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Nonlinear Torque Control for PMSM: A Lyapunov Technique Approach

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***Abstract*—** This study presents a novel means of designing a simple and effective torque controller for Permanent Magnet Synchronous Motor (PMSM). The overall stability of the system is shown using Lyapunov technique. The Lyapunov functions used contain a term penalizing the integral of the tracking error, enhancing the stability. The tracking error is shown to be globally uniformly bounded.

Simulation results are presented to show the effectiveness of the approach.

***Keywords*—** Integral action, Lyapunov Technique, Non Linear Control, Permanent Magnet Synchronous Motors, Torque Control, Stability.

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NOMENCLATURE

v_d, v_q	Stator *d*- and *q*- axes voltages.
i_d, i_q	Stator *d*- and *q*- axes currents.
ϕ_d, ϕ_q	Stator *d*- and *q*- axes flux linkages.
ϕ_m	flux created by rotor magnets.
R	stator resistance.
L_d, L_q	Stator *d*- and *q*- axes inductances.
T_e, T_L	electromagnetic and load torques.
J	moment of inertia.
B	viscous friction coefficient.
p	number of poles pairs.
ω_r	rotor speed in angular frequency.
ω	inverter frequency.

I. INTRODUCTION

DUE to its high torque to inertia ration, superior power density, high efficiency and many other advantages, the PMSM is the most widely used electrical motor in industrial applications.

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The most commonly used method of control for PMSM is field oriented control (FOC) [1]. The FOC represents the attempt to reproduce, for a PMSM, a dynamical behaviour similar to that of the dc machine, characterized by the fact that developed torque is proportional to the modulus of the stator current: to reach this objective, it is necessary to keep the rotor flux value constantly equal to the nominal value, so that, contemporarily, the optimal magnetic circuits exploitation guarantees the maximum power efficiency [2]. Regulation of currents i_d and i_q in closed loops leads indirectly to control of the motor developed torque according to the motor equation

$$T_e = \frac{3}{2} p\left[\phi_m i_q - \left(L_q - L_d\right) i_d i_q\right] \quad (1)$$

The conventional linear controllers such as PI, PID used with this structure are sensitive to plant parameter variation and load disturbance. The performance varies with operating conditions, ant it is also difficult to tune controller gain both on-line and off-line.

Fifteen years after the appearance of FOC, another technique to control the torque has been drawing increasing interest: the Direct Torque Control (DTC). The name of the DTC is derived from the fact that, on the basis of the errors between the reference and the estimated values of the torque and flux it is possible to directly control the inverter states in order to reduce the torque and flux errors within the prefixed band limits [3]. Current regulators followed by pulse width modulation (PWM) or hysteresis comparators and coordinate transformations to and from one of the rotating reference frames are not used in DTC systems. DTC controller is less sensible to the parameter detuning in comparison with FOC and he allows good torque control in steady state and transient operating conditions. Nevertheless DTC presents some drawbacks such as difficulty to control torque and flux at very low speed, high noise level at low speed, high current and torque ripple, and variable switching frequency behaviour [2].

The availability of high speed signal processing has stimulated increased interest in applying nonlinear control techniques to drives systems [4]. The mathematical model of an alternate current (ac) motor consists of coupled high-order nonlinear ordinary differential equations representing the dynamics of electrical and mechanical subsystems. Hence, a fully digitally controlled ac motor is a multi-input nonliear system where the inputs are the phase voltages and the outputs are the position, the velocity or the torque at the rotor shaft. Recent advances in modern control techniques suggest that the controllers for electrical motor should be designed directly from nonlinear models. Feedback linearization techniques have been

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extensively applied to the control of electrical machines. The essence of this technique is to first transform a nonlinear system into a linear on by a nonlinear feedback, and then uses the well-known linear design techniques to complete the controller design [4]. However, in many cases the feedback linearization method requires precise parameters plant and often cancels some useful non-linearities [5]. Non-linear sliding mode control has been applied to the control of synchronous motors [6]. This technique can offer many advantages, such as invariance condition, insensitivity or robustness, and fast dynamic response. Nevertheless, the sliding mode using switched controls produces chattering phenomena and torque perturbations.

Lyapunov theory has for a long time been an important tool in linear as well as nonlinear control. However, its use within nonlinear control has been hampered by the difficulties to find a Lyapunov function for a given system. If one can be found, the system is known to be stable, but the task of finding such a function has often been left to the imagination and experience of the designer.

The aim of this paper is the design of a control laws for a PMSM to achieve good torque control in steady state and transient operating conditions. The feedback system is globally asymptotically stable in the sense of the Lyapunov stability theory. Therefore we are interested in an extension of the Lyapunov function concept. This concept contains a term penalizing the integral of the tracking error, enhancing the stability.

The rest of this paper is organized as follows. In section II, we describe the PMSM model in a state space form suitable for Lyapunov-based control design. In section III, the nonlinear controller torque is derived. The stability of this controller is proven. Some illustrative simulation results are presented in section IV to validate the proposed controller and some concluding remarks are mentioned in the final section.

II. Modelling of PM Synchronous Motor

The mathematical model of a PMSM in a synchronous frame, or the so-called d-q frame, can be described as follows [1]:

$$v_d = Ri_d + \frac{d\phi_d}{dt} - \omega\phi_q \tag{2}$$

$$v_q = Ri_q + \frac{d\phi_q}{dt} + \omega\phi_d \tag{3}$$

$$\phi_d = L_d i_d + \phi_m \tag{4}$$

$$\phi_q = L_q i_q \tag{5}$$

The electric torque is

$$T_e = \frac{3}{2} p\left[\phi_m i_q - \left(L_q - L_d\right) i_d i_q\right] \tag{6}$$

And the equation for the motor dynamics is

$$T_e = T_L + B\omega_r + J\frac{d\omega_r}{dt} \tag{7}$$

The inverter frequency is related to the rotor speed as:

$$\omega = p\omega_r \tag{8}$$

Therefore the nonlinear state equations are given as

$$\frac{di_d}{dt} = \frac{v_d}{L_d} - \frac{R}{L_d} i_d + \omega\frac{L_q}{L_d} i_q \tag{9}$$

$$\frac{di_q}{dt} = \frac{v_q}{L_q} - \frac{R}{L_q} i_q - \omega\frac{L_d}{L_q} i_d - \omega\frac{\phi_m}{L_q} \tag{10}$$

$$\frac{d\omega_r}{dt} = \frac{3p\phi_m}{2J} i_q + \frac{3p}{2J}\left(L_q - L_d\right) i_d i_q - \frac{B}{J}\omega_r - \frac{1}{J} T_L \tag{11}$$

III. The Control Strategies and Stability Analyses

According to the electromagnetic torque of motor given in (6), it can be seen that the torque control can by achieved by regulation of currents i_d and i_q in closed loops. For surface magnets motors, the correspondence between torque and i_q is direct, while for interior magnets motors, this correspondence involves both i_d and i_q and is more complex. The proposed control system is designed to achieve torque-tracking objective. Voltages inputs are designed in order to guarantees the convergence of (i_d, i_d) to their desired trajectory (i_q^*, i_q^*). Since $i_d^* = 0$, the desired electromagnetic torque being directly proportional to the desired current i_q^*.

For the currents tracking objective, define the tracking errors as

$$e_d = i_d^* - i_d \tag{12}$$

$$e_q = i_q^* - i_q \tag{13}$$

And its dynamics derived from (9) and (10)

$$\frac{de_d}{dt} = -\frac{v_d}{L_d} + \frac{R}{L_d} i_d - \omega\frac{L_q}{L_d} i_q \tag{14}$$

$$\frac{de_q}{dt} = -\frac{v_q}{L_q} + \frac{R}{L_q} i_q + \omega\frac{L_d}{L_q} i_d + \omega\frac{\phi_m}{L_q} \tag{15}$$

In order to ensure the convergence of the tracking errors to zero, the Lyapunov function is chosen as:

$$V = \frac{1}{2} K_1 \vartheta_d^2 + \frac{1}{2} e_d^2 + \frac{1}{2} K_2 \vartheta_q^2 + \frac{1}{2} e_q^2 \tag{16}$$

Where K_1 and K_2 are positive constant

$$\vartheta_d = \int_0^t e_d(\tau) d(\tau) \tag{17}$$

is the integral of the d axe current tracking error, and

$$\vartheta_q = \int_0^t e_q(\tau) d(\tau) \tag{18}$$

is the integral of the d axe current tracking error.

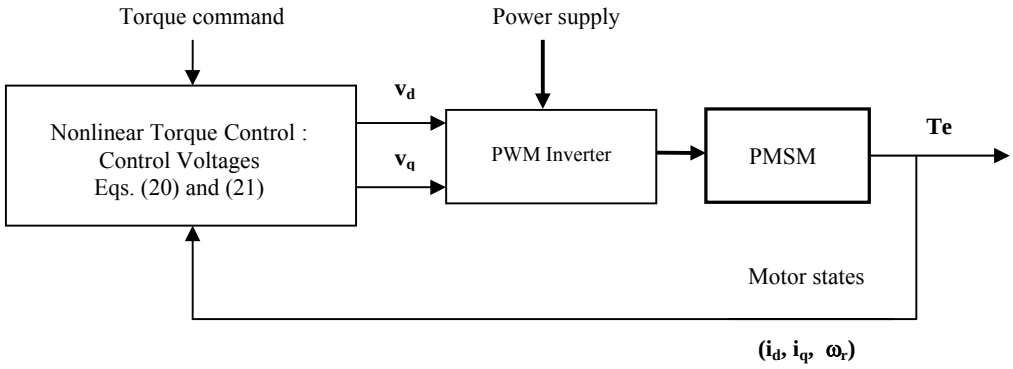


Fig. 1 A bloc diagram of Proposed Nonlinear Control system

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By integrating this integral actions into the Lyapunov function, we ensure the convergence of the tracking error to zero despite the presence of the disturbance and model uncertainty in the system.

Using (14) and (15), the derivative of the Lyapunov function (16) is computed as

$$\dot{V} = e_d\left(K_1\vartheta_d - \frac{v_d}{L_d} + \frac{R}{L_d}i_d - \omega\frac{L_q}{L_d}i_q\right) + e_q\left(K_2\vartheta_q - \frac{v_q}{L_q} + \frac{R}{L_q}i_q + \omega\frac{L_d}{L_q}i_d + \omega\frac{\phi_m}{L_q}\right) \quad (19)$$

To guarantee the global asymptotic stability in the current loop, the d-q axes control voltages are synthesized as

$$v_d = K_d L_d e_d + K_1 L_d \vartheta_d + Ri_d - \omega L_q i_q \quad (20)$$

$$v_q = K_q L_q e_q + K_1 L_q \vartheta_d + Ri_q + \omega L_d i_d + \omega\phi_m \quad (21)$$

Where K_d and K_q are a positive constant feedback gains.

Substituting (20) and (21) into (19) the derivative of the Lyapunov function becomes

$$\dot{V} = -K_d e_d^2 - K_q e_q^2 \le 0 \quad (22)$$

Define the following equation

$$W(t) = K_d e_d^2 + K_q e_q^2 \ge 0 \quad (23)$$

Furthermore, by using LaSalle Yoshizawa's theorem [7], its can be shown that $W(t)$ tend to zero as $t \to \infty$.

Therefore, e_d and e_q will converge to zero as.

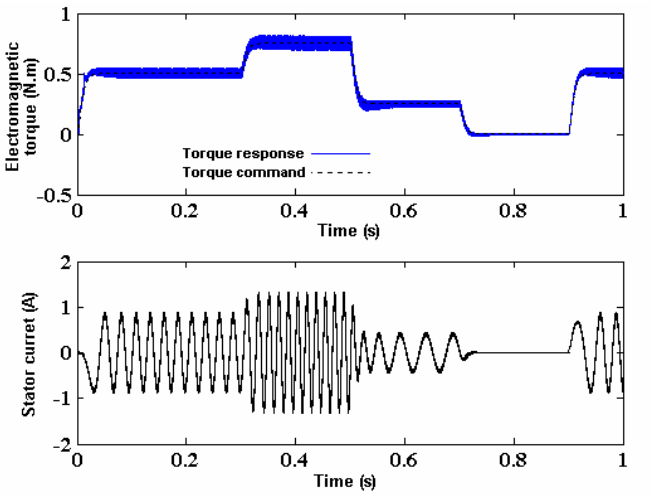


Fig.2 Tracking performance control of the drive system with nominal parameters

VI. Simulation Results

To show the validity of the mathematical analysis and, hence, to investigate the performance of the proposed nonlinear control scheme, Simulations works are carried out for the drive system.

The overall block diagram for proposed control scheme is shown in Fig.1. The overall system consists of a non-linear speed tracking controller, a PWM inverter, and a PM synchronous motor. Digital simulations have been carried out using MATLAB/SIMULINK. Table I shows the parameter values used in the ensuing simulation.

In Fig.2 the simulation results of proposed controller for PMSM are presented. It is shown electromagnetic torque, and stator current, respectively. We can see that the proposed controller can quickly and accurately tracks the desired reference.

Fig.3 shows the bidirectional performance tracking control. The torque command changed from 0.5 to – 0.5 N.m is given. It is obvious that with the derived control high and accuracy can be achieved for the nominal parameters.

In the next simulation, the influence on the servo-drive performance under parameter perturbations was investigated.

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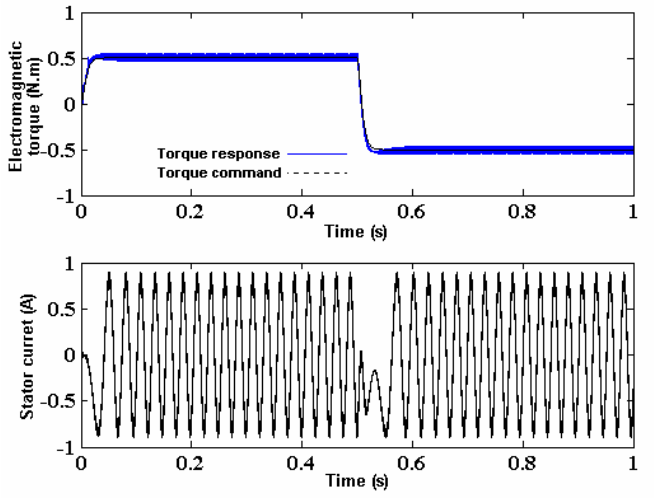


Fig.3 Bidirectional tracking performance control of the drive system with nominal parameters

The parameter perturbations introduced in the control were set to the following values: 30% in the stator resistance, 20% in the d and q axes stator inductances, 20% in the rotor flux linkage, and 400% in the mechanical inertia. The drive system is started at a constant load of 0.5N.m. A step load torque of $1.5T_0$ is applied at t = 0.3 s and removed at t = 0.6 s. The results corresponding are shown in Fig.4. We can see clearly that stable operation and good performances are preserved against the inaccuracy in the modelling of the parameter motor drive and their variations. In the same figure, motor torque quickly converges to the reference and recovers very will from the load disturbance. The proposed control scheme is virtually unaffected by these variations

From the results presented earlier, it is quite evident that the proposed technique gives good and high performances both in the torque tracking and regulation. The integral action have effectively enhancing the stability and eliminate the current tracking errors. It also reduces the effect of modelling inaccuracy or changes of the load parameters, including inertia and load disturbance.

V. Conclusion

In this study the torque control problem of a PMSM was addressed. The nonlinear behavior of the system limits the performance of classical linear controllers used for this purpose. This paper has successfully demonstrated the design, stability analysis, and simulation of a Lyapunov technique approach for the torque control system of the PMSM drive.

The feedback system is globally asymptotically stable in the sense of Lyapunov method. The transient state and steady state performances of the Lyapunov-based controller are enhanced via the introduction of an integral action in the control Lyapunov fonction. Consequently, high performance dynamics despite the presence of parameter uncertainties or disturbances are obtained. Some simulation results have been provided to verify the effectiveness of the proposed controller.

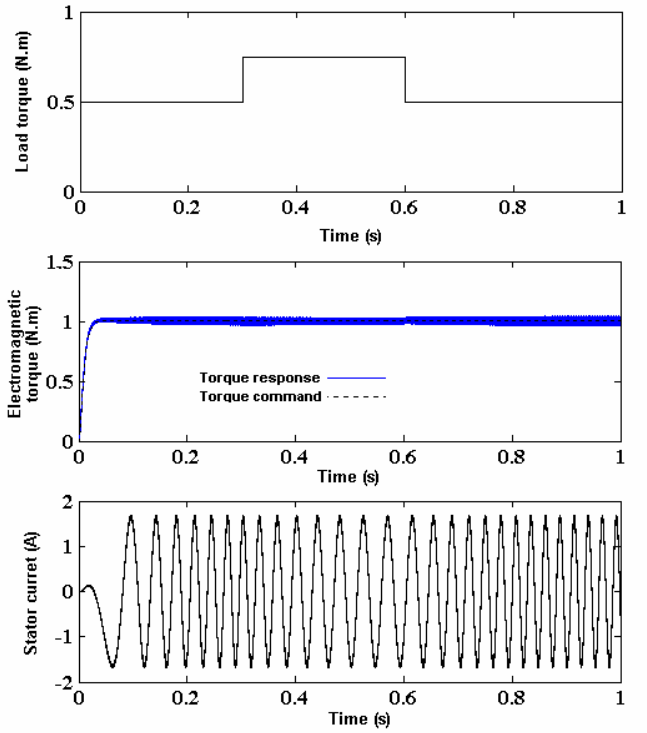


Fig.4 Simulation results of the proposed control scheme under load variation with perturbed parameters: +20% L_{d0} , +20% L_{q0} , +30% R_0 , and +400 J_0

TABLE I
PARAMETER VALUES OF PMSM USED IN SIMULATION

Number of pole pairs *P*	**2**
Armature resistance *R*	**3Ω**
Magnet flux linkage ϕ_m	**0.167*Web***
d-axis inductance L_d	**7*mΩ***
q-axis inductance L_q	**7*mΩ***
Moment inertia J	**0.134 10^{-3} *kgm²***
Rated power	**500*w***

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