. To describe the nonlinear system, we use the simultaneous use of both an easy decomposition technique and the triangular basis functions leads to a particular form of Hammerstein model. This model is then expressed in a parametric form and regression analysis is used for improving the relationship between input and output signals. To estimate the parameters, the RLS algorithm will be applied and SVD decomposition will be considered to separate the coupled parameters. The proposed approach allows to well describe the nonlinear dynamic system and to avoid obtaining a model with a great number of parameters.

The paper is organized as follows: in the second section we give a description of the Hammerstein model with static nonlinearity known as dead zone nonlinearity. The transfer function of the linear dynamic block is given. The third and the fourth section are devoted to the parameters estimation algorithm using **t**riangular basis functions. In the fifth section, we study in simulation the proposed estimation approach on academic examples. The simulation results are firstly carried out; secondly they are commented and discussed. A conclusion on the main works developed in this study ends the paper.

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II. Hammerstein model with Dead-zone nonlinearity

A. Description of block oriented nonlinear system

Hammerstein model is one of the most easy and known of the family of blocks oriented nonlinear systems [15]. The Hammerstein model is given by the cascade connection of a static nonlinearity block $N(.)$ followed by a linear dynamic system defined by a transfer function $T(q)$ shown in Fig 1. In this paper, a special form of mathematical Hammerstein model containing internal variable which is the output of the static nonlinear block is developed by introducing a general decomposition of the nonlinear function. The static nonlinearity is considered as dead zone nonlinearity characteristic which is approximated by Triangular basis functions.

We consider plants having the Hammerstein structure with ARX linear model shown in Fig 1.

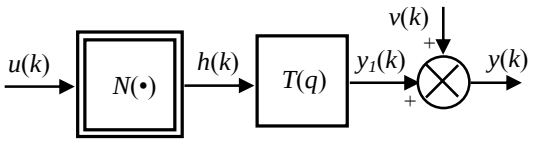


Fig. 1 Hammerstein model

The signals $u(k)$, $y(k)$ and $y_1(k)$ are respectively the model input, noisy model output and model output without noise, and the function $h(k)$, describing the nonlinear effects. The signal $h(k)$ is a nonavailable internal sequence related to the input only, is defined by:

$$h(k) = N(u(k)) \tag{1}$$

The transfer function $T(k)$ of the linear block is described by

$$T(q) = \frac{B(q^{-1})}{A(q^{-1})} \tag{2}$$

where q^{-1} is the backward shift operator, and

$$A(q^{-1}) = 1 + a_1 q^{-1} + \ldots + a_{n_A} q^{-n_A} \tag{3}$$

$$B(q^{-1}) = b_1 q^{-1} + b_2 q^{-2} + \ldots + b_{n_B} q^{-n_B} \tag{4}$$

The noise of measure in the output $v(k)$ defined by

$$v(k) = \frac{1}{A(q^{-1})} e(k) \tag{5}$$

where $e(k)$ is a bounded measurement disturbance which is supposed to be a zero-mean, white noise sequence.

B. Parameterization of static nonlinearity block

The function $N(.)$, describing the nonlinear effects, is supposed to be memoryless within some given finite interval $[u_{min}, u_{max}]$. The signal $h(k)$ is a nonavailable internal sequence related to the input only. Then we introduce a general decomposition of the nonlinear function such as

$$N(u) \cong \sum_{j=1}^{p} \mu_j \eta_j(u) = \mu^T \eta(u) \tag{6}$$

where $u \in [u_{min}, u_{max}]$, $\mu = [\mu_1 \ldots \mu_p]^T$ is a parameter vector, $\eta(u) = [\eta_1(u) \ldots \eta_p(u)]^T$ and $\eta_j(.)$, $j = 1, \ldots p$, represents the j[th] basis function $\eta_j(.)$. The basis functions is defined as follows

$$\eta_j(u) = \xi(\alpha_j(u - \gamma_j)) \tag{7}$$

where α_j and γ_j denote the dilation and translation parameters, respectively, and $\xi(.)$ represents a generator function which belongs to a large family of functions containing Gaussian functions, Generalized orthonormal basis function, triangular functions, trigonometric functions etc [18].

III. Hammerstein model Identification

The output of the nonlinear block $y(k)$ is written in the following way:

$$y(k) = (1 - A(q^{-1}))y(k) + B(q^{-1})N(u(k)) + e(k) \tag{6}$$

Substituting (3) and (4) into (6), we obtain:

$$\begin{aligned} y(k) = &-a_1 y(k-1) - \ldots - a_{n_A} y(k - n_{n_A}) \\ &+ b_1 N(u(k-1)) + \ldots + b_{n_B} N(u(k - n_{n_B})) + e(k) \end{aligned} \tag{7}$$

In a matrix form, we can write the system output equation as follows

$$y(k) = \theta^T \psi(k) + e(k) \tag{8}$$

where θ is the parameter vector given by $\theta = [a \quad \theta_b]^T$ where $a = [a_1 \ldots a_{n_A}]^T$, $b = [b_1 \ldots b_{n_B}]^T$, $\mu = [\mu_1 \ldots \mu_p]^T$ and $\theta_b = vec(\mu b^T)$ with $vec(.)$ is an operator which stacks the columns of a matrix into a vector. The observation vector $\psi(k)$ is described by:

$$\psi(k) = \begin{bmatrix} -\psi_y(k) \\ \psi_\eta(k) \end{bmatrix} \tag{9}$$

where $\psi_y(k) = \begin{bmatrix} y(k-1) \\ . \\ . \\ . \\ y(k-n_A) \end{bmatrix}$ and $\psi_\eta(k) = \begin{bmatrix} \eta(u(k-1)) \\ . \\ . \\ . \\ \eta(u(k-n_B)) \end{bmatrix}$

The estimated value of the parameter vector θ can be obtained by minimizing the following criterion.

$$\hat{\theta} = \arg\min \left\| Y - \psi\theta \right\|_2 \tag{10}$$

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where $Y = \begin{bmatrix} y(n_A) \\ . \\ . \\ . \\ y(r) \end{bmatrix}$ and $\psi = \begin{bmatrix} \psi^T(n_A) \\ . \\ . \\ . \\ \psi^T(r) \end{bmatrix}$ in considering input-output measurements pairs $[u(k); y(k)]$, $k = 1,\ldots,r$ where $r > n_A$.

To estimate the parameter vector θ, the Least Squares Estimation method and the Singular Value Decomposition (SVD) [3], [16], [17], is used by writing :

$$\hat{\mu}\hat{b}^T = (vec^{-1}(\hat{\theta}_b))u_1 v_1^T \tag{12}$$

where $\sigma_{\max}(.)$ is the maximum singular value of $vec^{-1}(\hat{\theta}_b)$, u_1 is the first left eigenvector of $vec^{-1}(\hat{\theta}_b)$, and v_1 is the first right eigenvector of $vec^{-1}(\hat{\theta}_b)$. Using (12), $\hat{\mu}$ and $\hat{b}$ can be given by:

$$\hat{\mu} = \beta\sigma_{\max}(vec^{-1}(\hat{\theta}_b))u_1 \tag{13}$$

$$\hat{b} = \frac{1}{\beta}v_1 \tag{14}$$

where β is an arbitrary parameter.

The nonlinear function $N(.)$ can be recovered from the estimate

$$\hat{N}(u(k)) = \hat{\theta}_b\psi_\eta(k) \tag{15}$$

IV. Approximation by Triangular Basis Function

To linearly parameterize the nonlinear function $N(.)$ we use the triangular basis functions given in Fig. 2. For simplicity, the input interval $[u_{\min}, u_{\max}]$ is evenly divided into p partitions separated by a set of points $\{u_1 \ldots u_p\}$, such that $u_{\min} = u_1 < u_2 < \ldots < up = u_{\max}$ with $\Delta = u_j - u_{j-1}$. The unit triangular generator function $\xi(.)$ is defined as

$$\xi(z) = \begin{cases} 1+z & if \quad -1 \le z < 0 \\ 1-z & if \quad 0 \le z < 1 \\ 0 & otherwise \end{cases} \tag{16}$$

Now, we write $\gamma_j = u_1 + j\Delta$, $\alpha_j = 1/\Delta$ and $\gamma_j = u_1 + j\Delta$

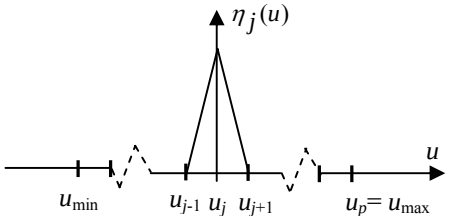


Fig. 2 Triangular basis function

V. Simulation Results

The proposed method for the parameter estimation of nonlinear dynamic system with discontinuous nonlinearities using the Hammerstein model was implemented and tested by means of MATLAB packages. The estimation of the model parameters (those of linear and nonlinear blocks) were carried out on the basis of input and output records. To illustrate the feasibility of the proposed identification method, the following example shows the parameters estimation process for the linear dynamic block, which is given by the following recursive equation:

$$y(k) = 1.6961y(k-1) - 0.8651y(k-2) + 0.5895h(k-1) + 0.4701h(k-2) \tag{17}$$

The discontinuous nonlinearity is described by the following equation:

$$N(u) = \begin{cases} u + 0.28 & u \le -0.28 \\ 0 & -0.28 < u < 0.28 \\ u - 0.28 & u \ge 0.28 \end{cases} \tag{18}$$

We choose input signal as a zero-mean, white noise sequence uniformly distributed between -1 and 1. The measurement noise was a zero-mean white noise Gaussian sequence. The Signal to Noise Ratio (the square root of the ratio of output and noise variances) was (SNR =25 and SNR=50). The SNR is defined as

$$SNR = \sqrt{\frac{\sum_{k=0}^{r} y^2(k)}{\sum_{k=0}^{r} v^2(k)}} \tag{19}$$

To taste the estimation quality of the model, the mean square error (MSE) is used. Indeed, the MSE values for the nonlinear identification method were calculated for two values of SNR, and the results are tabulated in table 2. The MSE is defined as

$$MSE = \frac{1}{N}\sum_{k=1}^{N}(y(k) - \hat{y}(k))^2 \tag{20}$$

where $\hat{y}(k)$ is the predicted output and N is the number of samples used in the identification process.

The evolution curves, for the output, the estimate values and the static nonlinearity (for SNR equal to 25) are given respectively in Figure 3, Figure 4 and Figure 5.

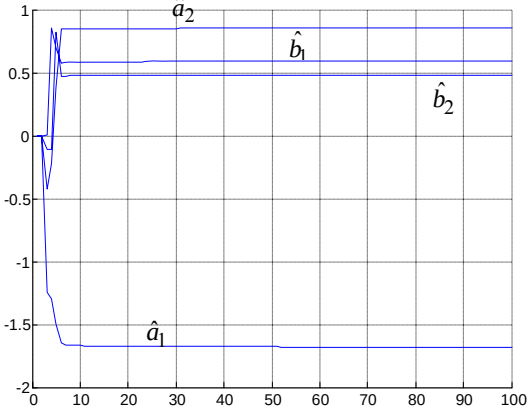


Fig. 3 Parameter estimates

The previous curves of estimates parameters Fig. 3 show a good convergence of the algorithm. Indeed, the estimate value converges to the true value quickly. The statistical average, for the estimate parameters of the last twenty samples with various values of SNR (25 and 50) is given in Table1. We note if the values of SNR increase, the estimates converge more towards the true values. Moreover, the mean square errors of proposed method given in Table 1, show that the quality of estimation is all better as the signal to noise ration (SNR) increases. It can be seen by comparing the estimate dead zone nonlinearity (red) and the real one (blue) Fig. 5 that the estimate curve coincides well with the real one.

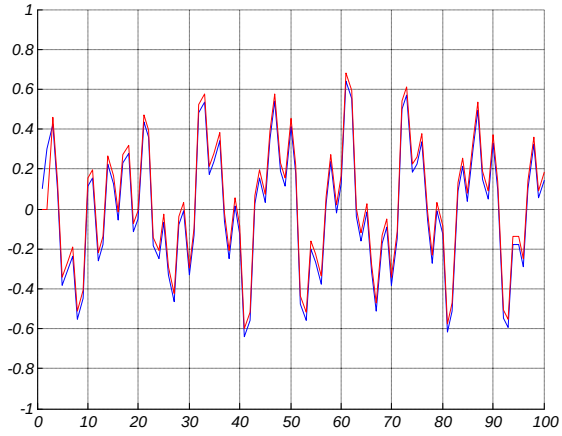


Fig. 4 Real system response (blue) and identified model response (red)

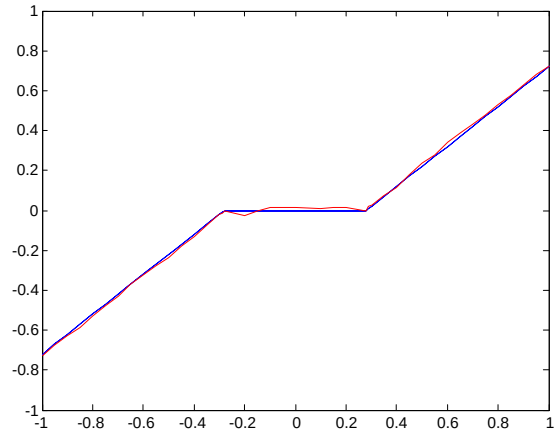


Fig. 5 The static nonlinearity function: actual $N(u)$ (blue) and estimated $N(u)$ (red)

TABLE I
ESTIMATE VALUES OF PARAMETERS

SNR	25	50
$\hat{a}_1$	-1.6864	-1.6959
$\hat{a}_2$	0.8561	0.8649
$\hat{b}_1$	0.5924	0.5893
$\hat{b}_2$	0.4773	0.4703

TABLE II
MEAN SQUARE ERROR

SNR	25	50
MSE	0.0302	0.0101

VI. Conclusion

In this paper, a new approach, in modeling and parameters identification of Hammerstein models with piecewise nonlinear characteristic. The contribution, in this work, has been dedicated to a new description of Hammerstein model by the introduction of simultaneous use of both an easy decomposition technique and a triangular basis functions to modeling the static nonlinear function. This method allow to parameterize the Hammerstein model leading to a linear regressed form so that least square techniques have been successfully used to estimate an oversized parameter matrix. Then, by recurring to SVD, optimal estimates of the parameter matrices characterizing the linear and nonlinear parts have been determined. The included example of the identification process has shown the feasibility and good convergence properties of the proposed technique. The presented method can be easily extended to Hammerstein systems with other types of nonlinearities, e.g., preload nonlinearity, saturation nonlinearity, ...

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