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Creating Streamtubes Based on Mass Conservative Streamlines

Nawin Raj, and Zhenquan Li

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***Abstract*—**Streamtube is used to visualize expansion, contraction and various properties of the fluid flow. These are useful in fluid mechanics, engineering and geophysics. The streamtube constructed in this paper only reveals the flow expansion rate along streamline. Based on the mass conservative streamline, we will show how to construct the streamtube.

***Keywords*—**Flow visualization, mass conservative, streamline, streamtube.

I. Introduction

STREAMLINES, streamtube and streamribbon are three of the most fundamental techniques for visualizing steady flow fields. Streamlines are paths of massless particles that are released in a steady flow. Streamtube and streamribbons show the expansion and rotation of the flow.

Formally a streamtube is defined as the surface formed by all streamlines passing through a given closed curve in the flow [1]. Such a streamtube can show expansion, contraction and deformation of flow fields. These are useful in fluid mechanics, engineering and geophysics. This paper follows the streamtube construction in [2, 3] where a streamtube is created by connecting the circular cross flow sections along a streamline. The radius of a cross flow section is determined by the local cross flow expansion rate. The process of creating a streamtube is: (1) generating a streamline; (2) connecting the circular cross flow sections along the streamline.

The accuracy of created streamtube is dependent on both the accuracies of the streamline and the circular cross flow sections. Li [4] introduced an adaptive streamline tracking method for three-dimensional CFD velocity fields based on the law of mass conservation. This method suits to the CFD velocity fields that more data is available. The advantages of the method introduced in [4] include that the accuracy of the tracked streamlines for a given mesh can be controlled by the threshold number T, i.e., the larger the threshold number T, the more accurate the tracked streamlines are, and the tracking process is stopped when not enough data is provided. The overall accuracy of tracked streamline depends on the initial mesh and the threshold number T. When CFD velocity fields are given as numerical solutions of mathematical models, more data of the velocity fields are possible to calculate.

The streamtube construction method in this paper is based on the adaptive streamline tracking method in [4] and is for incompressible flows but the method can also be used to compressible steady flows by replacing the CFD velocity fields with CFD momentum fields.

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Nawin Raj is with the school of Computing, Information and Mathematical Sciences, the University of the South Pacific, Suva, Fiji Islands.

Zhenquan Li is with the school of Computing, Information and Mathematical Sciences, the University of the South Pacific, Suva, Fiji Islands (e-mail: li_z@usp.ac.fj).

II. Streamline Construction

Assume that a CFD velocity field is given in a hexahedral mesh and the further data of the velocity field is available. In this paper, we subdivide a hexahedron into five tetrahedra (refer Fig. 2 in [4]) when we apply the conditions given in [4] to the hexahedron.

Assume that $\mathbf{V}_l = \mathbf{AX}+\mathbf{B}$ is the linear interpolation of a given CFD velocity field at the four vertices in a tetrahedron, where

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

are constant matrix and vertical vector respectively. $\mathbf{V}_l$ is unique if the four vertices of the tetrahedron are not on a plane [5]. Assume that f is a scalar function and $f\,\mathbf{V}_l$ satisfies the law of mass conservation on the tetrahedron. We will find the expressions of f for different Jacobean forms of matrix $\mathbf{A}$ and then describe the conditions about when more data of velocity fields is needed in a hexahedron. The expressions of f for all possible cases of a non-conservative 3D linear field are listed in Table I in [4].

A. Conditions for Seeking More Data in a Hexahedron

The conditions (CSFD) for seeking more data of the velocity fields in a hexahedron are: for a hexahedron, subdividing it into five tetrahedra, and then calculating the Jacobean forms of $\mathbf{A}$ in the linear interpolation of values of the velocity field and the coordinates at the vertices of each of the five tetrahedra respectively, if there exist at least one of the five expressions of f corresponding to the particular Jacobean of $\mathbf{A}$ in Table I in [4] equaling zero or infinity, after taking C as non-zero constant, at some points on the corresponding tetrahedra, further data are needed to be found inside the hexahedron. The locations of the more data are the

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vertices of the eight smaller hexahedra by connecting the mid points of opposite sides on each of the six faces (refer Fig. 3 in [4]).

B. Algorithm

The algorithm for the adaptive streamline tracking method is as follows.

1. Set $i = 0$.
2. Find the hexahedron that contains the seed point and divide the hexahedron into five tetrahedra. If the CSFD conditions are not satisfied in all five tetrahedra, draw the streamline segment that goes through the seed point in the hexahedron; otherwise go to Step 3. Take the intersection of the streamline segment with the boundary of the hexahedron as seed point or end point and go to Step 2. (for a new hexahedron).
3. Subdivide hexahedron into eight equal smaller hexahedra and calculate the values of the velocity field at the vertices of the smaller hexahedra and then let $i = i + 1$ and go to step 4.
4. Take the elements (smaller hexahedra) in the subdivided hexahedron as new elements of the mesh by replacing the initial element and go to step 2 if $i \le T$; Otherwise go to Step 1.

T is the threshold number. The bigger the threshold number T (or the more times of subdivisions), the more accurate the tracked streamlines are.

The streamlines constructed by the method reviewed above are very accurate as shown by the examples in [4]. The examples in [4] include closed streamlines and streamlines with asymptotic plane. The accuracy is shown by comparing the tracked streamlines with the exact streamlines.

III. Radius of Streamtube

We will create a streamtube by generating a mass conservative streamline and by connecting the circular cross flow sections along the streamline [3]. We are able to track accurate streamlines followed the review in last section. This section derives the formulae for the radius of streamtube. The radius of a streamtube, *r*, is governed by the following ordinary differential equation

$$\frac{1}{r}\frac{dr}{dt} = \frac{1}{2}\nabla_T \cdot \mathbf{u} \tag{1}$$

where $\nabla_T \cdot \mathbf{u}$ is the local cross flow divergence and is defined as

$$\nabla_T \cdot \mathbf{u} = \nabla \cdot \mathbf{u} - \frac{du^*}{d\xi}$$

in which $\frac{du^*}{d\xi}$ represents the change of the velocity magnitude along the streamline.

For the velocity field given in Section 2,

$$\mathbf{u} = \begin{pmatrix} u \\ v \\ w \end{pmatrix} = \mathbf{AX} + \mathbf{B}$$

The magnitude of the velocity field

$$m = \|\mathbf{u}\| = \sqrt{u^2 + v^2 + w^2}$$

The change of velocity magnitude along the streamline

$$D_{\mathbf{u}} m = \frac{d}{ds}\sqrt{u(\mathbf{Y})^2 + v(\mathbf{Y})^2 + w(\mathbf{Y})^2}$$

where $\mathbf{Y} = (\mathbf{X} + s\mathbf{u})^T = (x + su, y + sv, z + sw)$. Further calculation for the change of velocity magnitude gives

$$\begin{aligned} D_{\mathbf{u}} m &= \frac{1}{m}\left[\left(\frac{\partial u}{\partial x}u + \frac{\partial u}{\partial y}v + \frac{\partial u}{\partial z}w\right)u + \right. \\ &\quad \left(\frac{\partial v}{\partial x}u + \frac{\partial v}{\partial y}v + \frac{\partial v}{\partial z}w\right)v + \\ &\quad \left.\left(\frac{\partial w}{\partial x}u + \frac{\partial w}{\partial y}v + \frac{\partial w}{\partial z}w\right)w\right] \\ &= \frac{1}{m}\left[(a_{11}u + a_{12}v + a_{13}w)u + \right. \\ &\quad (a_{21}u + a_{22}v + a_{23}w)v + \\ &\quad \left.(a_{31}u + a_{32}v + a_{33}w)w\right] \\ &= \frac{1}{m}\mathbf{u}^T\mathbf{A}\mathbf{u} \end{aligned}$$

Because the streamtube we will create is based on the mass conservative streamline constructed in [4], the streamline has been viewed as the lines generated by a piecewise linear vector field that satisfy the law of mass conservation subject to the preset tolerance, i.e., even though the linear vector field **u** does not satisfy the law of mass conservation at some points in the domain, $\nabla \cdot \mathbf{u}$ is small enough to be considered as zero. Thus Eq. (1) can be written as

$$\frac{1}{r}\frac{dr}{dt} = -\frac{1}{2}\frac{du^*}{d\xi} = -\frac{1}{2}D_{\mathbf{u}} m . \tag{2}$$

Take integral on both sides of (2) with respect to *t* gives

$$r = r_0 e^{-\frac{1}{2m}\mathbf{u}^T\mathbf{A}\mathbf{u}(t - t_0)} . \tag{3}$$

IV. Streamtube Construction

Theoretically the streamtube can be created by drawing circles with radius in (3) and centre at the points on the streamline constructed by the method in [4]. A physical construction is introduced in [3] by drawing cylinders with the two ends of circles at two instants with the radiuses in [3] and centres at the points on streamlines. These hexahedra may be the original mesh elements or smaller hexahedra that obtained by subdividing the larger ones. The figure shows not only the expansion of the flow at the points but also the process of subdivision of the original mesh.

V. Examples

The following two examples show the adaptively tracked streamline by the method in [4] and the streamtube constructed based on the streamlines. Some of the figures are not high quality due to the current facilitates available. The

tracked streamline in red shown in Fig. 1 is for the threshold numbers T=6.

Example 1 Saddle-spiral flow

$$\mathbf{V} = \left(xz - y, yz + x, -z^2 \right)$$

with seed point (-0.8, 0.8, 1).

Fig. 1 shows the streamtube in three dimensions. Fig. 2 shows the projection of the streamtube in Fig. 1 on yz-plane. Fig. 3 shows the projection of the streamtube in Fig. 1 on xy-plane. The variation of the expansion rate for the saddle-spiral flow in Example 1 is very small in the time period shown in the figures.

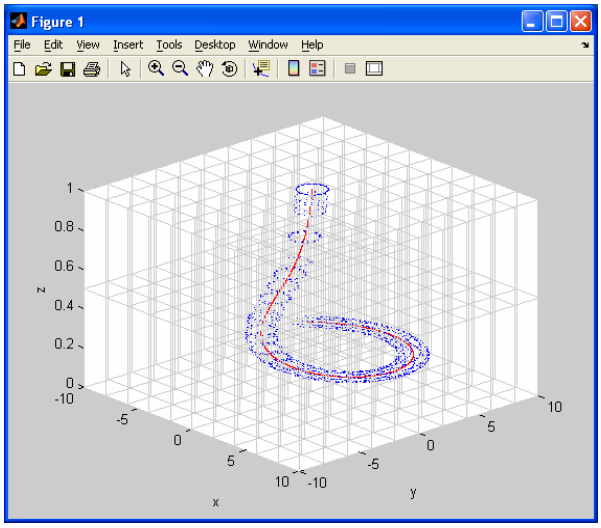


Fig. 1 Streamtube for Example 1 in 3D

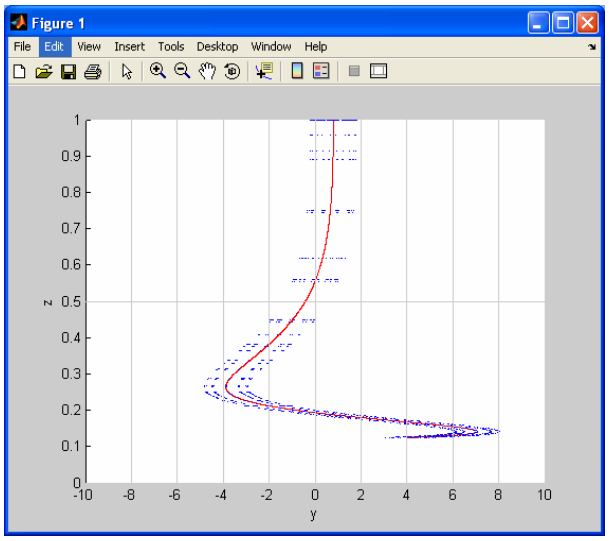


Fig. 2 Projection of the streamtube in Fig. 1 on yz-plane

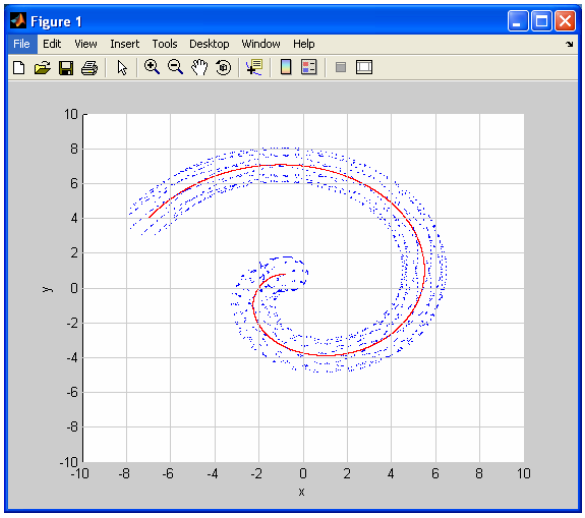


Fig. 3 Projection of the streamtube in Fig. 1 on xy-plane

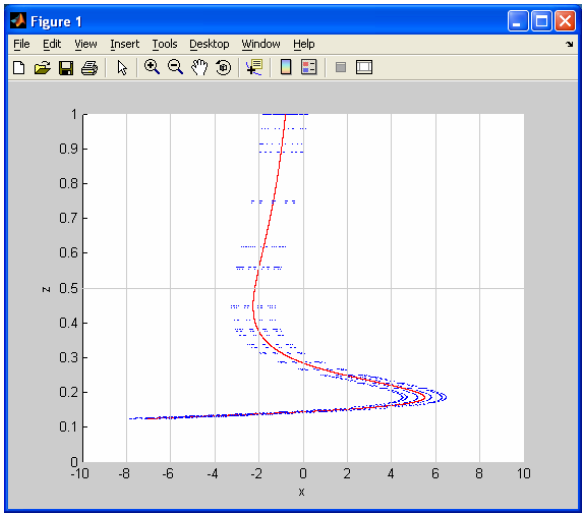


Fig. 4 Projection of the streamtube in Fig. 1 on xz-plane

Example 2 Toroidal flow velocity field

$$\mathbf{V} = \left(\frac{2x(z-1)}{r^2} - \frac{2y}{5r}, \frac{2y(z-1)}{r^2} + \frac{2x}{5r}, -\frac{2(r-9)}{r} \right)$$

with seed point (6.0006, 6.7076, 1), where $r = \sqrt{x^2 + y^2}$.

Fig. 5 shows the exact streamline in blue and tracked streamline in red used the method in [4]. This figure indicates that the tracked streamline is very accurate comparing with the exact streamline. When we drew the streamtube for this example, we were not allowed to draw circles like Example 1 due to the limit of the facilities available. We drew two dots instead of one circle.

Fig. 6 shows the streamtube in three-dimensions or more precisely, cross-section of the streamtube. The expansion rate for this example varies significantly from Fig. 6, and the projections of Fig. 6 on yz-plane in Fig. 7, on xy-plane in Fig. 8, and on xz-plane in Fig. 9.

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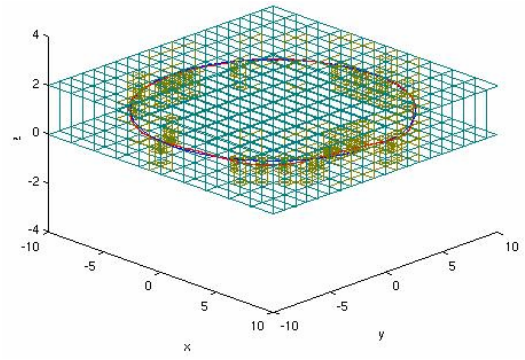


Fig. 5 Exact streamline in blue and tracked streamline in red for Example 2 in 3D.

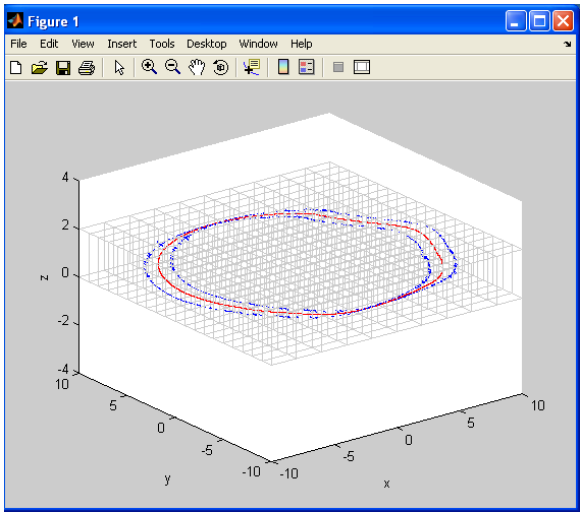


Fig. 6 Stream tube for Example 2 in 3D

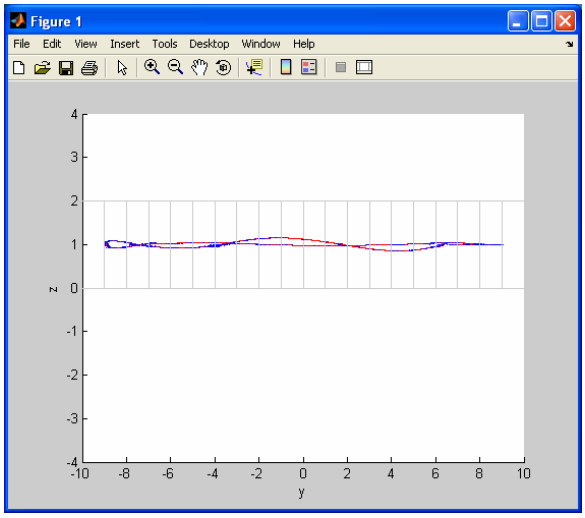


Fig. 7 Projection of the streamtube in Fig. 6 on yz-plane

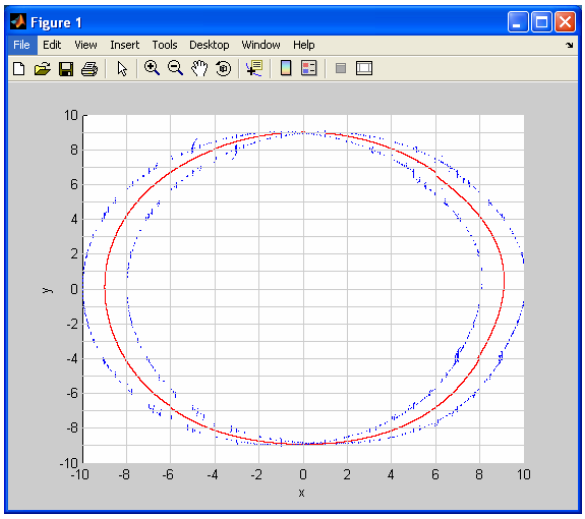


Fig. 8 Projection of the streamtube in Fig. 6 on xy-plane

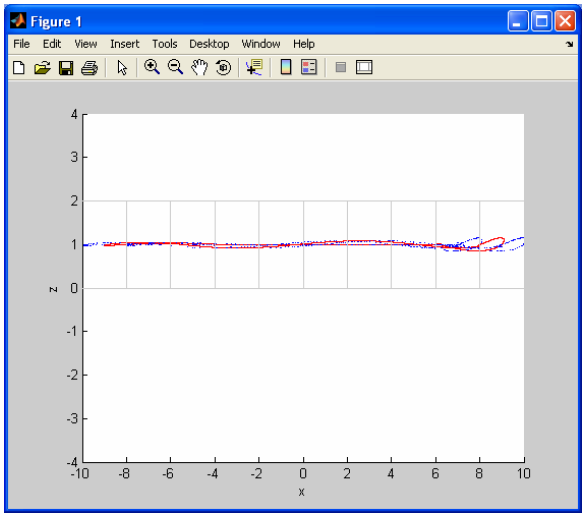


Fig. 9 Projection of the streamtube in Fig. 6 on xz-plane

VI. Discussion

This paper has introduced a streamtube construction by connecting the circular cross flow sections along a streamline. The streamtube created can show the local cross flow expansion rate. We still have some of following issues relating to the appearance of the streamtubes.

1. We used MATLAB in drawing the streamtubes. The streamtubes are not really tubes. We may need to write a function that draws a tube when the circles at the two ends are given.
2. The colors in the streamtubes may indicate the expansion rate.
3. Reduce the usage of the visual memory. For the computer we are using, the information of "low visual memory" was shown when we drew the figures.

These issues will be our future research topics.

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REFERENCES

[1] J.K. Kennard, and R.L. Street, *Elementary Fluid Mechanics*. John Wiley & Sons, 1975.

[2] D. Darmofal, and R. Haimes, "Visualization of 3-D Vector Fields: Variations on a Stream," in *Proceedings AIAA 30th Aerospace Science Meeting and Exibit*, Reno, Nev., Jan. 1992.

[3] S.K. Ueng, C. Sikorski, and K.L. Ma, "Efficient Streamline, Streamribbon, and Streamtube Constructions on Unstructed Grids," *IEEE Transaction on Visualization and Computer Graphics*, vol. 2, No., 2, pp. 100-109, 1996.

[4] Z. Li, "An Adaptive Streamline Tracking Method for Three-Dimensional CFD Velocity Fields Based on the Law of Mass Conservation," *Journal of Flow Visualization and Image Processing*, Vol. 13, No., 4, pp. 359-376, 2006.

[5] Z. Li, "A Mass Conservative Streamline Tracking Method for Three Dimensional CFD Velocity Fields", in *Proceedings of FEDSM'03*, Hawaii, 2003, FEDSM2003-45526, pp. 1-6.

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