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Interpolation of Geofield Parameters

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***Abstract*—**Various methods of geofield parameters restoration (by algebraic polynoms; filters; rational fractions; interpolation splines; geostatistical methods – kriging; search methods of nearest points – inverse distance, minimum curvature, local – polynomial interpolation; neural networks) have been analyzed and some possible mistakes arising during geofield surface modeling have been presented.

***Keywords*—** interpolation methods, geofield parameters, neural networks.

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I. Introduction

FOR many problems in sciences on Earth (geodesy, geology, geophysics, cartography, photogrammetry, etc.) the problem of modeling the geofields surface (height, depth, pressure, temperature, pollution factor, etc.), that is usually displayed on maps by means of isolines, is urgent. If represertation of geofields surface is possible as the function of two variables h=f (x, y), which has values $\hat{h}_i$ at (x_i, y_i), ($i = \overline{1,n}$) peaks, the digital model of this function is necessary for computer processing and storage.

We are going to consider the digital model of geofield (DMG) as a set of digital values of continuous objects in cartography (e.g. height of a relief) for which their spatial coordinates and the mean of structural description are specified. It will allow calculating the values of geofield in the given area. The important part of any DMG is the method of interpolating of its surface. For this, various ways of interpolation yield various results which can be estimated only from the point of view of practical applications. [1 - 12].

Since observations practically always have discrete character, so constructing maps is connected with necessity of solving geofield parameters continuous restoration task.

At the same time the task of geofield parameters restoration is far from being simple. So, if networks of supervision are essentially irregular, then it is necessary to obtain optimum decision. Therefore there was an expediency of creation of the automated databanks of various (geofield restoration) methods and their parameters mapping [2].

At present, more than ten methods of surface interpolation are known. They are the following; algebraic and orthogonal polynoms, filters, rational fractions; in some cases they take functions satisfying some apriori given conditions (e.g. positivity of f (x, y)) values; multi squardrik function, at which approximation is reached by means of square functions (squardrik), representing hyperboles; splines; geostatic methods (kriging), search methods of nearest points (inverse distance, minimum curvature, local – polynomial interpolation) etc. However, none of them is completely universal [1- 12].

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II. Problem Formulation and Solution

The use of statistical probability methods, such as the least-squares method, requires preliminary analysis of the data for normality of the sample distribution. A normality check assumes that the following four conditions are satisfied.

1. The intervals $\bar{x} \pm \sigma, \bar{x} \pm 2\sigma x$ and $\bar{x} \pm 3\sigma$ must contain 68, 95, and 100%, respectively, of the sample values $\bar{x}$ is the mean and o is the standard deviation).
2. The coefficient of variation *V* must not exceed 33%.
3. The kurtosis E_x and the asymmetry coefficient S_k must be close to zero.
4. $x \approx M$. where *M* is the sample median.

The analysis data [5], are used to model (1), showed that distribution has contradicted the normality assumption (Table 1).

It must be noted that in the early stage modeling of geofield, the data are not only limited and uncertain. It is therefore necessary to identify the parameters of a mathematical model of a multivariate object described by the regression equation.

$$H_m = \sum_{j=0}^{m} \sum_{k=0}^{m-j} c_{jk} x^j y^k \quad , (j = \overline{0,m}, k = \overline{0,m}, j+k \le m) \qquad (1)$$

We shall determine the fuzzy values of the parameters of equation (1) using experimentally statistical data of the process, i.e., the input x , y and output H coordinates of the model. Let us consider a solution of this problem using neural networks [13, 14].

Neural Network: A neural network consists of interconnected sets of neurons. When a neural network is used to solve equation (1), the input signals of the network are the values of the variable $B=(x , y)$, and the output is H. The values of the parameters are the network parameters. Neural-

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network training is the principal task in solving the problem of identification of the parameters of equation (1). We assume the presence of experimentally obtained fuzzy statistical data (see Table 1). From the input and output data, we compose training pairs for the network (B,T). To construct a model of a process, the input signals B are fed to the neural network input (Fig.1); the output signals are compared with standard output signals T.

After comparison, the deviation is calculated:

$$E = \frac{1}{2}\sum_{i=1}^{l}(H_i - T_i)^2$$

Training (correction) of the network parameters is concluded when the deviations E for all training pairs are less than the specified value (Fig.2). Otherwise, it is continued until *E* is minimized.

The network parameters for the left and right parts are corrected as follows:

$$c_{jk}^{n} = c_{jk}^{o} + \gamma\frac{\partial E}{\partial c_{jk}} \qquad (2)$$

Here c_{jk}^{o} and c_{jk}^{n} are the old and new values of the neural network parameters c_{jk}, and γ is the training rate.

Numerical Example

Let us consider the mathematical model is described by the regression equation (consider a special case (1) at m=2):

$$H=c_{00}+c_{10}x+c_{01}y+c_{20}x^2+c_{11}xy+c_{02}y^2. \qquad (3)$$

We shall construct a neural structure for solution of (1) in which the network parameters are the coefficients: $c_{00},c_{10},c_{01},c_{20},c_{11},c_{02}$. The structure has four inputs and one output (Fig. 3). Using a neural-network structure, we employ (2) to train the network parameters.

$$\frac{\partial E}{\partial c_{10}} = \sum_{i=1}^{l}(h_i - t_i)x \;; \qquad \frac{\partial E}{\partial c_{20}} = \sum_{i=1}^{l}(h_i - t_i)x^2$$

$$\frac{\partial E}{\partial c_{00}} = \sum_{i=1}^{l}(h_i - t_i); \qquad \frac{\partial E}{\partial c_{11}} = \sum_{i=1}^{l}(h_i - t_i)x\ y; \quad (4)$$

$$\frac{\partial E}{\partial c_{01}} = \sum_{i=1}^{l}(h_i - t_i)y \;; \qquad \frac{\partial E}{\partial c_{02}} = \sum_{i=1}^{l}(h_i - t_i)y^2$$

As a result of training (2) and (4), we find network parameters that satisfy the knowledge base with the required training quality. Statistical data were collected from experiments before the computer simulation was performed.

The network parameters were thus trained using the described neural network structure and experimental data. As a result, network-parameter values that satisfied the experimental statistical data were found:

C_{00}=1.42; C_{10}=2.11; C_{01}= -2.53; C_{20}= -1.10; C_{11}= -0.87; C_{02}=1.31 (5)

The coefficients were $c_{00},c_{10},c_{01},c_{20},c_{11},c_{02}$, the regression equation (5) was evaluated by a program written in Turbo Pascal on an IBM PC.

Let us compare obtained results (5) with results in [5]:

$\hat{C}_{00}$=1.45; $\hat{C}_{10}$=2.26; $\hat{C}_{01}$= -2.55; $\hat{C}_{20}$= -1.28; $\hat{C}_{11}$= -0.85; $\hat{C}_{02}$=1.33 (6)

Let us also note that application of neural nets in solving problems connected with identification of parameters of mathematic models of different by their nature geofields has active advantages if compared of traditional probability – statistical approaches. First of all, it is connected with the fact that offered methods can be used irrespective of the kind geofield parameters selective distribution.

The application of neural networks to solving problems that involve evaluation parameters of mathematical models of geofields has advantages over traditional statistical-probability approaches. Primary is the fact that the proposed procedure can be used regardless of the type of distribution of the geofield parameters. The more so, in the early stage of modeling, it is difficult to establish the type of parameter distribution due to insufficient data.

III. Conclusions

Taking into account the existence of various methods of geofields surfaces interpolation methods, and also packages of applied interpolation programs that allow, not possessing deep knowledge in interpolation theory and programming, to make necessary calculation for geofield parameters restoration, correct choice of the method and its governing parameters is one of the main stages in preparation for surfaces modelling and geofields parameters mapping.

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TABLE 1 ANALYSIS OF THE DATA FOR NORMALITY OF THE SAMPLE DISTRIBUTION

$\bar{x} \approx M$	68%	95%	100%	V<33%	$E_x \to 0$	$S_k \to 0$
0.71≠0.59 non – exe – cution	77.7% execu - tion	91.6 % non – exe – cution	100% execu - tion	47 % non – exe – cution	0.45 non – exe – cution	1.14 non – exe - cution

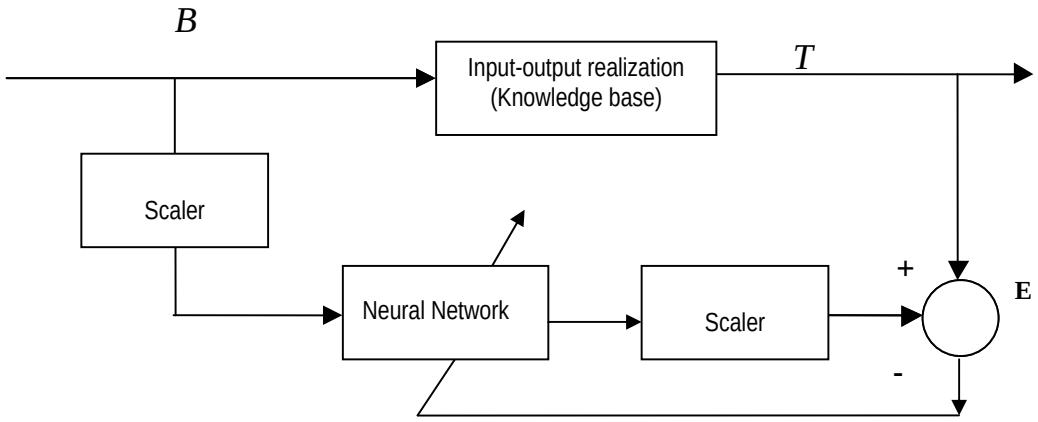


Fig. 1. Neural identification system.

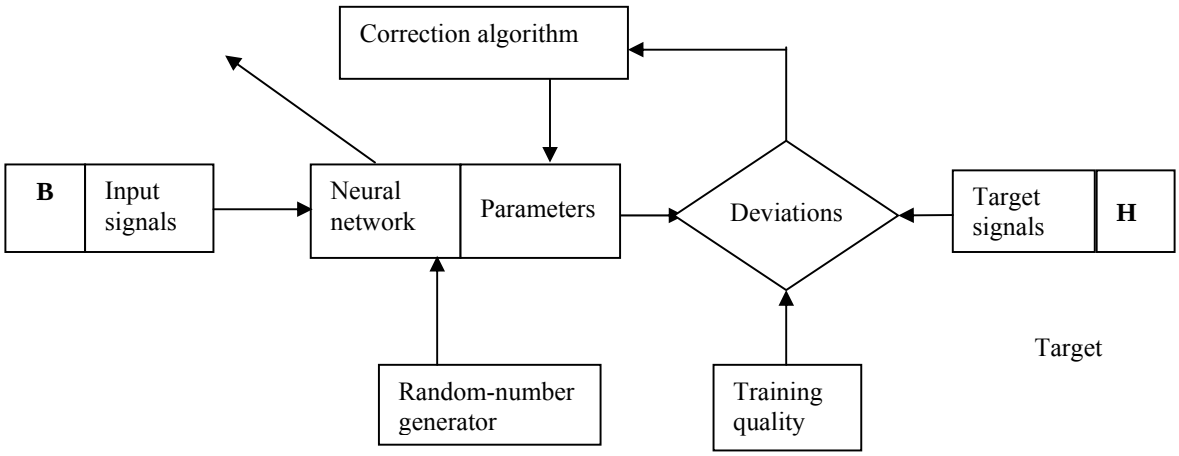


Fig. 2. System for network-parameter training (with backpropagation).

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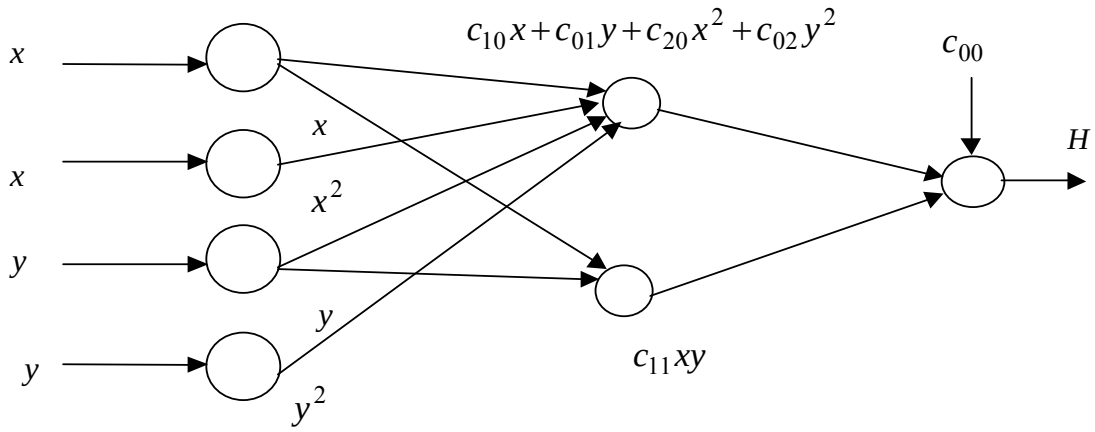


Fig. 3. Structure of neural network for second-order regression equation.

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