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A Mean–Variance–Skewness Portfolio Optimization Model

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***Abstract*—** Portfolio optimization is one of the most important topics in finance. This paper proposes a mean–variance–skewness (MVS) portfolio optimization model. Traditionally, the portfolio optimization problem is solved by using the mean–variance (MV) framework. In this study, we formulate the proposed model as a three-objective optimization problem, where the portfolio's expected return and skewness are maximized whereas the portfolio risk is minimized. For solving the proposed three-objective portfolio optimization model we apply an adapted version of the non-dominated sorting genetic algorithm (NSGAII). Finally, we use a real dataset from FTSE-100 for validating the proposed model.

***Keywords*—**Evolutionary algorithms, portfolio optimization, skewness, stock selection.

I. Introduction

THE classical MV model assumes that assets' returns tend to follow a Gaussian distribution, and therefore the characteristics of these assets can be described only by their first and second central moments of distributions. However, according to Chunhachinda et al. [1], asset's returns of the world's 14 major stock markets are not normally distributed. Moreover the same authors find that the correlation matrix of these stock markets was stable during January 1988 – December 1993. The authors utilize a polynomial goal programming approach in which investor preferences for skewness can be incorporated, to determine the optimal portfolio consisting of the choices of 14 international stock indexes. According to [1], the skewness alters drastically the composition of the derived portfolio. Moreover, according to the same study investors trade expected return of the portfolio for skewness.

According to Lai [2], when the skewness is taken into the portfolio optimization process the resulting optimal portfolio differs from the one that is obtained by the classical MV framework. Moreover, when we consider the three-objective optimal portfolio, investor preference can be incorporated as a polynomial goal programming problem. The authors conclude that a MV efficient portfolio is not necessarily efficient in terms of the MVS optimization framework.

Liu et al. [3] propose a MVS model for portfolio selection with transaction costs. The authors assume that the cost of buying and selling stocks can be represented with a function that calculates the difference between the existing investment choices and the updated portfolio after performing the necessary adjustments. Finally, the authors convert the aforementioned problem into a linear programming problem. According to [3], this technique can be used to solve large-scale portfolio selection problems. The authors provide a numerical example to illustrate that the method can be efficiently used in practice. Konno et al. [4] formulate a general portfolio optimization problem maximizing skewness subject to fixed expected return and variance constraints, whereby both the quadratic and cubic terms are linearly approximated to yield a mean-absolute deviation-skewness model.

The main contributions of this work are as follows. First, we propose a MVS model, which is an extension of the classical MV portfolio theory [5]-[7]. In particular, the expected return and skewness are maximized, while the risk is minimized. Second, for solving the examined MVS model we propose the application of a multi-objective evolutionary algorithm (MOEA) for handling the complexity that introduces the third central moment (i.e. the skewness) which is a non-concave function.

The remainder of the paper is organized as follows. Section II provides the formulation of the proposed MVS portfolio optimization model. The proposed MOEA for solving the examined model is presented in Section III. The design of experiments and the experimental results are presented and discussed in Section IV. Finally conclusions are drawn in Section V.

II. The MVS Portfolio Optimization Model

A. The Proposed Model

The MVS model is a direct extension of the classical MV portfolio model [8]-[10]. The MVS optimization framework is a reasonable choice to model investment situations where assets' returns do not follow the Gaussian distribution [11], and therefore the characteristics of these assets can be better described with the assistance of the third central moment (i.e. the skewness) [12]. For more general use the MVS optimization framework is given by the following relationships.

Optimize: $$f(w) = (f_1(w), f_2(w), f_3(w)) \quad (1)$$

Maximize portfolio return: $$f_1(w) = \sum_{i=1}^{N} w_i \bar{r}_i \quad (2)$$

Minimize portfolio risk: $$f_2(w) = \sum_{i=1}^{N} \sum_{j=1}^{N} w_i w_j \sigma_i \sigma_j \rho_{ij} \quad (3)$$

Maximize portfolio skewness: $$f_3(w) = \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{k=1}^{N} w_i w_j w_k S_{\mathrm{ijk}} \quad (4)$$

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Open Science Index, Computer and Information Engineering Vol:13, No:2, 2019 publications.waset.org/10010074/pdf

where w and $\bar{r}_i$ represent the weight of asset i in the portfolio and the return on the asset i, respectively. σ_i represents the standard deviation of stock returns i. ρ_{ij} is the correlation between asset i and j and $-1 \leq \rho_{ij} \leq 1$. Finally, S_{ijk} represents the coskewness between the returns of assets i, j, and k, respectively.

The MVS portfolio optimization model considers simultaneously three conflicting objectives namely the portfolio's expected return, variance and skewness [13]. Analytically, the portfolio's expected return and skewness are maximized whereas the portfolio's variance is minimized.

To address the limitations of the conventional optimization methods when generating the Pareto-optimal front, the metaheuristic optimization algorithms have been successfully applied to the multi-objective optimization problems [14], [15]. In particular, for solving the proposed MVS model we propose an adapted version of the NSGAII.

III. An Adapted Version of NSGAII for Solving the MVS Portfolio Optimization Problem

For the purposes of the present study we use an adapted version of NSGAII. In particular, the NSGAII [13] has been adapted to work well with the proposed MVS model and the imposed constraints. Also a reparation operator is utilized to make the infeasible solutions feasible. Fig. 1 presents the pseudocode of the adapted NSGAII.

step 1*:* Initialize population;
step 2: Apply the reparation operator for making the solutions feasible
step 3: Evaluate candidate solutions
step 4: Rank population
step 5: while termination condition is not true do
step 6: Selection
step 7: Crossover
step 8: Apply the reparation operator for making the solutions feasible
step 9: Mutation
step 10: Apply the reparation operator for making the solutions feasible
step 11: Evaluate objective functions
step 12: Combine parent and child populations, rank population
step 13: Select the winning individuals
step 14: Stopping criteria met?
step 15.a: Yes: Report the derived solutions.
step15.b: No: Go to step 5

Fig. 1 Pseudocode of the adapted NSGAII for solving the MVS model

As shown in Fig. 1 the adapted NSGAII starts by generating a population. Then the population is sorted based on Pareto conditions [16] into a number of fronts [17]. Thus, the first front is composed from the non-dominated solutions in the current population. The individuals of the second front are being dominated by the solutions of the first front. Respectively, the individuals of the third front are being dominated by the solutions of the second front and so on [18]. Next, the algorithm assigns to each solution a rank based either on fitness value of the particular individual or respectively on the front the particular individual belongs to. Furthermore, a parameter called crowding distance is calculated for each individual [19], [20]. The crowding distance is used to estimate the distance between the obtained solutions. The bigger the distance between the obtained solutions the better the derived front is. The selection between the obtained solutions is performed with the assistance of binary tournament selection [21]. In particular, an individual is selected if the rank is less than the other or if crowding distance is greater than the other. As soon as the selection process is done, the obtained solutions are subject to genetic operators, namely recombination and mutation operators [22]. Finally, we merge together the current population and the obtained offsprings and the resulting population is sorted again. Only the best N solutions are selected. These N solutions compose the new population [23], [24].

IV. Design of Experiments and Experimental Results

For the experiments we used one dataset from FTSE-100 in London, having 100 assets. The dimension of the available dataset is presented in Table I.

TABLE I
The Dimension Of The Test Problem

Index	Dimension
FTSE-100	100

We solve the proposed MVS model with the assistance of the NSGAII. For the fine-tuning of the algorithm we run two different configurations of the NSGAII: (a) in the first configuration of the NSGAII we use 50,000 function evaluations as the stopping condition and (b) in the second configuration of the NSGAII we use 100,000 function evaluations as the stopping condition.

TABLE II
The Metrics Obtained for Solving the MVS Portfolio Optimization Problem under Three Different Performance Metrics

Problem: The MVS model	NSGAII (50,000 eval.)
HV. Mean and Std	$2.70e\text{-}01_{8.7e\text{-}03}$
HV. Median and IQR	$2.68e\text{-}01_{1.8e\text{-}02}$
IGD. Mean and Std	$2.88e\text{-}03_{3.5e\text{-}04}$
IGD. Median and IQR	$3.08e\text{-}03_{6.3e\text{-}04}$
EPSILON. Mean and Std	$1.03e\text{-}03_{4.1e\text{-}04}$
EPSILON. Median and IQR	$8.67e\text{-}04_{6.4e\text{-}04}$
Problem: The MVS model	**NSGAII (100,000 eval.)**
HV. Mean and Std	$2.89e\text{-}01_{2.2e\text{-}03}$
HV. Median and IQR	$2.89e\text{-}01_{4.5e\text{-}03}$
IGD. Mean and Std	$2.26e\text{-}03_{1.6e\text{-}04}$
IGD. Median and IQR	$2.25e\text{-}03_{3.1e\text{-}04}$
EPSILON. Mean and Std	$6.90e\text{-}04_{6.7e\text{-}05}$
EPSILON. Median and IQR	$6.94e\text{-}04_{1.3e\text{-}04}$

In the NSGAII algorithm the mutation probability is set to 0.1 and the crossover probability is set to 0.9 for all test problems. The population size was set to 100. Also, for each problem we have executed 20 independent runs. The experimental results of the bi-objective problem, i.e. the

Open Science Index, Computer and Information Engineering Vol:13, No:2, 2019 publications.waset.org/10010074/pdf

averages, standard deviations, medians and the interquartile ranges (IQR) of all metrics, are given in Table II. Table II presents the results of the NSGAII under the two different configurations: (a) in the first configuration of the NSGAII we use 50,000 function evaluations as the stopping condition and (b) in the second configuration of the NSGAII we use 100,000 function evaluations as the stopping condition. Table II shows the derived values for the different performance measures used (i.e. HV, IGD and Epsilon indicator) for evaluating algorithm's performance. The higher the value of HV indicator the better is the computed front. The smaller the value of Inverted generational distance (IGD) and Epsilon indicator, the better is the distribution of the solutions.

TABLE III
BOXPLOTS FOR HV, IGD AND EPSILON: (A) NSGAII (50,000 EVAL.), (B) NSGAII (100,000 EVAL.) UNDER THREE DIFFERENT PERFORMANCE METRICS

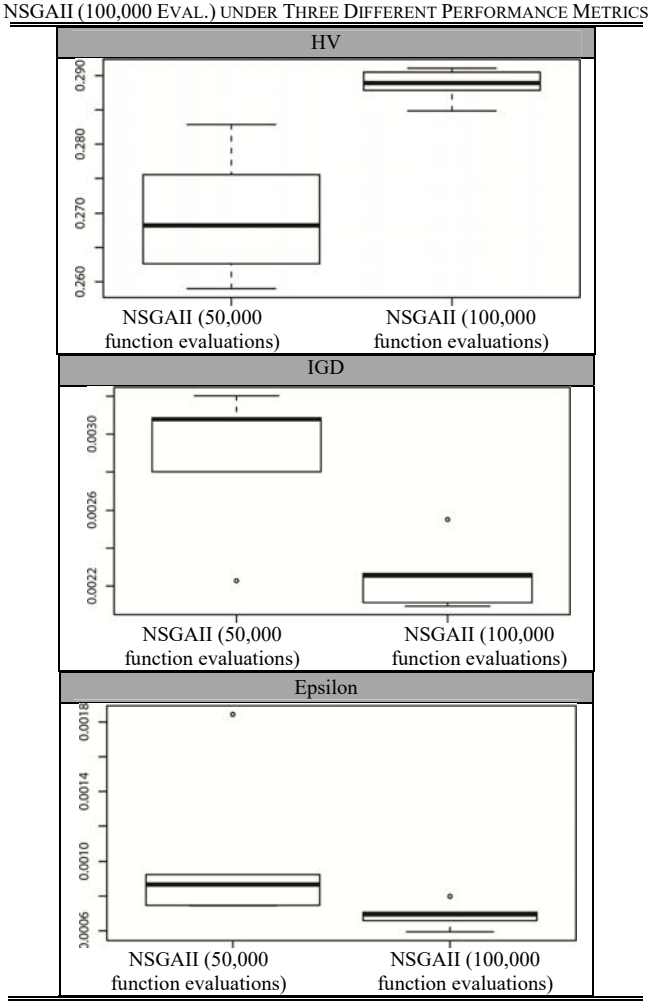


Table III uses boxplots to present graphically, the performance of the NSGAII for solving the MVS model under the two different configurations: (a) in the first configuration of the NSGAII we use 50,000 function evaluations as the stopping condition and (b) in the second configuration of the NSGAII we use 100,000 function evaluations as the stopping condition. The experimental results indicate that the second configuration of the NSGAII with the 100,000 function evaluations generates better results than the first configuration of the NSGAII with the 50,000 function evaluations for all different performance metrics.

Finally, Fig. 2 shows the trade-off fronts obtained for the different configurations of the MVS portfolio optimization model.

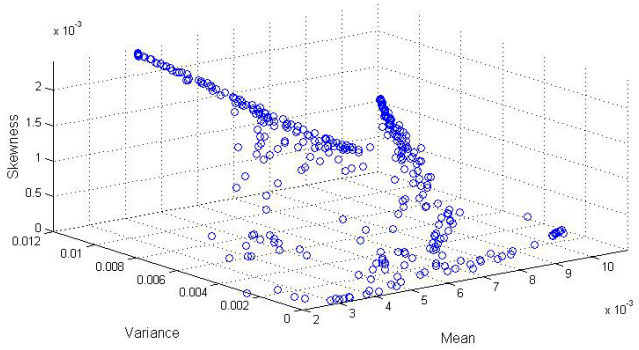


Fig. 2 The derived efficient frontiers for the MVS portfolio optimization model

V. CONCLUSIONS

In this paper we apply a well-known MOEA, namely the NSGAII for solving the MVS portfolio optimization model, under the two different configurations: (a) in the first configuration of the NSGAII we used 50,000 function evaluations as the stopping condition and (b) in the second configuration of the NSGAII we used 100,000 function evaluations as the stopping condition. The experimental results indicate that the second configuration of the NSGAII with the 100,000 function evaluations generates better results than the first configuration of the NSGAII with the 50,000 function evaluations for all different performance metrics.

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