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The Influence of Beta Shape Parameters in Project Planning

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***Abstract*—**Networks can be utilized to represent project planning problems, using nodes for activities and arcs to indicate precedence relationship between them. For fixed activity duration, a simple algorithm calculates the amount of time required to complete a project, followed by the activities that comprise the critical path. Program Evaluation and Review Technique (PERT) generalizes the above model by incorporating uncertainty, allowing activity durations to be random variables, producing nevertheless a relatively crude solution in planning problems. In this paper, based on the findings of the relevant literature, which strongly suggests that a Beta distribution can be employed to model earthmoving activities, we utilize Monte Carlo simulation, to estimate the project completion time distribution and measure the influence of skewness, an element inherent in activities of modern technical projects. We also extract the activity criticality index, with an ultimate goal to produce more accurate planning estimations.

***Keywords*—**Beta distribution, PERT, Monte Carlo Simulation, skewness, project completion time distribution.

I. Introduction

SUBSTANTIAL research has been carried out to develop methodologies to estimate the completion time distribution of a project. Methodologies can be roughly categorized into three distinct categories: deterministic analysis, analytical delimitation and simulation approaches. The latter ones provide a powerful framework enabling the elicitation of the required statistical measurements for any technical project. Reference [18] was the first to exploit Monte Carlo methods to analyze the time allocation of a project and then set a criticality index for each activity.

The estimation of the duration of a project (and consequently of its total cost) is a very important and challenging sector of technical project management, happening during the planning and commissioning contracting phases. In these cases, correct estimates allow project managers to minimize potential losses. In summary, a project's risk analysis has a major impact on the investment decisions [15].

The paper analyzes at first the features of nodal networks for technical project management. The description of the classic PERT and the Monte Carlo simulation methods are following. Finally, the reasons of the rejection of Markov-PERT methods for stochastic network analysis are demonstrated.

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II. Networks and PERT

Many scheduling problems can be represented as networks [12]. The original PERT $G=(V,A)$ network is a directed acyclic graph, where $V=\{v_1,v_2,\ldots,v_N\}$ is the set of nodes, constituting the activities of the project and A is the set of arcs, representing the priorities/limitations.

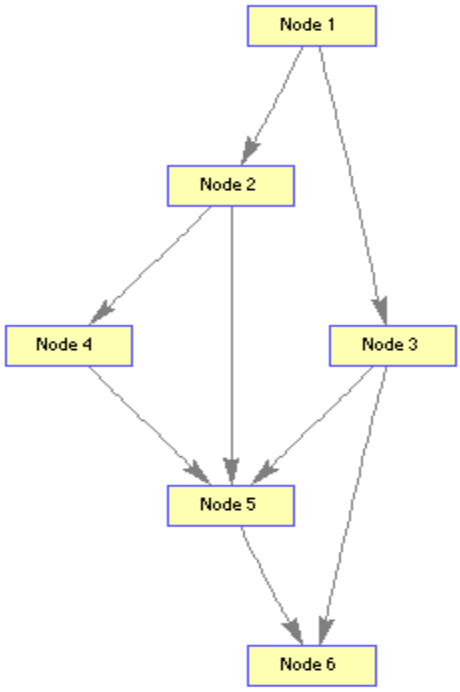


Fig. 1 An activity on node (AoN) network of six activities

The existing relationships between the activities are more complex. Finally, we observe cases where the initiation of an activity depends on percentage completion of another.

The adjacency list of a large network graph is more efficient due to its lower complexity comparing to the adjacency (connectivity) matrix. In this work, however, the adjacency matrix was employed to construct the PERT network and served as input when creating the list of all n paths from source to sink nodes.

The number of paths increases exponentially when nodes are added to the network. Thus, finding all possible paths that include both the source and the sink nodes is generally infeasible. In fact, the aforementioned problem is among the most famous ones in the scientific field of computational

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graph theory.

PERT, the first attempt to incorporate uncertainty into project networks, is a popular method with proven value when managing complex projects [14], [10], considered Beta (B) distribution as an adequate candidate to model the duration of an activity in the arbitrary interval $[c,d]$. Its probability density function is defined by

$$B(x;\alpha,\beta,c,d)=\frac{(x-c)^{\alpha-1}(d-x)^{\beta-1}}{B(\alpha,\beta)(d-c)^{\alpha+\beta-1}} \quad (1)$$

where $\alpha,\beta>0,\ a\le x\le b$. PERT mandates that the mean μ_x and the variance σ_x^2 of the duration x of an activity can be estimated by the formulas:

$$\mu_x=(c+4m+d)/6 \quad (2)$$

$$\sigma_x^2=(d-c)^2/36 \quad (3)$$

where c is the optimistic, m is the most likely, and d is the pessimistic duration of an activity.

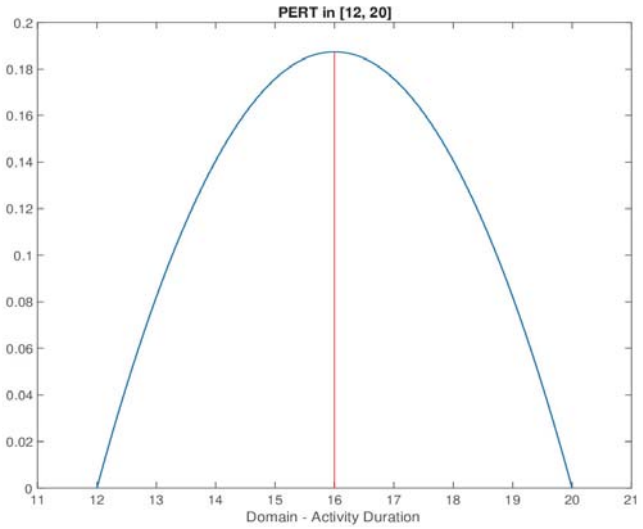


Fig. 2 Graphical representation of classical PERT for $c=12,\ m=16, d=20$

By the Central Limit Theorem, the distribution function of the duration of a project follows a normal distribution. Thus, if $X\in N(\mu,\sigma^2)$, the formula used to calculate the probability of completing a project in the time interval $[c,d]$ is:

$$P(c\le X\le d)=P\left(\frac{c-\mu}{\sigma}\le Z\le\frac{d-\mu}{\sigma}\right)=\Phi\left(\frac{d-\mu}{\sigma}\right)-\Phi\left(\frac{c-\mu}{\sigma}\right) \quad (4)$$

Despite the numerous efforts to defend this original approach [6], [15], [16] criticism was relentless and centered on PERT's estimated values. The claim was that formulas (2) and (3) did not originate directly from formula (1) of the generalized Beta distribution, implying thus lack of a sound theoretical basis, leading research to the 1959-1987 period where the main goal was to extract the relationship between the aforementioned formulas [14].

From 1987 and onwards, several researchers attempted either to modify the original PERT [7], [4], [5], or propose other approaches which estimated the activity time and consequently the project. Reference [9] reported that PERT's success is based on the process of assessing the timescales and not its estimated values. Based on the above, it is more than clear that the original PERT is not an "attractive" piece of research.

III. Monte Carlos Simulation

Monte Carlo simulation is a process in which random numbers are generated according to the probability distributions that are assumed to be associated with the source of uncertainty, such as the durations of the activities that constitute a project. The simulation takes into account potentially, all possible situations that may actually occur and estimates the probability that an event could become reality.

In stochastic PERT networks, it is assumed that random variables describing the duration of activities are connected via nodes. If the duration of each activity is considered to be linked to a probability distribution, then the problem of estimating the completion time of a project through simulation appears directly [12]. This approach is mostly suitable to cases where the probability of achieving a time goal and therefore a financial one is a prerequisite [19]. Therefore, given that this approach outperforms PERT [17], it had to be in the center of our research. We also aim to demonstrate that simulation techniques, in combination with other tools should be a first-line option when our goal is to get good estimates.

Probability density functions and random numbers are the important concepts in the Monte Carlo simulation. They define how likely an event can become reality by "attaching" a probability mass to an interval. The experiments are then conducted by randomly sampling from the probability density functions incorporated in the model. The result is thus a distribution.

A critical element of the Monte Carlo simulation is the choice of the number of iterations. The proper selection of the number of iterations balances between the numerical accuracy of the results and the reduction of the computational cost and the convergence. The convergence of the results is based on the Law of Large Numbers.

A. Probabilistic Modelling of the Activities

Simulation of technical projects duration, should usually firstly assess the underlying distribution of the random process by using a probability distribution. Typical choices involve gaussian, B, triangular and other. Although in many cases the underlying probabilistic properties of the project remain unknown to the researcher, they are usually deduced by the available data on similar procedures, or are elicited from professionals, experienced on the nature of the included activities. References [18], [1] showed through the creation of a $\beta_1-\beta_2$ plane (skewness – kurtosis) and its subsequent

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transformation to Θ_1-Θ_2, that earthmoving activities, an activity widely used in a variety of technical projects, can be modelled by the B distribution, a fact which provides the distribution with a particularly practical power, which is beyond the usual practice that stipulates its use because of its exceptional flexibility.

In this paper, we confine ourselves to the simple assumption that the time duration of the activities is independent and identically distributed (i.i.d) random variables coming from the B distribution.

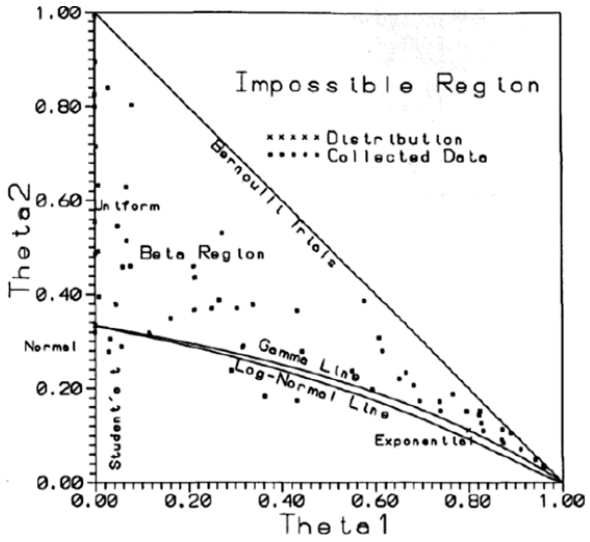


Fig. 3 Level Θ_1 - Θ_2 with known sampling distributions

B. Implementation of the Monte Carlo Simulation in PERT Networks

In deterministic activity-time projects, the total duration of the project is the length of the most time-consuming path in the network. We take advantage of this property, but our approach models the durations of activities as random variables derived from the generalized B distribution. The calculation of the duration of the project is achieved through the summation of the activity times on the critical path [11]. The time to complete all activities on the n^{th} path is a random variable X_n, [17]

$$X_n = \sum_{i \in P_n} T_i \tag{5}$$

where T_i is a scalar random variable. The network represents the priority relations between activities so that the length of the largest (critical) path through the network is the project's completion time, also described by the objective of finding the cumulative distribution function (CDF) of the random variable of the critical pathway, expressed by [17]:

$$T_{\text{project}} = \max X_n, \quad \text{where} \quad n = \{1, 2, \ldots, m\} \tag{6}$$

while the variance of this time, by the i.i.d. property, equals the sum of the variances of the duration of the critical activities and is given by the relation:

$$\sigma_T^2 = \sum_{i \in T} \sigma_i^2 \tag{7}$$

A realization of a network occurs when duration values have been assigned through sampling, for each of its activities [18]. This realization can be divided into three steps:

- Input of the nodal project network.
- Determination of the duration of each activity through sampling from the corresponding probability distribution function (B distribution).
- Calculation of the total project duration.

The application of the Monte Carlo technique to a PERT network is in plain words the combination of sampling with the use of the most time-consuming path algorithm for a large set of realizations.

C. Activity Criticality

By the term activity critical index, we define the probability of an activity to belong to the critical path, an extremely important index which points to the amount of attention that the project manager must pay to that particular activity [18]. The reason is simple and lies in the fact that the critical path is not unique in the stochastic analysis of the project as the completion time of the activity changes, which is another significant weakness of the original PERT.

D. Analysis of the Results

The simulation algorithm outputs the sample mean, standard deviation, minimum value, maximum value, median and percentiles of the time duration distribution, visualizes the results in a histogram, paired with the empirical cumulative distribution and finally the probability of an activity belonging to the critical path.

The first statistical parameter to be considered is the mean duration of the project. The calculation will be through:

$$\hat{\mu} = \frac{\sum_{i \in N} T_i}{N} \tag{8}$$

where N is the number of realizations of critical paths. In practice, for thousands of iterations we can be sure that the mean value of the sample converges to the real mean, due to the Law of Large Numbers which states that $E[X] \to \mu$, when $N \to \infty$. The formula

$$\sigma = \sqrt{\frac{\sum_{i \in N} \left(x_i - \hat{\mu}\right)^2}{N-1}} \tag{9}$$

gives the standard deviation of the sample and the formula

$$Var[T] = \sigma^2 \tag{10}$$

gives the sample variation.

$$\sqrt{\beta_1} = \frac{\sum_{i=1}^{N}\left(\frac{X_i - \hat{\mu}}{\sigma_x}\right)^3}{N} \tag{11}$$

$$\beta_2 = \frac{\sum_{i=1}^{N}\left(\frac{X_i - \hat{\mu}}{\sigma_x}\right)^4}{N} \tag{12}$$

provide the skewness and the kurtosis of the empirical distribution.

IV. PERT and Markov Chains

In all attempts to model PERT networks with Markov chains [3], [2], [8] the decisive assumption is that the duration of the activities are random variables modelled either by an exponential distribution with $\mu > 0$ and thus $1/\mu < \infty$ or a first class Erlang distribution [2] which is again a "masked" exponential distribution. This is happening because in state 0, the Markov chains of continuous time, one waits for an exponentially distributed time with parameter $\lambda \in (0, \infty)$ and then one is transferred to state 1. Therefore, the probability density function of the waiting time is given by $f_T(t) = \lambda e^{-\lambda t}$ for $t \geq 0$. This example is generalized for N states and is called Poisson process of rate λ [13]. Unfortunately, this allocation does not show any practical basis for modelling technical projects and consequently it is extremely important to look for different modeling methods including distributions having some practical advantages [20].

V. Implementation

The algorithmic implementation of the selected method is presented. The components of the algorithm are described, such as the selected PERT networks, the simulation scenarios are formulated, the algorithm is explained, and the results are presented. The computational complexity of the algorithm is discussed, and the required times are extensively presented.

A. Description of the Simulation Algorithm

Three nodal PERT networks were selected arbitrarily, the characteristics of which are summarized in the Table I.

TABLE I
PERT Node Network Properties

	Small	Medium	Large
Number of nodes	6	12	14
Number of edges	8	15	33
Density	0.222	0.104	0.168

Then four sets of four scenarios with the following key features were created through the input function.

Set 1. Each scenario has common estimated $[c,d]$ times, but combinations (α, β) change in each scenario.

Set 2. Each scenario has common estimated $[c,d]$ high fluctuation times but combinations (α, β) change in each scenario.

Set 3. Each scenario has common estimated times $[c,d]$ but combinations (α, β) change in each activity and in each scenario.

Set 4. Each scenario has common estimated $[c,d]$ high fluctuation times but combinations (α, β) change in each activity and in each scenario.

They are presented in detail in Tables IX-XII of the Annex. It is easy to see how combinations of the parameters (α, β) were chosen having in mind the qualitative distinction of skewness. Activities that usually occur in earthmoving technical projects were grouped by specifying sets of shape parameters, introduced mainly by domain experts.

Then, upon a user's instruction, the algorithm reads the selected number of files and stores them in a structure to ensure easy handling. Then, the number of iterations of the simulation is arbitrarily selected. In this research, 100, 10000 and 25000 iterations are performed to highlight the importance of an obvious sensible choice. The paths that contain both the source and the sink node are extracted and stored. For each selected scenario, the simulation begins by initially sampling the duration of each activity. Then the critical path is extracted and thus the duration of one realization is estimated, leading eventually to the project duration distribution. Criticality indices are calculated simultaneously.

B. Simulation Results

The stored simulation data are the input to the visualization function. The visualization includes histograms and the creation of the S-curve. The results are presented in two pairs, initially sets 1 and 3 and then 2 and 4.

The difference in the mean value is not significant inside the first set (close to 1%). The statistical parameter that appears to change substantially when increasing the number of samples (especially after rapid increase) is the standard deviation, whereas in the interval between 10,000 and 25,000 iterations it converges to a certain value. Therefore, it would be more appropriate to have a representation of the variance of the standard deviation for the interval between 100 and 25,000 iterations. This also indicates that the factor determining the required number of iterations is the standard deviation. Overall, the results seem to converge well between 10,000 and 25,000 iterations, with differences close to 1%.

The percentage is extracted with the formula

$$p = \frac{x - x_{ref}}{x_{ref}} \times 100\% \tag{13}$$

where x_{ref} is the time of classical PERT.

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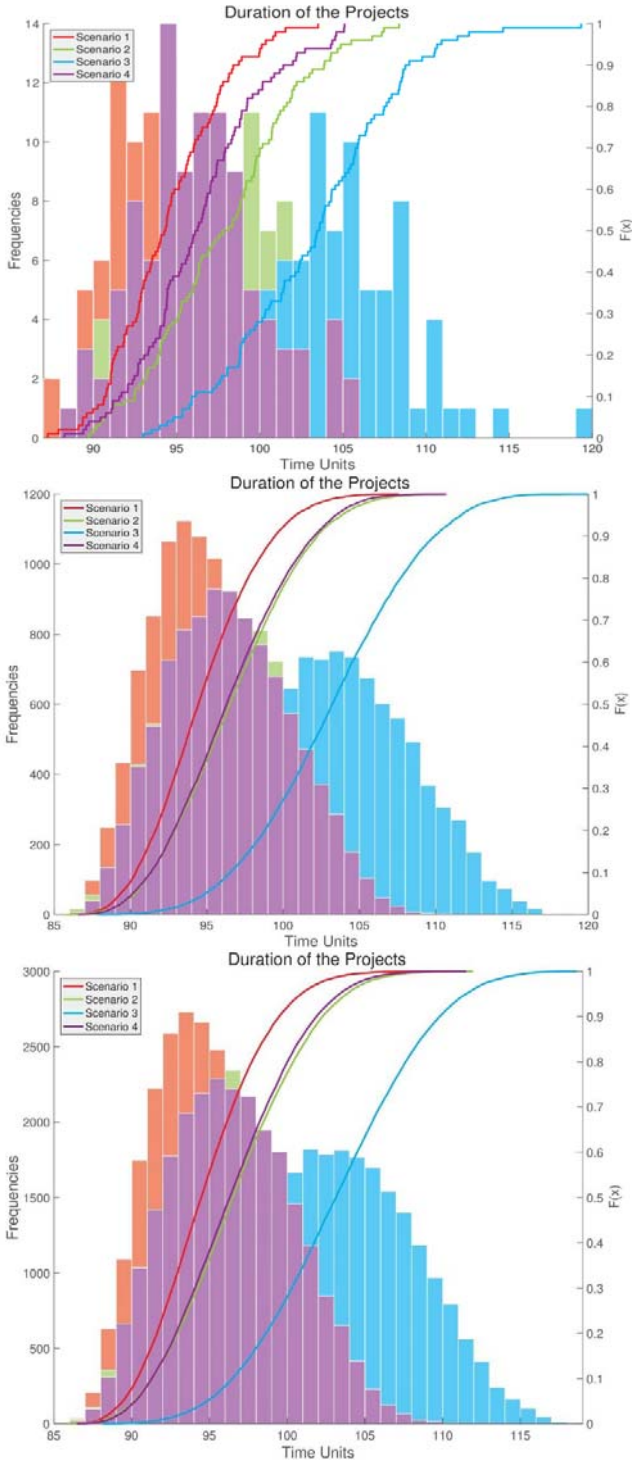


Fig. 4 Set 3: The first row presents a histogram of 4 scenarios for 100 reps, the second for 10,000 and the third for 25,000 reps. The difference between the second and the third scenario is minimal

Indicatively, the integration times are presented and compared with the completion times of the classical PERT, calculated using commercial software. The double dividing line separates the three selected networks.

TABLE II
SET 1: COMPARISON OF SIMULATION RESULTS

	Monte Carlo		PERT		
Set	μ	σ	μ	σ	**% diff in** μ
1	104.5	5.1	104.5	4.17	0.01
2	100.3	4.5	104.5	4.17	-4.05
3	95.4	3.5	104.5	4.17	-8.67
4	89.3	1.8	104.5	4.17	-14.58
1	166.0	6.1	166	4.58	-0.01
2	159.3	5.4	166	4.58	-4.02
3	151.9	4.2	166	4.58	-8.51
4	142.2	2.2	166	4.58	-14.33
1	164.5	5.8	164.5	3.89	-0.02
2	158.5	5.1	164.5	3.89	-3.63
3	151.9	4.0	164.5	3.89	-7.66
4	143.3	2.1	164.5	3.89	-12.86

TABLE III
SET 3: COMPARISON OF SIMULATION RESULTS

	Monte Carlo		PERT		
Set	μ	σ	μ	σ	**% diff in** μ
1	94.7	3.5	104.5	4.17	-9.36
2	96.7	4.2	104.5	4.17	-7.48
3	103.1	5.1	104.5	4.17	-1.31
4	96.5	4.0	104.5	4.17	-7.68
1	149.0	3.9	166	4.58	-10.23
2	151.9	4.5	166	4.58	-8.50
3	158.8	5.5	166	4.58	-4.37
4	149.9	4.2	166	4.58	-9.69
1	150.6	3.9	164.5	3.89	-8.46
2	151.3	4.2	164.5	3.89	-8.03
3	161.3	5.7	164.5	3.89	-1.94
4	150.9	4.0	164.5	3.89	-8.26

Concerning the criticality index, the results of the simulation show clearly (with approximately 100% probability) what activities are in this category. Table IV summarizes the results.

The first row refers to the small, the second to the medium one and the third to the large network. Results in this section, do not vary.

1. High Variance Set

It was deemed appropriate to carry out an experiment with a dataset of high variance. The pattern of fast convergence of the mean value was generally maintained, as was the slow convergence of the standard deviation. Generally, deviations follow the high variance of the input data and thus large variations (of the order of 10%) are observed in the resulting distribution tails.

Indicatively, the integration times are presented and compared with the completion times of the classical PERT, calculated using commercial software. The double dividing line separates the three selected networks.

In this experiment, there were some notable results in terms of the activity's criticality index. Due to the high variance of the input data and the nature of the project (through the selection the combinations (α, β), the simulation produced different results than the PERT method. Some examples are

listed.

TABLE IV
SETS 1 AND 3, CRITICAL ACTIVITIES

Repeats	*Scenarios 1.1 and 3.1*	*Scenarios 1.2 and 3.2*	*Scenarios 1.3 and 3.3*	*Scenarios 1.4 and 3.4*	*PERT*
100	1,3,5,6	1,3,5,6	1,3,5,6	1,3,5,6	1,3,5,6
10000	1,3,5,6	1,3,5,6	1,3,5,6	1,3,5,6	1,3,5,6
25000	1,3,5,6	1,3,5,6	1,3,5,6	1,3,5,6	1,3,5,6
100	1,3,4,6,9,11,12	1,3,4,6,9,11,12	1,3,4,6,9,11,12	1,3,4,6,9,11,12	1,3,4,6,9,11,12
10000	1,3,4,6,9,11,12	1,3,4,6,9,11,12	1,3,4,6,9,11,12	1,3,4,6,9,11,12	1,3,4,6,9,11,12
25000	1,3,4,6,9,11,12	1,3,4,6,9,11,12	1,3,4,6,9,11,12	1,3,4,6,9,11,12	1,3,4,6,9,11,12
100	1,3,6,9,13,14	1,3,6,9,13,14	1,3,6,9,13,14	1,3,6,9,13,14	1,3,6,9,13,14
10000	1,3,6,9,13,14	1,3,6,9,13,14	1,3,6,9,13,14	1,3,6,9,13,14	1,3,6,9,13,14
25000	1,3,6,9,13,14	1,3,6,9,13,14	1,3,6,9,13,14	1,3,6,9,13,14	1,3,6,9,13,14

TABLE V
SET 2: COMPARISON OF SIMULATION RESULTS

	Monte Carlo		PERT		
Set	μ	σ	μ	σ	**% diff in** μ
1	77.8	13.0	77.5	10.8	-0.45
2	62.6	11.5	77.5	10.8	19.19
3	45.8	9.0	77.5	10.8	40.95
4	23.8	4.7	77.5	10.8	69.25
1	115.1	14.8	108.5	16.7	-6.06
2	93.2	13.1	108.5	16.7	14.09
3	68.4	10.4	108.5	16.7	36.93
4	35.5	5.5	108.5	16.7	67.29
1	113.7	11.9	93	18.1	-22.23
2	93.1	11.0	93	18.1	-0.08
3	69.1	9.1	93	18.1	25.69
4	35.8	5.0	93	18.1	61.45

TABLE VI
SET 4: COMPARISON OF SIMULATION RESULTS

	Monte Carlo		PERT		
Set	μ	σ	μ	σ	**% diff in** μ
1	34.23	6.99	77.50	10.8	-55.83
2	47.84	9.63	77.50	10.8	-38.28
3	56.07	10.27	77.50	10.8	-27.65
4	45.07	8.95	77.50	10.8	-41.84
1	56.33	9.28	108.50	16.7	-48.08%
2	66.15	10.66	108.50	16.7	-39.04
3	81.01	11.78	108.50	16.7	-25.34
4	55.06	8.72	108.50	16.7	-49.25
1	56.35	8.67	93.00	18.1	-39.41
2	78.05	11.27	93.00	18.1	-16.08
3	89.91	12.07	93.00	18.1	-3.33
4	64.19	9.63	93.00	18.1	-30.98

TABLE VII
COMPARISON OF THE SMALL NETWORK FOR 25,000 REPS

Activity	%Monte Carlo Scenario 1	% Monte Carlo Scenario 3	PERT
1	100,00	100,00	X
2	54,60	57,74	X
3	**45,40**	**42,26**	
4	54,60	57,74	X
5	100,00	100,00	X
6	100,00	100,00	X

TABLE VIII
COMPARISON OF NETWORK CRITICAL ACTIVITY CRITERIA FOR 25,000 REPS

Activity	%Monte Carlo Scenario 3	PERT
1	100,00	X
2	30,17	X
3	66,80	X
4	**3,03**	X
5	59,22	X
6	10,05	X
7	30,72	X
8	**1,60**	X
9	79,60	X
10	18,80	X
11	35,10	X
12	**0,86**	X
13	64,05	X
14	100,00	X

C. Simulation Cycle Completion Times

All simulation cycles were carried out on a standard computer. The computational complexity of the sampler is $O\left(N^{1/2}\right)\cdot$

D. Summary

The components of the algorithm, such as the selected PERT networks, were described, the simulation scenarios were formulated, the structure of the algorithm analyzed, and the statistical analysis of the results were presented. The comparison of the results of the simulation with those of the classical PERT method was compared both at the time level and at the level of activity criticality. Finally, the computational complexity of the algorithm was discussed, and the required times were extensively presented.

VI. CONCLUSIONS

- **Computational Experience**

After repeated iterations of the simulation cycles it was clear that the results of the proposed method differ remarkably from those of the original PERT. Especially in cases where skewness is inherent in the nature of activity, which is nearly always the case nowadays, using Monte Carlo simulation to produce estimates for the duration of the project, is necessary. The Activity Criticality Index adds value to the model by

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incorporating an important element of project planning without adding more computational burden.

In terms of computational statistics, the developed algorithm is fully in line with the characteristics of the Monte Carlo simulation for technical projects developed by [18] explored in previous decades by many researchers until today. The framework is easily accessible to the average researcher, both due to the increased computational power and the availability of various commercial software packages. No unexpected values or results have been identified, trends were maintained and, in general, the method has successfully met every experiment. Value convergence is achieved quickly, and the 10000 iterations assertion, that is commonly found in the literature in all experiments except for one.

- **Further Research**

The estimation of a project's duration is part of risk analysis, an essential management sector, responsible to identify potential temporal and thus financial setbacks.

It would be very beneficial to observe the behavior of the particular method in cases where data gathered from earthmoving activities exist, where instead of manually creating the set of the (α,β) parameters, approaches such as the Maximum Likelihood Estimation (MLE) or Type II Maximum Likelihood estimation (also known as Empirical Bayes) could be employed to estimate them. Then, using domain expertise, a relevant grouping of the activities could be performed serving as input to the Monte Carlo simulation.

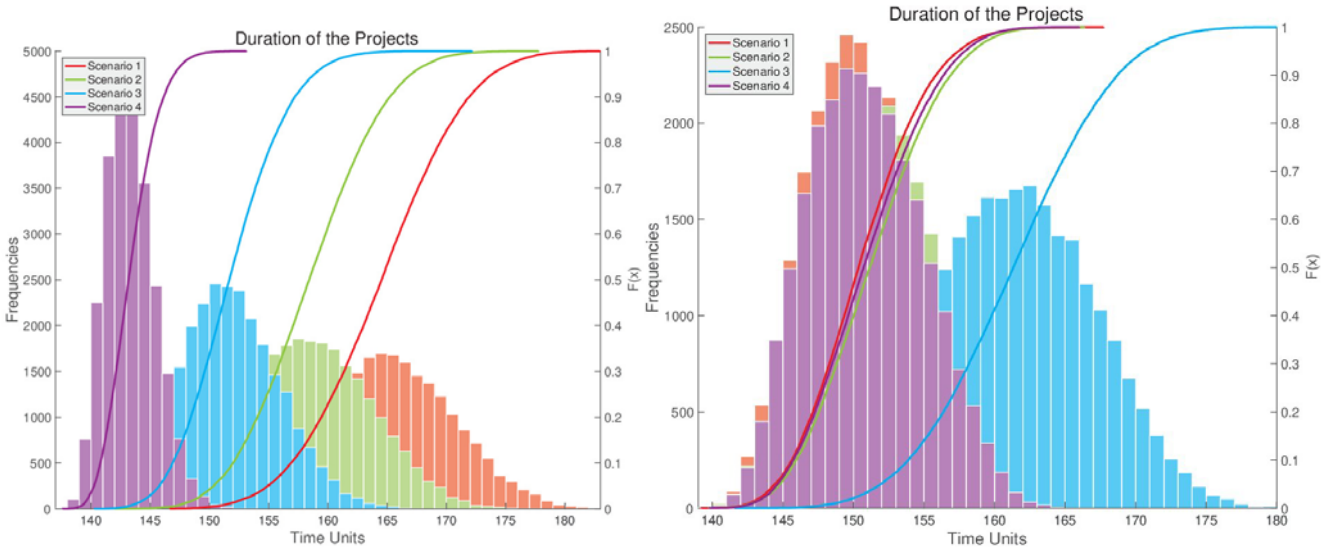


Fig. 5 Set 1 and 3. The correlation between the classic PERT (red on the left) and the other results

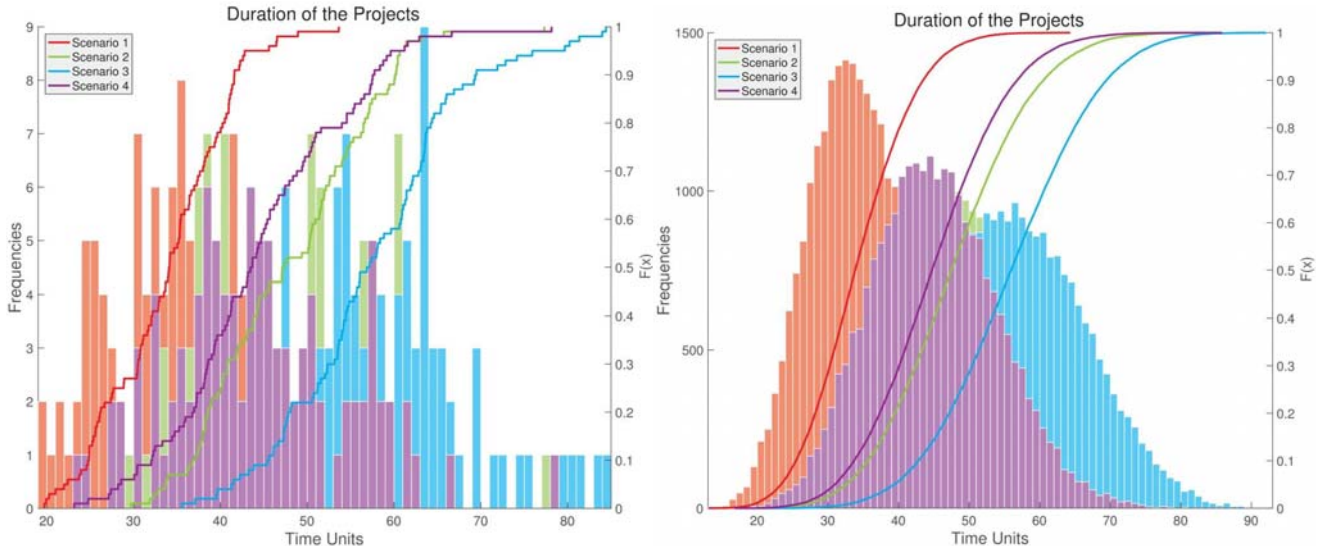


Fig. 6 Total 4, small network. Direct convergence of the mean value in scenarios 1 and 2 (red and green)

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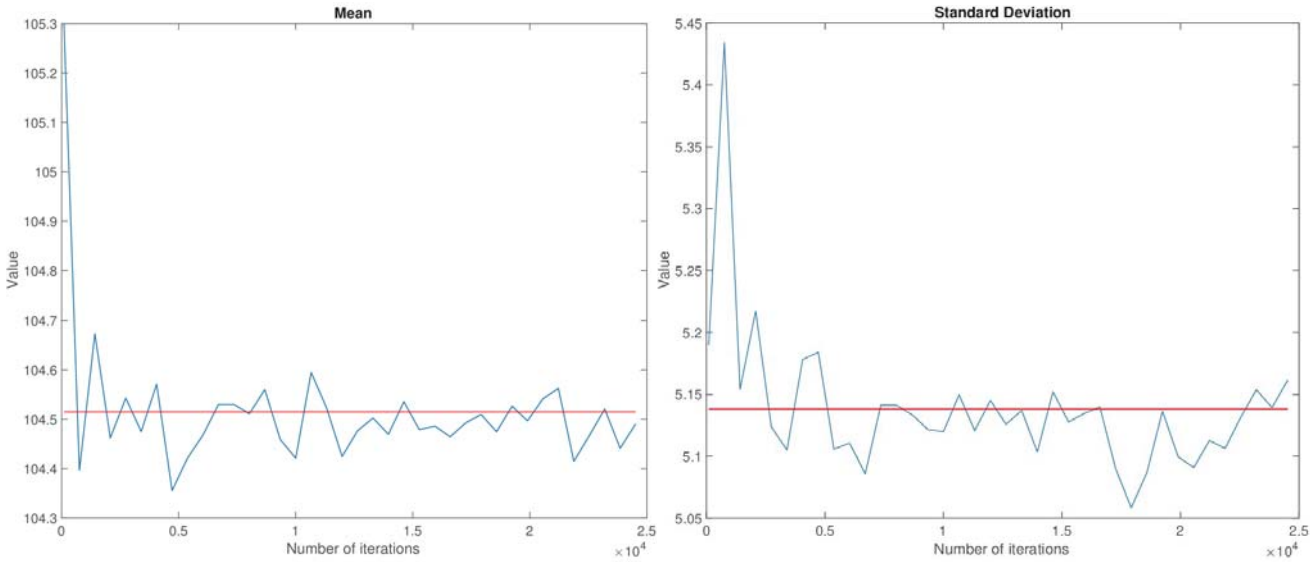


Fig. 7 Total 1, small network: Convergence of μ and σ values

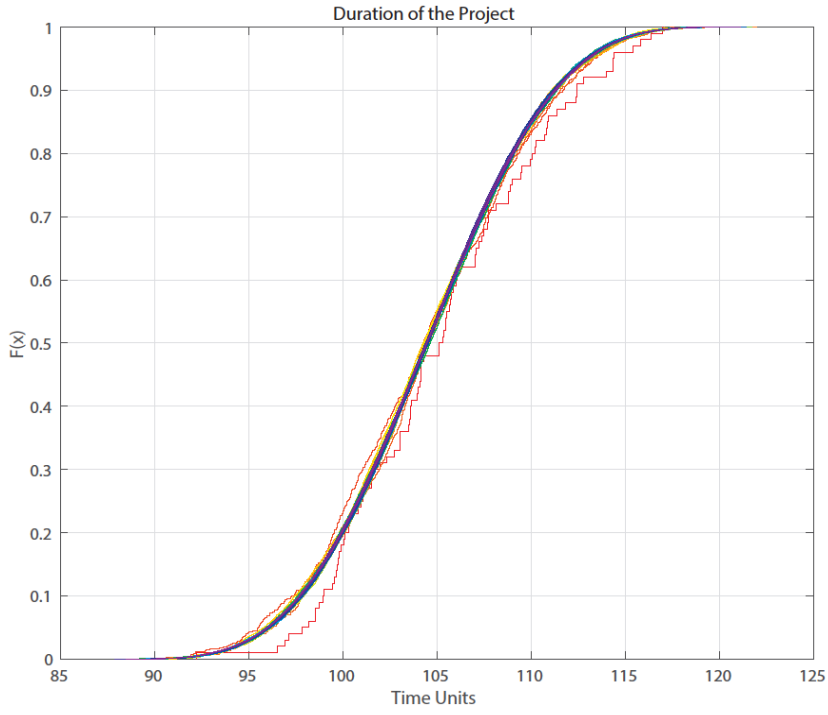


Fig. 8 Total 1, small network: Convergence of sigmoidal curves

In this paper, we have assumed that the duration of the activities are independent and identically distributed variables and relied on the central limit theorem for the distributional convergence of the results. This is a limiting assumption, often violated in real circumstances because some activities of the project may for example be affected by a common pool of resources. A simulation approach when there is dependence in the tasks of the project is therefore an interesting task. Nevertheless the next step would be to supplement this method with Grey Analysis methodologies, such as the GM(1, 1). Grey Analysis is made up of prediction models and is constantly gaining ground. In the event of a lack of real data, domain expert can supervise the creation of artificial data to simulate both the scheduling the execution phase of a project.

Finally, in the present study, we choose the duration limits $[c,d]$ of each activity using domain expertise, and of course this is a highly debatable proposition. The Grey Analysis' methodologies can go around this by predicting the $[c,d]$ field. This new piece of information is then a new input to the Monte Carlo simulation and will redefine the project completion time through new information. This powerful combination has not yet been explored in international literature and the authors believe that its results will be

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extremely interesting.

APPENDIX

TABLE IX
SCENARIOS FEATURES, SET 1

Tasks	Scenario 1		Scenario 2		Scenario 3		Scenario 4		Durations	
	α	β	α	β	α	β	α	β	c	d
1	2	2	2	3	2	5	2	13	10	30
2	2	2	2	3	2	5	2	13	8	12
3	2	2	2	3	2	5	2	13	29	31
4	2	2	2	3	2	5	2	13	4	16
5	2	2	2	3	2	5	2	13	5	19
6	2	2	2	3	2	5	2	13	40	45
7	2	2	2	3	2	5	2	13	12	28
8	2	2	2	3	2	5	2	13	7	21
9	2	2	2	3	2	5	2	13	32	48
10	2	2	2	3	2	5	2	13	7	13
11	2	2	2	3	2	5	2	13	10	17
12	2	2	2	3	2	5	2	13	9	11
13	2	2	2	3	2	5	2	13	15	25
14	2	2	2	3	2	5	2	13	10	14

TABLE X
SCENARIOS FEATURES, SET 2

Tasks	Scenario 1		Scenario 2		Scenario 3		Scenario 4		Durations	
	α	β	α	β	α	β	α	β	c	d
1	2	2	2	3	2	5	2	13	1	30
2	2	2	2	3	2	5	2	13	1	30
3	2	2	2	3	2	5	2	13	1	30
4	2	2	2	3	2	5	2	13	1	30
5	2	2	2	3	2	5	2	13	1	30
6	2	2	2	3	2	5	2	13	1	30
7	2	2	2	3	2	5	2	13	1	30
8	2	2	2	3	2	5	2	13	1	30
9	2	2	2	3	2	5	2	13	1	30
10	2	2	2	3	2	5	2	13	1	30
11	2	2	2	3	2	5	2	13	1	30
12	2	2	2	3	2	5	2	13	1	30
13	2	2	2	3	2	5	2	13	1	30
14	2	2	2	3	2	5	2	13	1	30

TABLE XI
SCENARIOS FEATURES, SET 3

Tasks	Scenario 1		Scenario 2		Scenario 3		Scenario 4		Durations	
	α	β	α	β	α	β	α	β	c	d
1	2	5	2	2	2	3	2	13	10	30
2	2	13	2	5	2	4	2	13	8	12
3	2	5	2	13	2	5	2	13	29	31
4	2	13	2	2	2	6	2	13	4	16
5	2	5	2	5	2	7	2	13	5	19
6	2	13	2	13	2	8	2	13	40	45
7	2	5	2	2	2	9	2	13	12	28
8	2	13	2	5	2	10	2	13	7	21
9	2	5	2	13	2	11	2	13	32	48
10	2	13	2	2	2	12	2	13	7	13
11	2	5	2	5	2	13	2	13	10	17
12	2	13	2	13	2	14	2	13	9	11
13	2	5	2	3	2	15	2	13	15	25
14	2	13	2	5	2	2	2	13	10	14

TABLE XII
SCENARIOS FEATURES, SET 4

Tasks	Scenario 1		Scenario 2		Scenario 3		Scenario 4		Durations	
	α	β	α	β	α	β	α	β	c	d
1	2	5	2	3	2	2	2	3	1	30
2	2	13	2	5	2	5	2	4	1	30
3	2	5	2	13	2	3	2	5	1	30
4	2	13	2	3	2	13	2	6	1	30
5	2	5	2	5	2	2	2	7	1	30
6	2	13	2	13	2	5	2	8	1	30
7	2	5	2	3	2	3	2	9	1	30
8	2	13	2	5	2	13	2	10	1	30
9	2	5	2	13	2	2	2	11	1	30
10	2	13	2	3	2	5	2	12	1	30
11	2	5	2	5	2	3	2	13	1	30
12	2	13	2	13	2	13	2	14	1	30
13	2	5	2	5	2	2	2	15	1	30
14	2	13	2	3	2	5	2	2	1	30

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