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Reflection of Plane Waves at Free Surface of an Initially Stressed Dissipative Medium

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***Abstract*—**The paper discuses the effect of initial stresses on the reflection coefficients of plane waves in a dissipative medium. Basic governing equations are formulated in context of Biot's incremental deformation theory. These governing equations are solved analytically to obtain the dimensional phase velocities of plane waves propagating in plane of symmetry. Closed-form expressions for the reflection coefficients of P and SV waves' incident at the free surface of an initially stressed dissipative medium are obtained. Numerical computations, using these expressions, are carried out for a particular model. Computations made with the results predicted in presence and absence of the initial stresses and the results have been shown graphically. The study shows that the presence of compressive initial stresses increases the velocity of longitudinal wave (P-wave) but diminishes that of transverse wave (SV-wave). Also the numerical results presented indicate that initial stresses and dissipation might affect the reflection coefficients significantly.

***Keywords*—**Dissipation medium, initial stress, longitudinal waves, reflection coefficients, reflection of plane waves, transverse waves.

I. Introduction

THE reflection coefficient of elastic waves from planar boundaries is important for calculations of amplitudes of various seismic signals. The coefficient has been studies for the case of homogenous and inhomogeneous media with several types of velocity distributions by Sinha [1], Tooly et al.,[2], Gupta [3,4,5], Acharya[6], Cerveny[7], Singh et al.,[8], Saini[9], Singh et al.,[10], Tomar et al.[11], Sharma[12] and others. In the existing literature the effects of dissipation and initial stresses present together in the medium has not been considered. A large amount of initial stresses may develop and may present in the medium caused by various factors, such as creep, gravity, external forces, difference of temperature etc. In fact, the Earth is an initially stressed and dissipative medium and the stresses are hydrostatic and compressive. Therefore, it is of interest to study the phenomenon of reflection of plane waves in the presence of initial stresses as well as dissipation.

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In this work, an attempt has been done to apply the theory of Biot's incremental deformation to derive closed-form algebraic expressions for the reflection coefficients when plane waves of P or SV type are incident at the plane free boundary of an initially stressed dissipative half-space. Basic governing equations are formulated in context of Biot's incremental deformation theory. Closed-form expressions for the reflection coefficients of P and SV waves incident at the free surface of an initially stressed dissipative medium are obtained. Numerical results presented indicate that the presence of compressive initial stresses increases the velocity of longitudinal wave (P-wave) but diminishes that of transverse wave (SV-wave). Also the initial stresses, present in the medium and dissipation, have an effect of the reflection coefficients.

II. Formulation of the Problem and its Solution

According to Biot [13], the basic dynamical equations of motion for an infinite, initially stressed medium, in the absence of external body forces, in case of plane strain are

$$\frac{\partial s_{11}}{\partial x}+\frac{\partial s_{12}}{\partial y}-P\frac{\partial \omega}{\partial y}=\rho\frac{\partial^2 u}{\partial t^2},$$
$$\frac{\partial s_{21}}{\partial x}+\frac{\partial s_{22}}{\partial y}-P\frac{\partial \omega}{\partial x}=\rho\frac{\partial^2 v}{\partial t^2} \quad (1)$$

where s_{ij} $(i, j=1,2)$ are the incremental stress components and ω is rational component given by

$$\omega=\frac{1}{2}\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right), \quad (2)$$

where u and v are the displacement components.

Assuming the anisotropy induced by initial stresses as orthotropic (in two dimensions) where the principal axes of initial stresses are identified with *x, y* axes, the stress-strain relations are taken as [13]

$$s_{11}=B_{11}e_{11}+B_{12}e_{22},$$
$$s_{22}=(B_{12}-P)e_{11}+B_{22}e_{22}, \quad (3)$$
$$s_{12}=2Qe_{12},$$

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where B_{ij} *(i, j = 1, 2)* and Q_3 are the incremental elastic coefficients and shear modulus, respectively.

These incremental elastic coefficients are related to Lame's coefficients λ and μ of the isotropic unstressed state. For the present case [13], these are:

$$B_{11}=(\lambda+2\mu+P),\quad B_{12}=(\lambda+P),$$
$$B_{21}=\lambda,\quad B_{22}=\lambda+2\mu,\quad Q=\mu \tag{4}$$

where $P=-S_{11}$ is the normal initial stress along the horizontal direction.

The incremental strain components e_{ij} *(i, j = 1, 2)* are related with the displacement components (*u* , *v*) through the relations

$$e_{11}=\frac{\partial u}{\partial x},\qquad e_{22}=\frac{\partial v}{\partial y},$$
$$e_{12}=\frac{1}{2}\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right). \tag{5}$$

For assuming dissipative medium, the two Lame's coefficients λ and μ for isotropic unstressed state of the medium are replaced by complex constants:

$$\lambda=\lambda_1+i\,\lambda_2\ ,\ \mu=\mu_1+i\,\mu_2\ , \tag{6}$$

where $i=\sqrt{-1}$, λ_2 and μ_2 are real and $\lambda_2 << \lambda_1$, $\mu_2 << \mu_1$.

The stress and strain components in dissipative medium are given by Fung [14],

$$s_{ij}=\bar{s}_{ij}\exp(i\varpi t),$$
$$e_{ij}=\bar{e}_{ij}\exp(i\varpi t),$$
$$\bar{e}_{ij}=\frac{1}{2}\left(\frac{\partial \bar{u}_i}{\partial x_j}+\frac{\partial \bar{u}_j}{\partial x_i}\right), \tag{7}$$

where $u_i=\bar{u}_i\exp(i\varpi t)$, ϖ being the angular frequency and $(i\ ,j=1,2)$.

From (4) – (7) in relation (3), the stress-strain relations become

$$\begin{aligned}\bar{s}_{11}&=[(\lambda_1+2\mu_1+P)+i(\lambda_2+2\mu_2)]\bar{e}_{11}+[(\lambda_1+P)+i\lambda_2]\bar{e}_{22},\\ \bar{s}_{22}&=(\lambda_1+i\lambda_2)\bar{e}_{11}+[(\lambda_1+2\mu_1)+i(\lambda_2+2\mu_2)]\bar{e}_{22},\\ \bar{s}_{12}&=2(\mu_1+i\mu_2)\bar{e}_{12}.\end{aligned} \tag{8}$$

From (2), (3), (5) and (8), the equation of motion in terms of the displacement components $\bar{u}$ and $\bar{v}$ can be written as

$$[(\lambda_1+2\mu_1+P)\frac{\partial^2\bar{u}}{\partial x^2}+(\lambda_1+\mu_1+\frac{P}{2}))\frac{\partial^2\bar{v}}{\partial x\,\partial y}+(\mu_1+\frac{P}{2})\frac{\partial^2\bar{u}}{\partial y^2}$$
$$+\rho\varpi^2\bar{u}]+i[(\lambda_2+2\mu_2)\frac{\partial^2\bar{u}}{\partial x^2}+(\lambda_2+\mu_2)\frac{\partial^2\bar{v}}{\partial x\,\partial y}+\mu_2\frac{\partial^2\bar{u}}{\partial y^2}]=0 \tag{9}$$

$$[(\lambda_1+2\mu_1)\frac{\partial^2\bar{v}}{\partial y^2}+(\lambda_1+\mu_1+\frac{P}{2}))\frac{\partial^2\bar{u}}{\partial x\partial y}+(\mu_1-\frac{P}{2})\frac{\partial^2\bar{v}}{\partial x^2}+\rho\varpi^2\bar{v}]$$
$$+i[(\lambda_2+2\mu_2)\frac{\partial^2\bar{v}}{\partial y^2}+(\lambda_2+\mu_2)\frac{\partial^2\bar{u}}{\partial x\partial y}+\mu_2\frac{\partial^2\bar{v}}{\partial x^2}]=0. \tag{10}$$

The displacement Vector $\vec{U}^{(n)}=(u^{(n)},v^{(n)},0)$ is given by

$$\vec{U}^{(n)}=A_n\vec{d}^{(n)}\exp(i\,\Omega_n), \tag{11}$$

where the index n assigns an arbitrary direction of propagation of waves, $\vec{d}^{(n)}=(d_1^{(n)},d_2^{(n)})$ is the unit displacement vector and

$$\Omega_n=k_n[c_n t-(\vec{X}\bullet\vec{\Gamma}^{(n)})], \tag{12}$$

is the phase factor in which $\vec{\Gamma}^{(n)}=(\Gamma_1^{(n)},\Gamma_2^{(n)})$ is the unit propagation vector, c_n is the velocity of propagation, $\vec{x}=(x,y)$ and k_n is the corresponding wave number, which is related to the angular frequency by

$$\varpi=k_n\,c_n. \tag{13}$$

In matrix form, the displacement components (11) may be expressed as

$$\begin{pmatrix}u^{(n)}\\ v^{(n)}\end{pmatrix}=\begin{pmatrix}A_n\,d_1^{(n)}\\ A_n\,d_2^{(n)}\end{pmatrix}\exp[-ik_n(x\Gamma_1^{(n)}+y\Gamma_2^{(n)}-c_n t)]. \tag{14}$$

This can be written in the form

$$\begin{pmatrix}u^{(n)}\\ v^{(n)}\end{pmatrix}=\begin{pmatrix}A\\ B\end{pmatrix}\exp[-ik_n(x\Gamma_1^{(n)}+y\Gamma_2^{(n)}-c_n t)]. \tag{15}$$

Omitting, for convenience, the bared notations of displacement components and angular frequency, and inserting relations (13) and (15) in equations(9) and (10),one gets

$$A\{[(\lambda_1+2\mu_1+P)\Gamma_1^{(n)2}+(\mu_1+\frac{P}{2})\Gamma_2^{(n)2}-\rho c_n^2]+i[(\lambda_2+2\mu_2)\Gamma_1^{(n)2}$$
$$+\mu_2\Gamma_2^{(n)2}]\}+B[(\lambda_1+\mu_1+\frac{P}{2})\Gamma_1^{(n)}\Gamma_2^{(n)}+i(\lambda_2+\mu_2)\Gamma_1^{(n)}\Gamma_2^{(n)}]$$
$$=Aa_1+Bb_1=0, \tag{16}$$

$$A[(\lambda_1+\mu_1+\frac{P}{2})\Gamma_1^{(n)}\Gamma_2^{(n)}+i[(\lambda_2+\mu_2)\Gamma_1^{(n)}\Gamma_2^{(n)}]+B\{[(\lambda_1+2\mu_1)\Gamma_2^{(n)2}$$
$$+(\mu_1-\frac{P}{2})\Gamma_1^{(n)2}-\rho c_n^2]+i[(\lambda_2+2\mu_2)\Gamma_2^{(n)2}+\mu_2\Gamma_1^{(n)2}]\}$$
$$=Aa_2+Bb_2=0. \tag{17}$$

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For non-zero solution of A and B, from equations (16) and (17), one must have

$$\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = 0 . \tag{18}$$

The last equation gives two values of c_n^2 may be obtained which give the square of velocities of a plane wave propagating in the direction $\Gamma^{(n)}$. These are given by

$$\begin{aligned} c_n^2 &= c_P^2 = \frac{1}{2\rho}(I + \sqrt{J_n}), \\ c_n^2 &= c_{SV}^2 = \frac{1}{2\rho}(I - \sqrt{J_n}), \end{aligned} \tag{19}$$

where

$$I = \{\lambda_1 + \mu_1(3+\zeta) + i(\lambda_2 + 3\mu_2)\},$$

$$\begin{aligned} J_n = &-8(\lambda_2+\mu_2)^2\Gamma_1^{(n)4} + 8\zeta\mu_1[\lambda_1 + \mu_1(1+\zeta)] + 8(\lambda_2+\mu_2)^2\Gamma_1^{(n)2} \\ &+ [\lambda_1 + \mu_1(1-\zeta)]^2 - (\lambda_2+\mu_2)^2 \\ &+ i\left(8\zeta\mu_1(\lambda_2+\mu_2)\Gamma_1^{(n)2} + 2(\lambda_2+\mu_2)(\lambda_1+\mu_1) - 2\zeta\mu_1(\lambda_2+\mu_2)\right). \end{aligned} \tag{20}$$

where $\zeta = \frac{P}{2\mu_1}$ is the initial stress parameter and $\Gamma_1^{(n)2} + \Gamma_2^{(n)2} = 1$.

Equation (19) gives the square of velocities of propagation as well as damping. Real parts of the right hand sides correspond to phase velocities and the respective imaginary parts correspond to damping velocities of P and SV waves, respectively. It is observed that both C_P^2 and C_{SV}^2 depend on initial stresses, damping and direction of propagation Γ_1^n. In absence of initial stresses and damping, the above analysis corresponds to the case for classic dynamics of elastic solid.

When the effect of initial stresses is absent and the medium is non-dissipative (i.e. $\zeta = \frac{P}{2\mu_1} = 0, \lambda_2 = \mu_2 = 0$).

In this case, equation (19) in non-dimensional form can be written as

$$\begin{aligned} \frac{c_p^2}{\alpha^2} &= \frac{1}{2}\left[1 + \frac{\beta^2}{\alpha^2} + \left(4\zeta\left(1 - \frac{\beta^2}{\alpha^2} + \frac{\zeta}{2}\right)\sin^2 e_1 + \left(1 - \frac{\beta^2}{\alpha^2} - \frac{\zeta}{2}\right)\right)^{\frac{1}{2}}\right], \\ \frac{c_{SV}^2}{\beta^2} &= \frac{1}{2}\frac{\alpha^2}{\beta^2}\left[1 + \frac{\beta^2}{\alpha^2} - \left(4\zeta\left(1 - \frac{\beta^2}{\alpha^2} + \frac{\zeta}{2}\right)\sin^2 e_2 + \left(1 - \frac{\beta^2}{\alpha^2} - \frac{\zeta}{2}\right)\right)^{\frac{1}{2}}\right]. \end{aligned} \tag{21}$$

where $\alpha = \left[\frac{(\lambda+2\mu)}{\rho}\right]^{\frac{1}{2}}$, $\beta = \left[\frac{\mu}{\rho}\right]^{\frac{1}{2}}$ which coincides with the result of Selim et al.[15].

Also, from equations (16) and (17), we obtain

$$\frac{d_1^{(n)}}{d_2^{(n)}} = \frac{\Delta_n}{\rho c_n^2 - T_n} = \frac{\rho c_n^2 - \Theta_n}{\Delta_n}, \tag{22}$$

where

$$\begin{aligned} T_n &= \left[(\lambda_1 + 2\mu_1(1+\zeta) + i(\lambda_2 + 2\mu_2)\right]\Gamma_1^{(n)2} + \left[\mu_1(1+\mu_1\zeta) + i\mu_2\right]\Gamma_2^{(n)2}, \\ \Delta_n &= \left[(\lambda_1 + \mu_1(1+\mu_1\zeta)) + i(\lambda_2 + \mu_2)\right]\Gamma_1^{(n)}\Gamma_2^{(n)}. \\ \Theta_n &= \left[(\lambda_1 + 2\mu_1) + i(\lambda_2 + 2\mu_2)\right]\Gamma_2^{(n)2} + \left[\mu_1(1-\mu_1\zeta) + i\mu_2\right]\Gamma_1^{(n)2}. \end{aligned} \tag{23}$$

Equation (23)may used to find $\vec{d}^{(n)}$ in terms of $\vec{\Gamma}^{(n)}$.

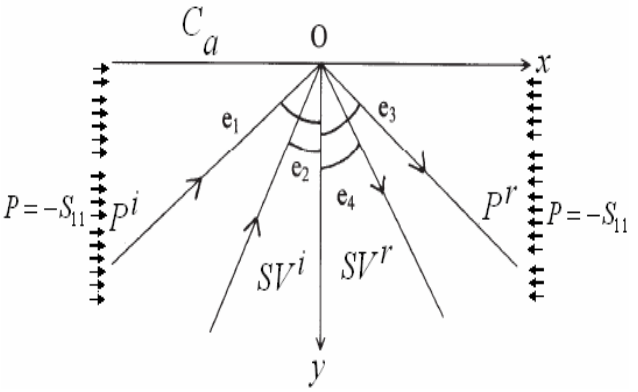


Fig. 1 Reflection of P and SV waves at free surface of initially stressed medium

III. Reflection of Plane Waves at Stress-Free Surfaces

We consider an initially stressed dissipative half-space occupying the region $y \geq 0$ (Fig. 1). The plane of elastic symmetry is taken as the xy plane. In this section, we shall derive the closed-form expressions for the reflection coefficients, when plane (P or SV) waves incident at the traction-free boundary $y = 0$. We consider plane strain case in which the displacement components are independent of z and are of the type

$$U^{(n)} = \left(u^{(n)}(x,y,t),\, v^{(n)}(x,y,t),\, 0\right). \tag{24}$$

Incident P or SV waves will generate reflected P and SV waves as shown in Fig. 1. Accordingly, the total displacements field may be represented by

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$$u(x,y,t) = \sum_{j=1}^{4} A_j d_1^{(j)} e^{i\Omega_j},$$
$$v(x,y,t) = \sum_{j=1}^{4} A_j d_2^{(j)} e^{i\Omega_j}, \qquad (25)$$

where

$$\Omega_1 = k_1[c_1 t - (x \sin e_1 - y \cos e_1)],$$
$$\Omega_2 = k_2[c_2 t - (x \sin e_2 - y \cos e_2)],$$
$$\Omega_3 = k_3[c_3 t - (x \sin e_3 + y \cos e_3)],$$
$$\Omega_4 = k_4[c_4 t - (x \sin e_4 + y \cos e_4)]. \qquad (26)$$

The subscripts (1) be assumed for incident P waves , (2) for incident SV waves, (3) for reflected P waves and (4) for reflected SV waves respectively.

In the plane $y=0$, the displacement and stress components due to incident P-wave $(\Gamma_1^{(1)} = \sin e_1,\ \Gamma_2^{(1)} = -\cos e_1)$ may be written as

$$u^{(1)} = A_1 d_1^{(1)} e^{i\Omega_1},\ v^{(1)} = A_1 d_2^{(1)} e^{i\Omega_1},$$
$$s_{12}^{(1)} = iA_1 k_1 Q_1 (d_1^{(1)} \cos e_1 - d_2^{(1)} \sin e_1) e^{i\Omega_1},$$
$$s_{22}^{(1)} = iA_1 k_1 \left(Q_3 d_2^{(1)} \cos e_1 - Q_2 d_1^{(1)} \sin e_1\right) e^{i\Omega_1}, \qquad (27)$$

where

$$Q_1 = (\mu_1 + i\mu_2),$$
$$Q_2 = (\lambda_1 + i\lambda_2),$$
$$Q_3 = [(\lambda_1 + 2\mu_1) + i(\lambda_2 + 2\mu_2)]. \qquad (28)$$

In the plane $y=0$, the displacement and stress components due to incident SV-wave $(\Gamma_1^{(2)} = \sin e_2,\ \Gamma_2^{(2)} = -\cos e_2)$ may be written as

$$u^{(2)} = A_2 d_1^{(2)} e^{i\Omega_2},$$
$$v^{(2)} = A_2 d_2^{(2)} e^{i\Omega_2},$$
$$s_{12}^{(2)} = iA_2 k_2 Q_1 (d_1^{(2)} \cos e_2 - d_2^{(2)} \sin e_2) e^{i\Omega_2},$$
$$s_{22}^{(2)} = iA_2 k_2 \left(Q_3 d_2^{(2)} \cos e_2 - Q_2 d_1^{(2)} \sin e_2\right) e^{i\Omega_2}, \qquad (29)$$

In the plane $y=0$, the displacement and stress components due to reflected P-wave $(\Gamma_1^{(3)} = \sin e_3,\ \Gamma_2^{(3)} = \cos e_3)$ may be written as

$$u^{(3)} = A_3 d_1^{(3)} e^{i\Omega_3},\ v^{(3)} = A_3 d_2^{(3)} e^{i\Omega_3},$$
$$s_{12}^{(3)} = -iA_3 k_3 Q_1 (d_1^{(3)} \cos e_3 + d_2^{(3)} \sin e_3) e^{i\Omega_3},$$
$$s_{22}^{(3)} = -iA_3 k_3 \left(d_1^{(3)} Q_2 \sin e_2 + Q_3 d_2^{(3)} \cos e_3\right) e^{i\Omega_3}, \qquad (30)$$

In the plane $y=0$, the displacement and stress components due to reflected SV-wave $(\Gamma_1^{(4)} = \sin e_4,\ \Gamma_2^{(4)} = \cos e_4)$ may be written as

$$u^{(4)} = A_4 d_1^{(4)} e^{i\Omega_4},$$
$$v^{(4)} = A_4 d_2^{(4)} e^{i\Omega_4},$$
$$s_{12}^{(4)} = -iA_4 k_4 Q_1 (d_1^{(4)} \cos e_4 + d_2^{(4)} \sin e_4) e^{i\Omega_4}, \qquad (31)$$
$$s_{22}^{(4)} = -iA_4 k_4 \left(Q_2 d_1^{(4)} \sin e_4 + Q_3 d_2^{(4)} \cos e_3\right) e^{i\Omega_4},$$

IV. Boundary Conditions

The boundary conditions appropriate for the free surfaces are vanishing of incremental boundary forces. So, the two boundary conditions required to be satisfied at the plane $y = 0$, are

$$\Delta f_x = s_{12}^{(n)} + e_{12}^{(n)} P = 0, \qquad \Delta f_y = s_{22}^{(n)} = 0. \qquad (32)$$

These equations can be written as:

$$s_{12}^{(1)} + s_{12}^{(2)} + s_{12}^{(3)} + s_{12}^{(4)} + 2\mu_1 \zeta \left(e_{12}^{(1)} + e_{12}^{(2)} + e_{12}^{(3)} + e_{12}^{(4)}\right) = 0,$$
$$s_{22}^{(1)} + s_{22}^{(2)} + s_{22}^{(3)} + s_{22}^{(4)} = 0. \qquad (33)$$

Inserting equations (27), (29), (30) and (31) in equations (33), we obtain

$$\begin{aligned} &iA_1 k_1 Q_1 (d_1^{(1)} \cos e_1 - d_2^{(1)} \sin e_1) e^{i\Omega_1} \\ &+ iA_2 k_2 Q_1 (d_1^{(2)} \cos e_2 - d_2^{(2)} \sin e_2) e^{i\Omega_2} \\ &- iA_3 k_3 Q_1 (d_1^{(3)} \cos e_3 + d_2^{(3)} \sin e_3) e^{i\Omega_3} \\ &- iA_4 k_4 Q_1 (d_1^{(4)} \cos e_4 + d_2^{(4)} \sin e_4) e^{i\Omega_4} \\ &+ i\mu_1 \zeta \begin{pmatrix} A_1 k_1 (d_1^{(1)} \cos e_1 - d_2^{(1)} \sin e_1) e^{i\Omega_1} \\ + A_2 k_2 (d_1^{(2)} \cos e_2 - d_2^{(2)} \sin e_2) e^{i\Omega_2} \\ - A_3 k_3 (d_1^{(3)} \cos e_3 + d_2^{(3)} \sin e_3) e^{i\Omega_3} \\ - A_4 k_4 (d_1^{(4)} \cos e_4 + d_2^{(4)} \sin e_4) e^{i\Omega_4} \end{pmatrix} = 0, \end{aligned} \qquad (34)$$

and

$$\begin{aligned} &iA_1 k_1 \left(Q_3 d_2^{(1)} \cos e_1 - Q_2 d_1^{(1)} \sin e_1\right) e^{i\Omega_1} \\ &+ iA_2 k_2 \left(Q_3 d_2^{(2)} \cos e_2 - Q_2 d_1^{(2)} \sin e_2\right) e^{i\Omega_2} \\ &- iA_3 k_3 \left(Q_2 d_1^{(3)} \sin e_2 + Q_3 d_2^{(3)} \cos e_3\right) e^{i\Omega_3} \\ &- iA_4 k_4 \left(Q_2 d_1^{(4)} \sin e_4 + Q_3 d_2^{(4)} \cos e_3\right) e^{i\Omega_4} = 0. \end{aligned} \qquad (35)$$

Since equations (34) and (35) are to be satisfied for all values of x and t, hence

$$\Omega_1(x,0) = \Omega_2(x,0) = \Omega_3(x,0) = \Omega_4(x,0), \qquad (36)$$

which means

$$k_1(C_p^i\, t - x\sin e_1) = k_2(C_{SV}^i\, t - x\sin e_2),$$
$$= k_3(C_p^r\, t - x\sin e_3) = k_4(C_{SV}^r\, t - x\sin e_4). \quad (37)$$

where C_P^i, C_{SV}^i, C_P^r, C_{SV}^r are phase velocities of incident P-wave, incident SV-wave, reflected P-wave and reflected SV-wave, respectively.
The above equation gives

$$k_1\sin e_1 = k_2\sin e_2 = k_3\sin e_3 = k_4\sin e_4,$$

and

$$k_1 C_P^i = k_2 C_{SV}^i = k_3 C_P^r = k_4 C_{SV}^r. \quad (38)$$

From relation (13) equation.(38) can be written as

$$\frac{\sin e_1}{C_p} = \frac{\sin e_2}{C_{SV}} = \frac{\sin e_3}{C_p} = \frac{\sin e_4}{C_{SV}} = \frac{1}{C_a}, \quad (39)$$

where C_a is the apparent phase velocity. The above relation represents Snell's Law for orthotropic medium.

Equations (34) and (35) after using relations (36)-(39), may be written as,

$$A_1\delta_1 + A_2\delta_2 + A_3\delta_3 + A_4\delta_4 = 0,$$
$$A_1\delta_5 + A_2\delta_6 + A_3\delta_7 + A_4\delta_8 = 0, \quad (40)$$

where

$$\delta_1 = k_1 L(d_1^{(1)}\cos e_1 - d_2^{(1)}\sin e_1),$$
$$\delta_2 = k_2 L(d_1^{(2)}\cos e_2 - d_2^{(2)}\sin e_2),$$
$$\delta_3 = -k_3 L(d_1^{(3)}\cos e_3 + d_2^{(3)}\sin e_3),$$
$$\delta_4 = -k_4 L(d_1^{(4)}\cos e_4 + d_2^{(4)}\sin e_4),$$
$$\delta_5 = k_1\left(Q_3 d_2^{(1)}\cos e_1 - Q_2 d_1^{(1)}\sin e_1\right),$$
$$\delta_6 = k_2\left(Q_3 d_2^{(2)}\cos e_2 - Q_2 d_1^{(2)}\sin e_2\right), \quad (41)$$
$$\delta_7 = -k_3\left(Q_2 d_1^{(3)}\sin e_3 + Q_3 d_2^{(3)}\cos e_3\right),$$
$$\delta_8 = -k_4\left(Q_2 d_1^{(4)}\sin e_4 + Q_3 d_2^{(4)}\cos e_4\right),$$
$$L = (Q_1 + \zeta\mu_1).$$

A. Incident P-Waves

In the case of incident P-waves, $A_2 = 0$ and equation (40) becomes

$$A_1\delta_1 + A_3\delta_3 + A_4\delta_4 = 0,$$
$$A_1\delta_5 + A_3\delta_7 + A_4\delta_8 = 0. \quad (42)$$

Solving equation(42), we obtained the amplitude ratios in the form

$$\frac{A_3}{A_1} = \frac{(\delta_4\delta_5 - \delta_1\delta_8)}{(\delta_3\delta_8 - \delta_4\delta_7)}, \quad \frac{A_4}{A_1} = -\frac{(\delta_3\delta_5 - \delta_1\delta_7)}{(\delta_3\delta_8 - \delta_4\delta_7)} \quad (43)$$

B. Incident SV-Waves

In the case of incident SV-waves, $A_1 = 0$ and equation (40) becomes

$$A_2\delta_2 + A_3\delta_3 + A_4\delta_4 = 0,$$
$$A_2\delta_6 + A_3\delta_7 + A_4\delta_8 = 0, \quad (44)$$

Solving equation(44), we obtained the amplitude ratios in the form

$$\frac{A_3}{A_2} = \frac{(\delta_4\delta_6 - \delta_2\delta_8)}{(\delta_3\delta_8 - \delta_4\delta_7)}, \quad \frac{A_4}{A_2} = -\frac{(\delta_3\delta_6 - \delta_2\delta_7)}{(\delta_3\delta_8 - \delta_4\delta_7)} \quad (45)$$

System of equations (43) and (45) contain both real and imaginary parts. Real parts of expressions (43) and (45) allow one to determine the reflection coefficients of the reflected P and SV-waves at a given incident P and SV waves amplitudes, respectively.

V. Numerical Calculations and Conclusions

For the purpose of numerical computations, the following Physical constants are considered for the infinite medium as Aswan geological crustal structures given by Kebeasy et al.[16].

$\rho = 2.15$ g/cm^3, $\mu_1 = 1.90930 \times 10^{11}$ dyne/cm^2, $\lambda_1 = 2.22075 \times 10^{11}$ dyne/cm^2,
$\mu_2 = 0.436 \times 10^{11}$ dyne/cm^2, $\lambda_2 = 0.305 \times 10^{11}$ dyne/cm^2,
$\alpha = 5.3 \times 10^5$ cm/s, $\beta = 2.98 \times 105$cm/s.

Using all these data, the dimensional velocities of P-wave($\frac{c_p^2}{\alpha^2}$) and SV-wave($\frac{c_s^2}{\alpha^2}$) are calculated for different angles of propagation under different initial stress parameter(ζ). To avoid instability created by the compressive initial stress, the values of ζ have been taken within the range 0.6 [17]. The real parts of the values obtained from equation (19) give the phase velocities and imaginary parts give the corresponding damping in P and SV waves for an dissipative medium under initial stresses. In the case of ζ=0, this gives the results for initial stress–free medium. The results of computations are presented in Figs. 2 to 5. Fig. 2 shows the effect of initial compressive stresses on the velocity

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of propagation of longitudinal wave (P-wave). The velocity of longitudinal wave is clearly depends on the initial compressive stress present on the medium. The curves also, show that initial compressive stress increases the velocity of longitudinal wave and it is different at different direction of propagation. The velocity of propagation is independent of initial stresses at $\theta = 0^0$ i.e., along the x- direction (direction of initial stress) and effect is more prominent along perpendicular direction.

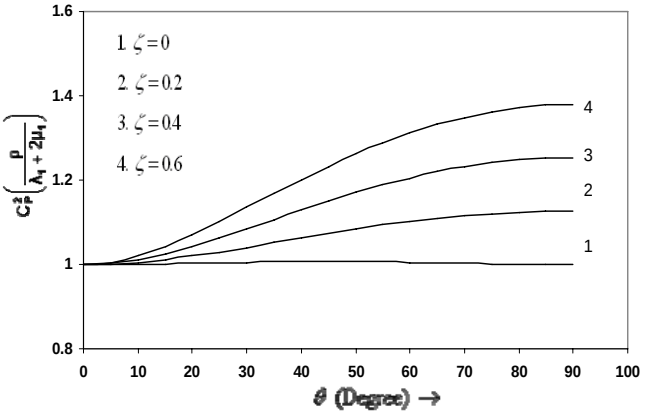


Fig. 2 Variation of C_P^2 / α^2 with the direction of propagation θ for different values of ζ

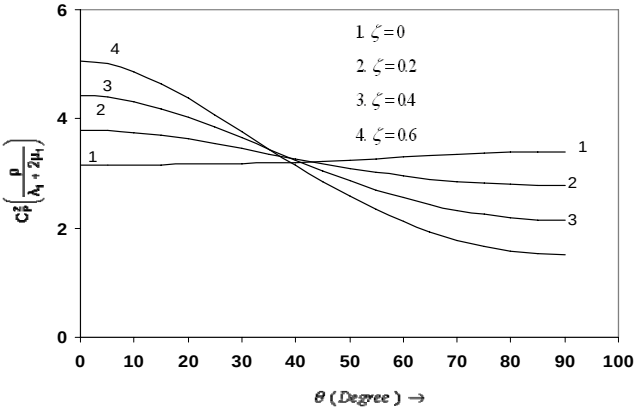


Fig. 3 Variation of C_{SV}^2 / α^2 with the direction of propagation θ for different values of ζ

Fig. 3 gives the variation of C_{SV}^2 / α^2 with direction of propagation θ^0 for different values of initial stress parameter ζ. It is clear that from curves, the square of velocity of SV-wave is higher nearer to the y-direction and it goes on decreasing as the direction changes towards x-axis. It is also observed that phase velocity increases with an increase in initial stress parameter for any particular angle of incidence within the range 0^0 to 40^0 and reverse is the case within the range 40^0 to 90^0 approximately.

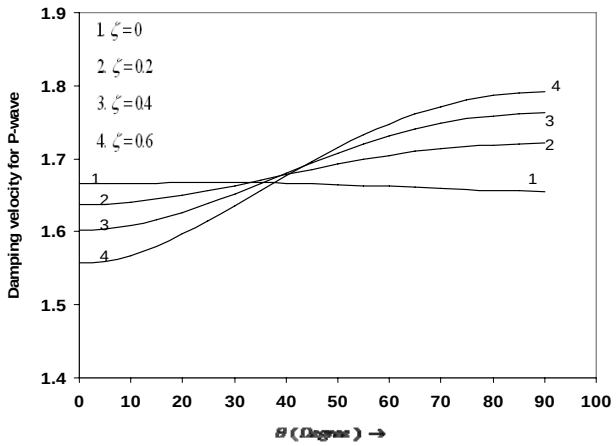


Fig. 4 Variation of damping velocity of P-waves with the direction of propagation θ for different values of ζ

Figs. 4 and 5 give the variations of the square of damping velocities corresponding to C_P and C_{SV}, respectively. From Fig. 4 it is seen that damping corresponding to C_P is minimum nearer to the y-direction and it goes on increasing as the direction changes towards x-axis, the damping velocity decreases with an increase in initial stress parameter for any particular angle of incidence within the range 0^0 to 40^0 and reverse is the case within the range 40^0 to 90^0 approximately.

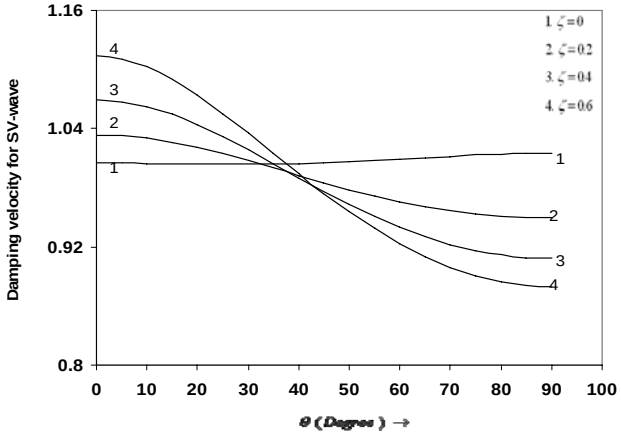


Fig. 5 Variation of damping velocity of SV-waves with the direction of propagation θ for different values of ζ

Fig. 5 exhibits the variations of damping velocity corresponding to C_{SV} waves with direction of propagation for different values of initial stress parameter ζ. It is clear that the variations are just the reverse as discussed in the case of Fig. 4.

The reflection coefficients of various reflected waves are computed for a certain range of angle of incidence of P and SV for different values of the initial stress parameter $(\zeta = 0.0,\ 0.2,\ 0.4,\ 0.6)$ to observe the impact of initial stress at each angle of incidence. The

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variations of the reflection coefficients A_3 / A_1 $\{|R_{PP}|\}$ and A_4 / A_1 $\{|R_{PS}|\}$ for the case of incident P-waves as the absolute values of the real parts of expressions (42) with the angle of incidence are shown graphically in Figs. 6 and 7. The numbers shown in the curves of these figure denotes the reflection coefficients with initial stress parameter {1 means $\zeta=0.0$, 2 means $\zeta=0.2$, 3 means $\zeta=0.4$ and 4 means $\zeta=0.6$}.

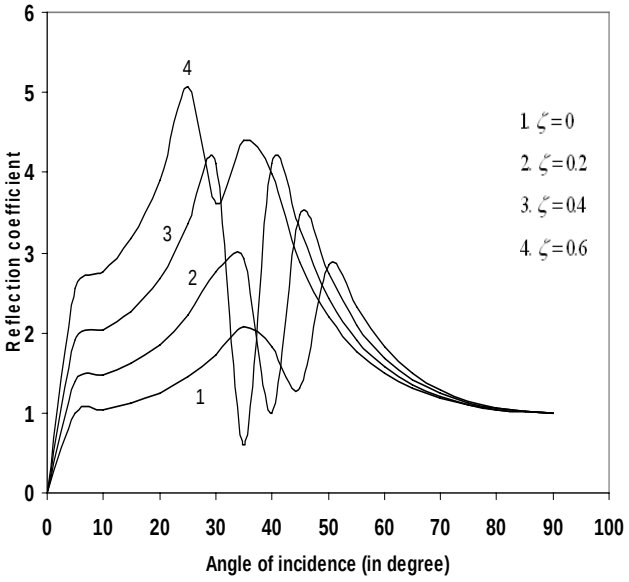


Fig. 6 Reflection coefficient $|R_{PP}|$ under different values of ζ for the incident of P- wave

Figs. 6 and 7 show that when the incidence angle of P-wave $e_1 = 0^0$ (vertical incidence) there is no reflection of SV-wave and there only exists one reflected P-wave. And in the case of horizontal incidence ($e_1 = 90^0$), there exist two reflected waves (P-wave and SV-wave). From these figures it is observed that the reflection coefficients $\{|R_{PP}|\}$ and $\{|R_{PS}|\}$ increases with an increase in initial stress parameter for any particular angle of incidence within the range 0^0 to 45^0 and reverse is the case within the range 50^0 to 90^0. The reflection coefficient $|R_{PP}|$ has its maximum value near $e_1 = 25^0$ at the value of stress parameter $\zeta = 0.4$. And the reflection coefficient $|R_{PS}|$ of has its maximum value at

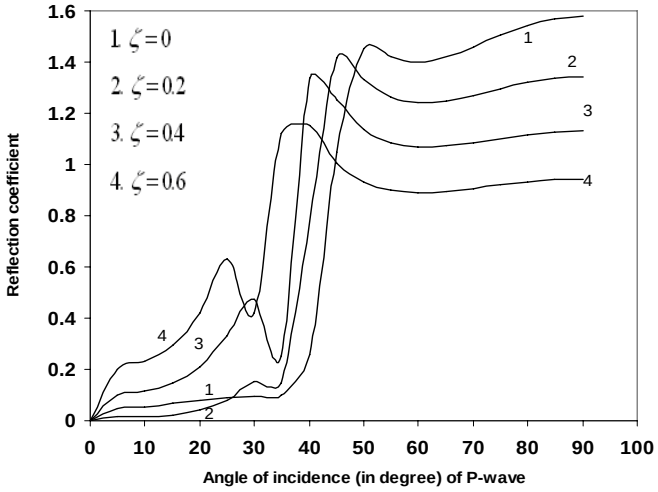


Fig. 7 Reflection coefficient $|R_{PS}|$ under different values of ζ for the incident of P- wave

normal incidence $e_1 = 90^0$ at the absence of the initial stress ($\zeta = 0.0$).

The variations of the reflection coefficients A_3 / A_2 $\{|R_{PS}|\}$ and A_4 / A_2 $\{|R_{SS}|\}$ for the case of incident SV-waves as the absolute values of the real parts of expressions (442) with the angle of incidence are shown graphically in Figs. 8 and 9.

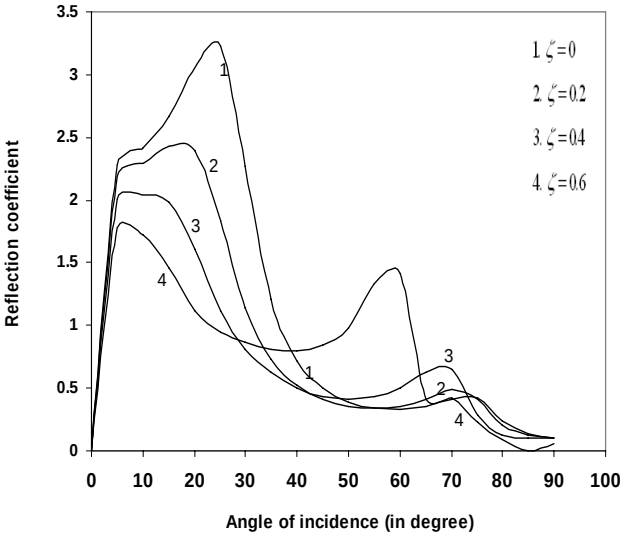


Fig. 8 Reflection coefficient $|R_{SP}|$ under different values of ζ for the incident of SV- wave

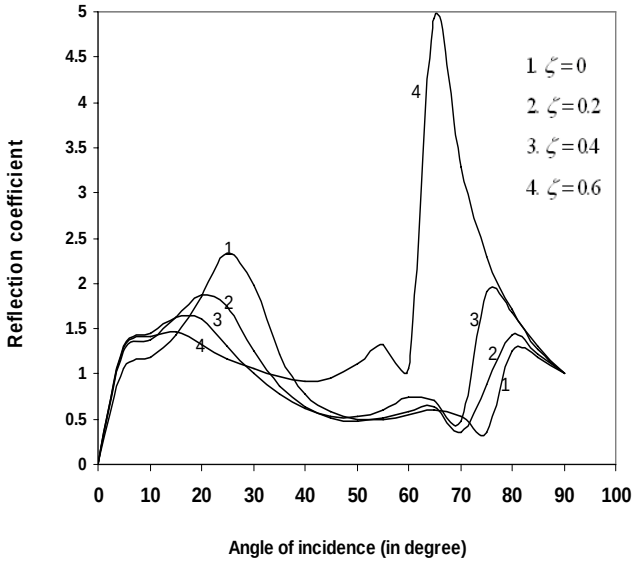


Fig. 9 Reflection coefficient $|R_{SS}|$ under different values of ζ for the incident of SV- wave

From Figs. 8 and 9 we can see that for the incidence SV-wave $|R_{PS}|=0$ and $|R_{SS}|=1$ at incidence $e_2 = 0^0$ and $e_2 = 90^0$. This means, for the vertical and horizontal incidence, there is only one reflected SV-wave. From these figures it is observed that the reflection coefficients $\{R_{SS}\}$ and $\{|R_{SP}|\}$ decreases with an increase in initial stress parameter for any particular angle of incidence within the range 0^0 to 45^0 and reverse is the case within the range 50^0 to 90^0 approximately. The reflection coefficient $|R_{SP}|$ has its maximum value near $e_1 = 30^0$ at the absence of the initial stress $(\zeta = 0.0)$. the value of stress parameter ζ=0.4. And the reflection coefficient $|R_{PS}|$ of has its maximum value at normal incidence $e_1 = 90^0$ at the absence of the initial stress $(\zeta = 0.0)$. And the reflection coefficient $|R_{SS}|$ has its maximum value near $e_1 = 65^0$ at the value of stress parameter ζ=0.4. It is also observed from Figs. 6 -9 that the change of the reflection coefficients does not go smoothly, may due to the effect of a dissipation of the medium.

VI. Conclusion

From the above theoretical and numerical study, it can be concluded that both the velocities and reflection coefficients change with the initial stresses parameter. Also, it is observed that the damping of the medium has strong effect in the propagation of plane waves and reflection coefficients. Since every medium has damping so it is more realistic to take in account the dissipation of the medium instead of the non-dissipation for the problem of reflection plane waves in the elastic medium.

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