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Redundancy Component Matrix and Structural Robustness

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***Abstract*—**We introduce the redundancy matrix that expresses clearly the geometrical/topological configuration of the structure. With the matrix, the redundancy of the structure is resolved into redundant components and assigned to each member or rigid joint. The values of the diagonal elements in the matrix indicates the importance of the corresponding members or rigid joints, and the geometrically correlations can be shown with the non-diagonal elements. If a member or rigid joint failures, reassignment of the redundant components can be calculated with the recursive method given in the paper. By combining the indexes of reliability and redundancy components, we define an index concerning the structural robustness. To further explain the properties of the redundancy matrix, we cited several examples of statically indeterminate structures, including two trusses and a rigid frame. With the examples, some simple results and the properties of the matrix are discussed. The examples also illustrate that the redundancy matrix and the relevant concepts are valuable in structural safety analysis.

***Keywords*—**Structural robustness, structural reliability, redundancy component, redundancy matrix.

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I. Introduction

FOR the sake of the safety, the structure is usually needed to meet the specified requirements in strength, durability, stability and anti-seismic performance. In addition, robustness or vulnerability is considered as another important factor for structural safety, especially in the cases of unforeseen events. Robustness is defined as an ability of a structure and its members to resist excessive deterioration [1]. The ability is often regarded as one of the structural advantages. However, it should be understood as a kind of property because it may bring some adverse factors, such as reduction of sensitivity about monitoring the damages in the structure, cost increase for construction, and so on. Optimization of structures based on system reliability has computational and modeling complexities [2]. Therefore, it might be difficult to seek a balance between the pros and cons. The search for a more effective method is meaningful in structural design.

Systems are assumed to be an assembly of components. The robustness and reliability of structures are related to the mechanical properties of the components and the geometrical or topological relationship between these components [2]. Redundancy is a typical parameter reflecting geometric/ topological configuration of structures. For this reason, analysis based on redundancy is regarded as one of the ways to optimize the structure. There are different definitions of redundancy proposed in the literature [3]. The commonly conventional definition is degree of indeterminacy, which is defined by remainder between number of unknown reactive forces and number of independent equilibrium equations [4]. In general, the technical term "degree of indeterminacy" is just for overall structural system. Highly indeterminacy does not mean more safe structure. Even though degree of redundancy is, intuitively, an ideal system safety performance metric [5], the metric only provides a necessary but not sufficient condition in guaranty of structural system stability [1]. Every member plays a certain role about its importance in the structure. In order to express this relationship, we intend to introduce the concept of redundancy components and illustrate its some applications. The concept is suitable to several types of structures but this paper treats the problems only about plane truss and frame for the reason of simplicity.

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II. Redundancy Matrix

In a designed structure, there is no geometric contradiction among the components. Actually, after a statically indeterminate structure is built, the ideally geometric harmony between the components does not exist due to the effects from various factors such as manufacture errors, temperature, load acted on the structure, etc. The redundant links, internally and externally, impose constraints that must be satisfied in order to resolve geometrical disagreements or inconsistencies, by adjusting the geometric values of the members and joints. These adjusted values are closely related with the distribution of the redundancy components in the structure. Therefore, according to the observation about adjusted values, this distribution can be understood.

There are different geometric constraints in a structure system. For example, the geometrical constraint is only the length of member in the truss. In addition to the length, the angle of between two members is another geometrical constraint for the rigid frame.

In order to introduce the concept of redundancy component, consider a truss with n members. The *designed length* of the members in the truss is denoted by the n-vector $\widetilde{\mathbf{L}}$:

$$\widetilde{\mathbf{L}} = \begin{pmatrix} \widetilde{L}_1 & \widetilde{L}_2 & \cdots & \widetilde{L}_n \end{pmatrix}^T \tag{1}$$

Suppose a set of virtual increment is added to the geometric constraints, which can caused by different sources such as manufactured errors, temperature, etc.. Therefore the length vector is expressed as

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$$\mathbf{L} = \widetilde{\mathbf{L}} + \mathbf{\Delta} \tag{2}$$

where $\mathbf{\Delta} = \begin{pmatrix}\Delta_1 & \Delta_2 & \cdots & \Delta_n\end{pmatrix}^T$ is a virtual random vector, and its mathematical expectation and covariance matrix are assumed as

$$\left.\begin{aligned} E(\mathbf{\Delta}) &= 0 \\ Cov(\mathbf{\Delta}) &= diag\begin{pmatrix}\sigma_1^2 & \sigma_2^2 & \cdots & \sigma_n^2\end{pmatrix}\end{aligned}\right\} \tag{3}$$

This stochastic model is not the unique form, but it will simplify the following derivation. Because every member must be subjected to the geometrical constraints, after being installed, the *actual length* vector may be written as

$$\hat{\mathbf{L}} = \mathbf{L} + \mathbf{V} \tag{4}$$

where $\mathbf{V} = \begin{pmatrix}V_1 & V_2 & \cdots & V_n\end{pmatrix}^T$ is the deformation vector of the members.

The designed coordinates of the *t* joints are expressed with 3*t*-vector:

$$\mathbf{X}^0 = \begin{pmatrix}X_1^0, Y_1^0, Z_1^0, \cdots, X_t^0, Y_t^0 Z_t^0\end{pmatrix}^T \tag{5}$$

For member L_i with tow joints a and b, the designed coordinates of the joints are defined as:

$$\mathbf{X}_i^0 = \begin{pmatrix}X_a^0 & Y_a^0 & Z_a^0 & X_b^0 & Y_b^0 & Z_b^0\end{pmatrix}^T \tag{6}$$

The coordinate increments of the joints are

$$\mathbf{\Delta X}_i = \begin{pmatrix}u_a & v_a & w_a & u_b & v_b & w_b\end{pmatrix}^T \tag{7}$$

After the structure is constructed, hence, the actual coordinates of the joints of the member may be written as follows

$$\hat{\mathbf{X}}_i = \mathbf{X}_i^0 + \mathbf{\Delta X}_i = \begin{pmatrix}\hat{X}_a & \hat{Y}_a & \hat{Z}_a & \hat{X}_b & \hat{Y}_b & \hat{Z}_b\end{pmatrix}^T \tag{8}$$

For the member, the geometrical condition which needs to be met in the established structure is

$$\hat{\mathbf{L}}_i = \sqrt{\left(\hat{X}_b - \hat{X}_a\right)^2 + \left(\hat{Y}_b - \hat{Y}_a\right)^2 + \left(\hat{Z}_b - \hat{Z}_a\right)^2} \tag{9}$$

By linearizing (9) about $\hat{\mathbf{X}}_i$, we obtain the deformation equation for member *i*.

$$\mathbf{V}_i = \mathbf{A}_i \mathbf{\Delta X}_i - \mathbf{\Delta}_i$$

in which $\mathbf{A}_i$ is a coefficient matrix that is given by $\mathbf{L}_i$ derivation of $\hat{\mathbf{X}}_i$.

For every geometrical constraint, the corresponding deformation equation may be obtained. Summing up all the equations, the global deformation equation can be expressed as

$$\mathbf{V} = \mathbf{A\Delta X} - \mathbf{\Delta} \tag{10}$$

The potential energy of the structure caused by the geometrically virtual increments is

$$\chi = \frac{1}{2}\mathbf{V}^T\mathbf{WV} \tag{11}$$

where W is defined as weight matrix which depends on the geometrical and mechanical parameters of the members. For the truss, the weight matrix is

$$\mathbf{W} = \begin{pmatrix} \frac{E_1 A_1}{L_1} & 0 & \cdots & 0 \\ 0 & \frac{E_2 A_2}{L_2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \frac{E_n A_n}{L_n} \end{pmatrix} \tag{12}$$

Differentiate χ by $\mathbf{\Delta X}$ and equating the first derivative to zero we obtain the normal equations as follows [6]

$$\mathbf{A}^T\mathbf{WA\Delta X} = \mathbf{A}^T\mathbf{W\Delta} \tag{13}$$

Substituting the solution into (10), the deformations vector about the members is given by

$$\mathbf{V} = \left(\mathbf{A}\left(\mathbf{A}^T\mathbf{WA}\right)^{-1}\mathbf{A}^T\mathbf{W} - \mathbf{I}\right)\mathbf{\Delta} \tag{14}$$

For *n*-dimensional random vector Y with mathematical expectation $\boldsymbol{\eta}$ and covariance matrix $Cov(\mathbf{Y})$, the mathematical expectation of quadratic form about Y can be written as:

$$E\left(\mathbf{YBY}^T\right) = tr\left(\mathbf{B}Cov\left(\mathbf{Y}\right)\right) + \boldsymbol{\eta}^T\mathbf{B}\boldsymbol{\eta} \tag{15}$$

where B is a invertible matrix. Substituting V for Y, W for B, $Cov(\mathbf{V})$ for $Cov(\mathbf{Y})$and E（V） for $\boldsymbol{\eta}$ in (15), the mathematical expectation of quadratic form about V is

$$E\left(\mathbf{V}^T\mathbf{WV}\right) = tr\left(\mathbf{W}Cov\left(\mathbf{V}\right)\right) + \left(E\left(\mathbf{V}\right)\right)^T\mathbf{W}E\left(\mathbf{V}\right) \tag{16}$$

Take note of the assumption shown in (3), we have

$$E\left(\mathbf{V}^T\mathbf{WV}\right) = tr\left(\mathbf{W}Cov\left(\mathbf{V}\right)\right) \tag{17}$$

The equation for estimating variance may be written as

$$E\left(\frac{\mathbf{V}^T\mathbf{WV}}{R}\right) = \hat{\sigma}_0^2 \tag{18}$$

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in which R represents the number of extra restraints taken to determine the values in (10). Actually, it is the degree of redundancy about the structure. Combining (17) and (18), we obtain the result as

$$E\left(\mathbf{V}^T\mathbf{W}\mathbf{V}\right) = tr\left(\mathbf{W}Cov\left(\mathbf{V}\right)\right) = R\ \hat{\sigma}_0^2 \tag{19}$$

Let

$$\frac{Cov\left(\mathbf{V}\right)}{\hat{\sigma}_0^2} = \mathbf{Q}_V$$

Then

$$\frac{1}{\sigma^2}\mathbf{W}Cov\left(\mathbf{V}\right) = \mathbf{W}\mathbf{Q}_V \tag{20}$$

We define matrix $\mathbf{WQ}_V$ as redundancy matrix of the structure. the matrix possesses the following properties:

Property 1. $\mathbf{WQ}_V$ is idempotent. That is

$$\left(\mathbf{W}\mathbf{Q}_V\right)^2 = \mathbf{W}\mathbf{Q}_V \tag{21}$$

Property 2. As an idempotent matrix, the eigenvalues of $\mathbf{WQ}_V$ are 0 or 1 and its rank equals to its trace. That is

$$tr\left(\mathbf{W}\mathbf{Q}_V\right) = rank\left(\mathbf{W}\mathbf{Q}_V\right) \tag{22}$$

Property 3. The trace equals to the redundant degree of the structure. The mathematical expression can be given as

$$tr\left(\mathbf{W}\mathbf{Q}_V\right) = R \tag{23}$$

Property 4. The value of the *i*th diagonal element in matrix $\mathbf{WQ}_V$ indicates the redundancy component r_i is allotted to member *i* and the interval of its value is [0,1]. The expressions are

$$r_i = \left(\mathbf{W}\mathbf{Q}_V\right)_{ii} \tag{24}$$

and

$$0 \le r_i \le 1 \tag{25}$$

From (23) and (24), we may find the redundant degree equals the sum of the all components

$$R = \sum_{i=1}^{n} r_i \tag{26}$$

The redundancy component matrix shows a geometrical property of the structure. The redundancy component r_i demonstrates the importance of member *i*. Generally speaking, The smaller the value of component r_i, the more important the member. If the value equals 0, the corresponding member is essential. On the contrary, if its value equal to zero, the corresponding member possess no structural significance.

Two types of geometrical constraint are included in a rigid frame. Besides the length, the angle of between two members is the other one. For an approximate rigid joint *i* connecting joints a and b, the geometrical condition which need to be meet is

$$\arccos\alpha_i = \frac{\mathbf{M}_i{}^T\mathbf{N}_i}{\left(\mathbf{M}_i{}^T\mathbf{M}_i\right)^{\frac{1}{2}}\left(\mathbf{N}_i{}^T\mathbf{N}_i\right)^{\frac{1}{2}}} \tag{27}$$

where $\mathbf{M}_i$ and $\mathbf{N}_i$ are direction numbers of the two members. The relationship between direction numbers and the coordinates of the joints can be expressed as

$$\begin{cases} \mathbf{M}_i = \left(\hat{X}_a - \hat{X}_i \quad \hat{Y}_a - \hat{Y}_i \quad \hat{Z}_a - \hat{Z}_i\right)^T \\ \mathbf{N}_i = \left(\hat{X}_b - \hat{X}_i \quad \hat{Y}_b - \hat{Y}_i \quad \hat{Z}_b - \hat{Z}_i\right)^T \end{cases} \tag{28}$$

The corresponding redundancy matrix can be derived by the same method described above.

III. Examples

A. Example 1

In order to verify the above results, a statically indeterminate truss with 8 members is used as an example (See Fig. 1). Obviously, its degree of redundancy is 2, and no redundant restraint is provided to members 7 and 8. In addition，member 6 completely redundant because it is restrained with two hinge supports.

The redundancy matrix is as

$$\left(\mathbf{W}\mathbf{Q}_V\right) = \begin{pmatrix} 0.1155 & 0.1155 & 0.0000 & 0.1155 & -0.1634 & -0.1634 & 0.0000 & 0.0000 \\ 0.1155 & 0.1155 & 0.0000 & 0.1155 & -0.1634 & -0.1634 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.1155 & 0.1155 & 0.0000 & 0.1155 & -0.1634 & -0.1634 & 0.0000 & 0.0000 \\ -0.2310 & -0.2310 & 0.0000 & -0.2310 & 0.3267 & 0.3267 & 0.0000 & 0.0000 \\ -0.2310 & -0.2310 & 0.0000 & -0.2310 & 0.3267 & 0.3267 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \end{pmatrix}$$

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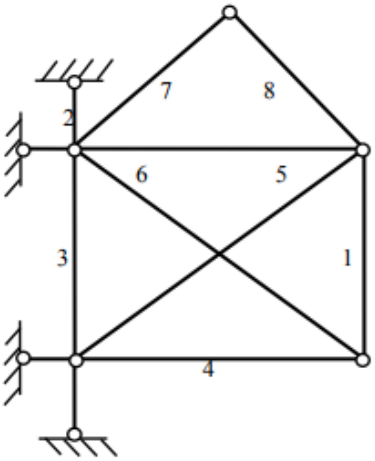


Fig. 1 The truss with 8 members

The redundant degree for the truss is equal to the trace of the matrix, which can be calculated by the sum of the all components.

$$R = \sum_{i=1}^{n} r_{ii} = 2$$

The values in matrix (29) illustrate the distribution of redundancy components and the correlations between them. It is seen that the diagonal element corresponding to member 3 is 1 and other elements are zeros. In this case, the member is completely redundant. Its failure has no effect on the structure if the purpose of use function is not taken into account. The zeros of the non-diagonal elements indicate that after the member is removed, the redundancy components previously allocated to other members will not be changed.

The values of all elements corresponding to members 7 and 8 are equal to zero. This situation shows that the two members are entirely essential. If any one of them is destroyed, the sub-structure associated it will failure.

The redundancy components with values between 0 and 1 are unequally assigned to the other five members. The sum is equal to 1, and the smaller redundancy value corresponds to the more essential member. If the non-diagonal elements are not equal to zero, as we have seen from the matrix, failure of any member will result in changes of redundancy components assigned previously to other related members.

B. Example 2

Fig. 2 shows a statically determinate truss with 26 members, which is employed for the further specification of the properties about the redundancy matrix.

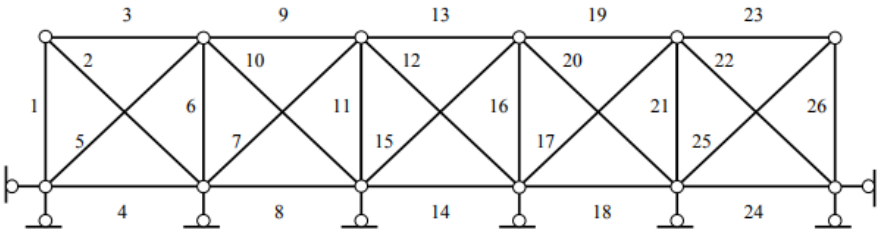


Fig. 2 The truss with 26 members

The calculated values of redundancy components for all members are shown in Table I.

TABLE I
THE CALCULATED REDUNDANCY COMPONENTS

Number	Redundancy component	Number	Redundancy component
1	0.2356	14	0.3784
2	0.3332	15	0.3942
3	0.2356	16	0.5091
4	0.3823	17	0.3689
5	0.3587	18	0.3914
6	0.5120	19	0.4551
7	0.3977	20	0.3977
8	0.3914	21	0.5120
9	0.4551	22	0.3587
10	0.3689	23	0.2356
11	0.5091	24	0.3823
12	0.3942	25	0.3332
13	0.4741	26	0.2356

The redundancy degree for the truss is equal to the trace of the matrix, which can be calculated by the sum of the all components:

$$R = \sum_{i=1}^{n} r_{ii} = 9.9989 \approx 10$$

The deviation is caused by the rounding errors. The redundancy components indicate the dependence of the structure on a member. If a member which has smaller redundancy components is damaged, the safety of the structure may be subjected to greater impact, entirely or partly. The elements in redundancy matrix for the next stage can be calculated with the recursive method when a member fails:

$$r_{ij} = r_{ij} - \frac{r_{il}}{r_{ll}} r_{lj} \tag{30}$$

Assume members 3 be removed in the truss, for example, the changed redundancy components are listed in Table II. It can be seen from the table that members 1 and 2 become necessary in this case because of their value zeros, and the sum of the

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redundancy components is reduced to 9.

TABLE II
THE REDUNDANCY COMPONENTS FOR 25 MEMBERS

Number	Redundancy component	Number	Redundancy component
1	0	14	0.3775
2	0	15	0.3941
3	/	16	0.5090
4	0.3490	17	0.3689
5	0.3250	18	0.3911
6	0.5120	19	0.4551
7	0.3797	20	0.3977
8	0.3552	21	0.5119
9	0.4231	22	0.3586
10	0.3362	23	0.2356
11	0.5049	24	0.3818
12	0.3909	25	0.3332
13	0.4738	26	0.2356

C. Statically Indeterminate Rigid Rigid Frame

For a rigid frame, the same matrix may be obtained. The geometrical restrictions include the length and angle. Redundancy components are assigned to approximate rigid joints and members in the structure.

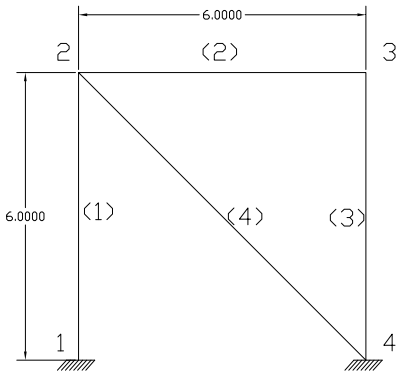


Fig. 3 The fame with 4 members

For the rigid frame shown in Fig. 3, the redundancy component matrix is calculated as

$$WQ_r = \begin{pmatrix} 0.1341 & 0.2596 & 0.0489 & 0.0402 & -0.0217 & 0.0152 & 0.0011 & -0.0152 & 0.2564 & 0.1489 & 0.0707 & 0.2888 \\ 0.0947 & 0.8064 & -0.1060 & -0.0441 & 0.1150 & 0.0973 & -0.0354 & 0.0973 & -0.1672 & -0.0933 & -0.0900 & -0.0710 \\ -0.0596 & -0.2288 & 0.5608 & 0.0070 & -0.2778 & -0.0365 & 0.0524 & 0.0365 & 0.0055 & -0.0390 & -0.2830 & -0.1342 \\ 0.0402 & -0.1624 & -0.0602 & 0.1163 & 0.0234 & -0.2589 & 0.0393 & 0.2589 & 0.4390 & -0.1122 & 0.0368 & -0.0355 \\ -0.0098 & -0.0846 & -0.3019 & 0.0509 & 0.5588 & -0.1765 & -0.0637 & 0.1765 & 0.1292 & 0.0615 & -0.2569 & -0.1033 \\ 0.0087 & 0.1214 & 0.1631 & -0.0902 & -0.2472 & 0.4174 & -0.0284 & -0.4174 & -0.1238 & 0.0442 & 0.0841 & 0.0546 \\ 0.0011 & -0.0368 & 0.1388 & 0.0393 & -0.3116 & -0.2409 & 0.0921 & 0.2409 & -0.0054 & -0.1056 & 0.1728 & 0.0487 \\ -0.0478 & 0.0042 & 0.0358 & 0.0131 & -0.0878 & -0.3994 & 0.0812 & 0.3994 & -0.3206 & -0.0376 & 0.0519 & 0.0296 \\ 0.0880 & -0.1666 & -0.0960 & 0.1032 & 0.1111 & 0.1405 & -0.0419 & -0.1405 & 0.7596 & -0.0746 & -0.0151 & -0.0651 \\ -0.2105 & -0.0638 & -0.4844 & -0.1587 & 0.4727 & 0.4237 & -0.1494 & -0.4237 & -0.5283 & 0.4990 & 0.0117 & -0.0968 \\ -0.0348 & -0.1536 & -0.3821 & 0.0257 & -0.3336 & -0.0864 & 0.0700 & 0.0864 & 0.0678 & 0.0200 & 0.7156 & -0.1228 \\ 0.1195 & -0.1185 & -0.0489 & -0.0254 & 0.0593 & 0.0474 & -0.0178 & -0.0474 & -0.0150 & -0.0342 & -0.0104 & 0.9404 \end{pmatrix}$$

The degree of structural redundancy can be verified with the diagonal values in the matrix:

$$tr(WQ_V) = 6$$

IV. AN INDEX FOR STRUCTURAL ROBUSTNESS

On the basis of the redundancy matrix, we can further explore some problems related to the structural safety. Every redundancy component reflects importance of the corresponding member or rigid joints. According to the distribution of the redundancy components in a statically indeterminate structure, we may acquire the general feeling about the robustness of some sub-structures or the whole structure. Generally speaking, the importance of a member mates with both structure form and the load condition. The failure of any unit will affects the structural system by changing the force conduction path and causes redistribution of redundancy components.

We may define an index about structural robustness based on the geometrical/topological configuration and the reliabilities of geometrical constrains (member or rigid joint) as follows

$$Rob = \frac{\sum_{i=1}^{n} r_{ii}\left(1-\Phi\left(-\beta_i\right)\right)}{\sum_{i=1}^{n}\left(1-r_{ii}\right)} \tag{31}$$

The interval of this value is $[0,1]$, and it depends on failure probabilities of geometric constraints, redundancy components, and the number of members and rigid joints in the structure. As an example, we assumed the geometrical and mechanical parameters for the truss, and obtained the values of reliability indexes for all members shown in Table III. According to (31) and values in the table, the calculated result is 0.6251. It is important to employ simple mathematical expressions in engineering. However, this index may not be the best form because the non-diagonal elements have not been used in the formula. The further research is required on the applicability and rationality.

TABLE III
THE VALUES OF RELIABILITY INDEX

Number	Reliability index	Number	Reliability index
1	1.529	14	2.818
2	2.526	15	2.510
3	2.659	16	1.878
4	2.822	17	2.577
5	2.337	18	2.827
6	1.900	19	2.772
7	2.444	20	2.444
8	2.827	21	1.900
9	2.772	22	2.337
10	2.577	23	2.659
11	2.878	24	2.822
12	2.510	25	2.526
13	2.792	26	1.529

V. CONCLUSIONS

Based on the geometric/topological configuration of the structure, the redundancy matrix expresses clearly the distribution of redundancy components. With the matrix, the redundancy of the structure is resolved into components and distributed to every number or rigid joint. In addition, the matrix also indicates the rule about the redundancy component redistribution when a member or rigid joint failure. Hence, it presents a convenient way to analyze the importance of members and structural robustness.

By combining the index of reliability and redundancy components, we define the new index concerning the structural robustness. The examples that are given in the paper shows that the redundancy matrix and the relevant concepts is valuable in analysis of structural safety.

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