World Academy of Science, Engineering and Technology
International Journal of Industrial and Manufacturing Engineering
Vol:11, No:6, 2017

Optimal Opportunistic Maintenance Policy for a Two-Unit System

Nooshin Salari, Viliam Makis, Jane Doe

Open Science Index, Industrial and Manufacturing Engineering Vol:11, No:6, 2017 publications.waset.org/10007253/pdf

***Abstract*—**This paper presents a maintenance policy for a system consisting of two units. Unit 1 is gradually deteriorating and is subject to soft failure. Unit 2 has a general lifetime distribution and is subject to hard failure. Condition of unit 1 of the system is monitored periodically and it is considered as failed when its deterioration level reaches or exceeds a critical level N. At the failure time of unit 2 system is considered as failed, and unit 2 will be correctively replaced by the next inspection epoch. Unit 1 or 2 are preventively replaced when deterioration level of unit 1 or age of unit 2 exceeds the related preventive maintenance (PM) levels. At the time of corrective or preventive replacement of unit 2, there is an opportunity to replace unit 1 if its deterioration level reaches the opportunistic maintenance (OM) level. If unit 2 fails in an inspection interval, system stops operating although unit 1 has not failed. A mathematical model is derived to find the preventive and opportunistic replacement levels for unit 1 and preventive replacement age for unit 2, that minimize the long run expected average cost per unit time. The problem is formulated and solved in the semi-Markov decision process (SMDP) framework. Numerical example is provided to illustrate the performance of the proposed model and the comparison of the proposed model with an optimal policy without opportunistic maintenance level for unit 1 is carried out.

***Keywords*—**Condition-based maintenance, opportunistic maintenance, preventive maintenance, two-unit system.

I. Introduction

MAINTENANCE policies can be classified into two categories: Corrective and preventive maintenance. Corrective maintenance (CM) is performed after failure of the system and preventive maintenance (PM) is performed before the system fails [1]. Most systems are subject to deterioration and preventive maintenance of these systems can reduce the occurrence of failures and the resulting high cost. Preventive maintenance strategies are defined as time-based and condition-based preventive maintenance [2]. Condition-based maintenance suggests the required maintenance actions based on the information obtained from inspection data. Lam and Yeh [3], determined an optimal preventive threshold to minimize the expected long run cost rate for a Markovian deteriorating system under continuous and sequential inspection.

The deterioration process is not the only cause of failure and systems may fail also due to shocks. A system with failures caused by deterioration process and shocks was considered in [4], [5]. Huynh et al. proposed a maintenance model for a system considering dependent shock and deterioration processes [6].

N. Salari and V. Makis are with the Department of Mechanical and Industrial Engineering, University of Toronto, ON, CA, M5S 3G8 (e-mail: nsalari@mie.utoronto.ca).

J. Doe is with the Anonymous University.

Many research papers have been published dealing with maintenance models for single unit systems. In a single-unit system, the entire system is considered as one component. Maintenance models for single unit systems can be applied to multi-unit systems if there is no dependency between units [7]. Dependencies among units are categorized as: Economic, stochastic and structural dependencies. Examples of the systems with economic dependency among units are: Aircrafts, power generators, chemical plants. Shi and Zeng [8] studied a multi-unit system with economic and stochastic dependency, and optimal group structure and opportunistic maintenance zone were determined to minimize the long-run expected average cost.

Castanier et al. [9] proposed a condition based maintenance policy for a two- unit deteriorating system, where each unit is subject to gradual deterioration and is monitored at non-periodic inspection times. Zhu et al. [10] proposed a condition-based maintenance model for a multi-component system subject to continuous stochastic deterioration. They determined the optimal preventive maintenance limits for components and optimal joint maintenance interval by minimizing the long-run expected average cost rate. A replacement model for a two-unit system with failure rate interaction was introduced in [11]. Laggoune [12] proposed a preventive maintenance approach for a multi-component series system subjected to random failures, where the cost rate is minimized under general lifetime distribution. Wang [13] introduced a geometric process repair model to develop maintenance policy for a series repairable system consisting of two non-identical components and one repairman. A replacement policy is considered based on the number of failures of components 1 and 2.

This paper studies maintenance policy for a system consisting of two units with economic dependency. Unit 1 is subject to deterioration and unit 2 has a general life time distribution. The objective is to determine an optimal preventive and opportunistic maintenance levels for unit 1 and optimal preventive maintenance level for unit 2, minimizing the total long run expected average cost per unit time. We formulate and solve this problem using SMDP modeling framework.

In [14], authors assumed preventive and opportunistic maintenance levels for both units, but in this paper we consider preventive and opportunistic maintenance levels for unit 1 and preventive maintenance level for unit 2 and compare this policy with the policy which does not take into account opportunistic maintenance for unit 1.

Numerical example is presented to illustrate the optimal maintenance policy. We also compare the results with

Open Science Index, Industrial and Manufacturing Engineering Vol:11, No:6, 2017 publications.waset.org/10007253/pdf

an optimal maintenance policy which does not consider opportunistic replacement for unit 1.

II. Model Description

Consider a system consisting of 2 units. Unit 1 is gradually deteriorating and unit 2 has a general lifetime distribution. We assume that the deterioration level of unit 1 is hidden and can be known only by inspections performed at discrete equidistant time epochs $(k\Delta)$.

Let $\{X_t\}_{t\geq 0}$ be a continuous time Markov process describing the deterioration process of unit 1 with a finite state space $\Omega = \{0, 1, ..., N\}$. State 0 represents a new unit, and state N, which is absorbing, represents failure state of unit 1. The intermediate states $1, 2, 3, ..., N-1$ represent the increasing degree of deterioration.

Let $S = \{(x, m)|x \in \Omega, m \in \{0, \Delta, ..., L\Delta\}\}$ be the state space of the whole system at inspection times where x is the deterioration level of unit 1, and m represents the age of unit 2 with maximum useful age of $L\Delta$.

To model monotonic system deterioration, we assume that the state process of unit 1 is non-decreasing with probability 1. The instantaneous transition rates q_{ij}, $i, j \in \Omega$, are defined by:

$$q_{ij} = lim_{u\to 0}\frac{P(X_{t+u} = j|X_t = i)}{u} < +\infty,\ i \neq j$$
$$and \quad q_{ii} = -\sum_{j\neq i} q_{ij} \tag{1}$$

The transition probability matrix, $P_{ij}(t) = P(X_{s+t} = j|X_s = i)$ is obtained by solving the Kolmogorov backward differential equations [15].

Unit 2 has a general lifetime density function denoted by $f_2(t)$ and ξ_2 represents its failure time. We assume that inspections are perfect and inspection and replacement times are negligible.

If at an inspection time, deterioration level of unit 1 is in $\{N_1, N_1 + 1, ..., N - 1]$ or exceeds N, it is preventively or correctively replaced, respectively. Unit 2 is correctively replaced by the next inspection time if it fails in an inspection interval. It is preventively replaced at the end of its useful life (age $M = L\Delta$). Unit 2 is also preventively replaced when its age reaches or exceeds preventive maintenance level $M_1 \leq M$. If unit 2 is replaced at an inspection epoch, there is an opportunity to replace unit 1, if its deterioration level reaches or exceeds an opportunistic maintenance level $N_2 < N_1$.

Our objective is to find the values of N_1, N_2, M_1, minimizing the long run expected average cost per unit time. We will describe the system states for this particular system and derive the formulas for the cost components and the transition probabilities.

III. Formulation and Solution of the Problem in the SMDP Framework

In this section, we formulate and solve the maintenance optimization problem in the SMDP framework.

A. State Definition

1) State $(0, 0)$: Both units are as good as new.
2) State (x, m): Both units are operating, x represents deterioration level of unit 1 and m is the age of unit 2.
3) State $(x, 0)$: Unit 1 is working with deterioration level x below N_2 and unit 2 is replaced due to failure.

For the cost minimization problem, the SMDP is determined by the following quantities [15]:

- $P_{i,j}$ = the probability that the system will be in state $j \in S$ at the next decision epoch given the current state is $i \in S$.
- τ_i = the expected sojourn time until the next decision epoch given the current state is $i \in S$.
- C_i = the expected cost incurred until the next decision epoch given the current state is $i \in S$.

We note that each of these components depends also on the action taken in the current state i. Once transition probabilities, costs and sojourn times for each state are defined, the long-run expected average cost can be obtained for selected parameters M_1, N_1, N_2 by solving the following system of linear equations [15]:

$$V_m = C_m - g(M_1, N_1, N_2)\cdot\tau_m + \sum_{k\in S} P_{m,k}\cdot V_k \tag{2}$$
$$V_j = 0 \ for\ an\ arbitrarily\ selected\ single\ state\ j \in S$$

The optimal preventive and opportunistic maintenance levels (M_1^*, N_1^*, N_2^*) and the corresponding minimum long-run expected average cost per unit time can be found by iteratively solving the system of linear equations in (2).

B. Derivation of the Transition Probabilities

- If there is no replacement at the current inspection time:
  1) Transition from state (x, m), to state $(x', m + \Delta)$ occurs when unit 2 does not fail in the next inspection interval and the next deterioration level of unit 1 is x'. This transition probability is equal to:

$$\begin{aligned} &P_{(x,m),(x',m+\Delta)} = \\ &P(X_{n\Delta}=x', \xi_2 > m+\Delta|\xi_2 > m, X_{(n-1)\Delta}=x) = \\ &P(X_{n\Delta}=x'|X_{(n-1)\Delta}=x)\times P(\xi_2 > m+\Delta|\xi_2 > m) \\ &= P_{(x,x')}(\Delta)\cdot\frac{R_2(m+\Delta)}{R_2(m)} \end{aligned} \tag{3}$$

  2) Transition from state (x, m), to state $(x', 0)$, occurs when unit 2 fails and unit 1 deterioration level x' is below N_2, and the corresponding transition probability is given by:

Open Science Index, Industrial and Manufacturing Engineering Vol:11, No:6, 2017 publications.waset.org/10007253/pdf

$$P_{(x,m),(x',0)} = P(X_{n\Delta} = x', \xi_2 < m+\Delta | \xi_2 > m, X_{(n-1)\Delta} = x) = P(X_{n\Delta} = x' | X_{(n-1)\Delta} = x) \times P(\xi_2 < m+\Delta | \xi_2 > m) = P_{(x,x')}(\Delta) \cdot (1 - \frac{R_2(m+\Delta)}{R_2(m)}) \quad (4)$$

3) Transition from state (x, m), to state $(0, 0)$, occurs when both units are replaced. Unit 2 is correctively replaced due to failure and unit 1 next deterioration level exceeds N_2. The transition probability is equal to:

$$P_{(x,m),(0,0)} = P(X_{n\Delta} \in \{N_2, N_2+1, ..., N\}, \xi_2 < m+\Delta | \xi_2 > m, X_{(n-1)\Delta} = x) = \sum_{x'=N_2}^{N} P_{(x,x')}(\Delta) \cdot (1 - \frac{R_2(m+\Delta)}{R_2(m)}) \quad (5)$$

- Both units are replaced when the age of unit 2 exceeds M_1 and the deterioration level of unit 1 exceeds N_2 at the current inspection time. Then, there are three possibilities which can happen till the next inspection time:

$$P_{(x,m),(x',\Delta)} = P_{(0,x')}(\Delta) \cdot R_2(\Delta) \quad (6)$$

$$P_{(x,m),(x',0)} = P_{(0,x')}(\Delta) \cdot (1 - R_2(\Delta)), \quad for\ x' < N_2 \quad (7)$$

$$P_{(x,m),(0,0)} = \sum_{x'=N_2}^{N} P_{(0,x')}(\Delta) \cdot (1 - R_2(\Delta)) \quad (8)$$

- Only unit 1 is replaced at the current inspection time when deterioration level of unit 1 exceeds N_1 and the age of unit 2 is less than M_1. Then, there are three possibilities which can happen till the next inspection time:

$$P_{(x,m),(x',m+\Delta)} = P_{(0,x')}(\Delta) \cdot \frac{R_2(m+\Delta)}{R_2(m)} \quad (9)$$

$$P_{(x,m),(x',0)} = P_{(0,x')}(\Delta) \cdot (1 - \frac{R_2(m+\Delta)}{R_2(m)}), \quad for\ x' < N_2 \quad (10)$$

$$P_{(x,m),(0,0)} = \sum_{x'=N_2}^{N} P_{(0,x')}(\Delta) \cdot (1 - \frac{R_2(m+\Delta)}{R_2(m)}) \quad (11)$$

- Only unit 2 is replaced at the current inspection time when the age of unit 2 exceeds M_1 and deterioration level of unit 1 is less than N_2. Then there are three possibilities which can happen till the next inspection time:

$$P_{(x,m),(x',\Delta)} = P_{(x,x')}(\Delta) \cdot R_2(\Delta) \quad (12)$$

$$P_{(x,m),(x',0)} = P_{(x,x')}(\Delta) \cdot (1 - R_2(\Delta)), \quad for\ x' < N_2 \quad (13)$$

$$P_{(x,m),(0,0)} = \sum_{x'=N_2}^{N} P_{(x,x')}(\Delta) \cdot (1 - R_2(\Delta)) \quad (14)$$

C. Expected Cost and Sojourn Time

The following cost components are considered in the model:
C_I: Inspection cost
C_{Fi}: Failure replacement cost of unit $i, i \in \{1, 2\}$.
C_{Pi}: Preventive replacement cost of unit $i, i \in \{1, 2\}$.
C_{O1}: Opportunistic replacement cost of unit 1.
C_K: Set-up cost incurred every time when one or two replacements are performed

- Expected cost for state (x, m), when there is no replacement at the current inspection time is equal to:

$$\begin{aligned} &E(Cost | \xi_2 > m, X_{(n-1)\Delta} = x) = \\ &E(Cost | m < \xi_2 \le m+\Delta, X_{(n-1)\Delta} = x) \\ &\times P(\xi_2 \le m+\Delta | \xi_2 > m, X_{(n-1)\Delta} = x) \\ &+ E(Cost | \xi_2 > m+\Delta, X_{(n-1)\Delta} = x) \\ &\times P(\xi_2 > m+\Delta | \xi_2 > m, X_{(n-1)\Delta} = x) \\ &= C_I + [(C_{F1} + C_{F2} + C_K) P_{x,N}(\Delta) \\ &+ (C_{P1} + C_{F2} + C_K) \sum_{x'=N_1}^{N-1} P_{x,x'}(\Delta) \\ &+ (C_{O1} + C_{F2} + C_K) \sum_{x'=N_2}^{N_1-1} P_{x,x'}(\Delta) \\ &+ (C_{F2} + C_K) \sum_{x'=x}^{N_2-1} P_{x,x'}(\Delta)] \\ &\times (1 - \frac{(R_2(m+\Delta))}{R_2(m)}) \end{aligned} \quad (15)$$

- Expected cost for state (x, m), when both units are replaced preventively (deterioration level of unit 1 is in the set $\{N_1, N_1+1, ..., N-1\}$ and the age of unit 2 reaches or exceeds M_1) at the current inspection time is equal to:

$$\begin{aligned} &E(Cost | \xi_2 > m, X_{(n-1)\Delta} = x) \\ &= E(Cost | \xi_2 \le \Delta, X_0 = 0) P(\xi_2 \le \Delta | X_0 = 0) \\ &+ E(Cost | \xi_2 > \Delta, X_0 = 0) P(\xi_2 > \Delta | X_0 = 0) \\ &= C_I + C_{P1} + C_{P2} + C_K \\ &+ \{(C_{F1} + C_{F2} + C_K) P_{0,N}(\Delta)((1 - R_2(\Delta)) \\ &+ (C_{P1} + C_{F2} + C_K) \sum_{x'=N_1}^{N-1} P_{0,x'}(\Delta)(1 - R_2(\Delta)) \\ &+ (C_{O1} + C_{F2} + C_K) \sum_{x'=N_2}^{N_1-1} P_{0,x'}(\Delta)(1 - R_2(\Delta)) \\ &+ (C_{F2} + C_K) \sum_{x'=x}^{N_2-1} P_{0,x'}(\Delta)(1 - R_2(\Delta)) \end{aligned} \quad (16)$$

Open Science Index, Industrial and Manufacturing Engineering Vol:11, No:6, 2017 publications.waset.org/10007253/pdf

To simplify expressions for the cost components we define C_Z as follows:

$$\begin{aligned} C_Z = \{&(C_{F1}+C_{F2}+C_K)P_{0,N}(\Delta) \\ &+(C_{P1}+C_{F2}+C_K)\sum_{x'=N_1}^{N-1} P_{0,x'}(\Delta) \\ &+(C_{O1}+C_{F2}+C_K)\sum_{x'=N_2}^{N_1-1} P_{0,x'}(\Delta) \\ &+(C_{F2}+C_K)\sum_{x'=x}^{N_2-1} P_{0,x'}(\Delta)\}\times(1-R_2(\Delta)) \end{aligned} \tag{17}$$

Then, the expected cost $E(Cost|\xi_2 > m, X_{(n-1)\Delta} = x)$ can be written as:

$$C_I + C_{P1} + C_{P2} + C_K + C_Z \tag{18}$$

- Expected cost for state (x, m) when unit 2 is preventively replaced and unit 1 is opportunistically replaced (age of unit 2 exceeds M_1 and deterioration level of unit 1 is in $\{N_2, N_2+1, ..., N_1-1\}$) at the current inspection time is equal to:

$$E(Cost) = C_I + C_{O1} + C_{P2} + C_K + C_Z \tag{19}$$

- Expected cost for state (x, m) when unit 2 is preventively replaced and unit 1 is correctively replaced (age of unit 2 exceeds M_1 and deterioration level of unit 1 reaches N) at the current inspection time is equal to:

$$E(Cost) = C_I + C_{F1} + C_{P2} + C_K + C_Z \tag{20}$$

- Expected cost for state (N, m) when unit 1 is correctively replaced because its deterioration level reaches N and unit 2 is not replaced because its age is below M_1 is equal to:

$$\begin{aligned} E(Cost|\xi_2 > m, X_{(n-1)\Delta} = N) = \\ C_I+C_{F1}+C_K+(C_{F1}+C_{F2}+C_K)P_{0,N}(\Delta) \\ \times(1-\frac{(R_2(m+\Delta))}{R_2(m)}) \\ +(C_{P1}+C_{F2}+C_K)\sum_{x'=N_1}^{N-1}P_{0,x'}(\Delta) \\ \times(1-\frac{(R_2(m+\Delta))}{R_2(m)}) \\ +(C_{O1}+C_{F2}+C_K)\sum_{x'=N_2}^{N_1-1}P_{0,x'}(\Delta) \\ \times(1-\frac{(R_2(m+\Delta))}{R_2(m)}) \\ +(C_{F2}+C_K)\sum_{x'=x}^{N_2-1}P_{0,x'}(\Delta) \\ \times(1-\frac{(R_2(m+\Delta))}{R_2(m)}) \end{aligned} \tag{21}$$

- Expected cost for state (x, m) when unit 1 is preventively replaced because its deterioration level exceeds N_1 and unit 2 is not replaced because its age is below M_1 is equal to:

$$\begin{aligned} E(Cost) = C_I + C_{P1} + C_K \\ +(C_{F1}+C_{F2}+C_K)P_{0,N}(\Delta) \\ \times(1-\frac{(R_2((m+1)\Delta))}{R_2(m\Delta)}) \\ +(C_{P1}+C_{F2}+C_K)\sum_{x'=N_1}^{N-1}P_{0,x'}(\Delta) \\ \times(1-\frac{(R_2((m+1)\Delta))}{R_2(m\Delta)}) \\ +(C_{O1}+C_{F2}+C_K)\sum_{x'=N_2}^{N_1-1}P_{0,x'}(\Delta) \\ \times(1-\frac{(R_2((m+1)\Delta))}{R_2(m\Delta)}) \\ +(C_{F2}+C_K)\sum_{x'=x}^{N_2-1}P_{0,x'}(\Delta) \\ \times(1-\frac{(R_2((m+1)\Delta))}{R_2(m\Delta)}) \end{aligned} \tag{22}$$

- Expected cost for state (x, m) when unit 2 is preventively replaced because its age exceeds M_1 and unit 1 is not replaced because its deterioration level is below N_2:

$$\begin{aligned} E(Cost) = C_I + C_{P2} + C_K + \\ (C_{F1}+C_{F2}+C_K)P_{x,N}(\Delta)(1-R_2(\Delta)) \\ +(C_{P1}+C_{F2}+C_K)\sum_{x'=N_1}^{N-1}P_{x,x'}(\Delta)(1-R_2(\Delta)) \\ +(C_{O1}+C_{F2}+C_K)\sum_{x'=N_2}^{N_1-1}P_{x,x'}(\Delta)(1-R_2(\Delta)) \\ +(C_{F2}+C_K)\sum_{x'=x}^{N_2-1}P_{x,x'}(\Delta)(1-R_2(\Delta)) \end{aligned} \tag{23}$$

We assume that replacement time is negligible and the expected sojourn time for any state is equal to Δ.

D. Numerical Example

In this section we illustrate the presented model and the maintenance policy with a numerical example. We assume that unit 1 deterioration follows a continuous-time homogeneous Markov chain $(X_t : t \in R^+)$ with state space, $\Omega = \{0, 1, .., 8\}$. State 0 indicates that the unit is new and state 8 indicates the failure state. We consider the following probability matrix P:

$$P = \begin{bmatrix} 0.35 & 0.21 & 0.16 & 0.12 & 0.07 & 0.05 & 0.03 & 0.01 \\ 0 & 0.47 & 0.19 & 0.12 & 0.1 & 0.06 & 0.04 & 0.02 \\ 0 & 0 & 0.53 & 0.17 & 0.12 & 0.1 & 0.03 & 0.05 \\ 0 & 0 & 0 & 0.6 & 0.15 & 0.10 & 0.05 & 0.1 \\ 0 & 0 & 0 & 0 & 0.5 & 0.2 & 0.15 & 0.15 \\ 0 & 0 & 0 & 0 & 0 & 0.4 & 0.3 & 0.3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.4 & 0.6 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{24}$$

The lifetime distribution of unit 2 follows Gamma distribution

Open Science Index, Industrial and Manufacturing Engineering Vol:11, No:6, 2017 publications.waset.org/10007253/pdf

with parameters $k = 2$ and $\theta = 10$ with the probability density function of the form:

$$f_2(x) = \frac{1}{\Gamma(k)\theta^k} x^{k-1} e^{\frac{-x}{\theta}}; \qquad k > 0,\ \theta > 0 \tag{25}$$

The following parameters are considered for the proposed model:

$$C_I = 10,\ C_{F1} = 400,\ C_{P1} = 80,\ C_{O1} = 70,$$

$$C_{F2} = 350,\ C_{P2} = 80,\ C_K = 100,\ \Delta = 5,\ M = 100$$

Considering these parameters, we compute the optimal preventive and opportunistic maintenance levels (N_1^*, N_2^*, M_1^*) in the SMDP framework, minimizing the long-run expected average cost per unit time. We applied the model and used (2) to obtain the optimal results shown in Table I. Each run took 1.278 seconds on an Intel Core (TM) i5 CPU with 2.27 GHz.

TABLE I
OPTIMAL CONTROL LIMITS AND THE AVERAGE COST

	PM unit1	OM unit1	PM unit2	Average
Optimal	(N_1)	(N_2)	(M_1)	$Cost$
Values	6	4	45	31.9662

The optimal policy is to replace unit 1 preventively when its deterioration level reaches or exceeds $N_1 = 6$ and replace unit 2 when its age exceeds $M_1 = 45$. At the time of replacement of unit 2, there is an opportunity to replace unit 1. Unit 1 is opportunistically replaced when its deterioration level reaches or exceeds $N_2 = 4$.

To investigate the effect of the opportunistic replacement limit of unit 1 on the average cost for the defined system, we compare the results with the optimal policy which does not consider the opportunistic replacement for unit 1. The resulting minimum long-run expected average cost and the optimum levels are given in Table II.

TABLE II
MINIMUM LONG RUN AVERAGE COST PER UNIT TIME AND THE OPTIMAL REPLACEMENT LEVELS

Maintenance policy	Average cost	N_1	N_2	M_1
New Policy	31.9662	6	4	45
Policy without OM	32.5879	6	–	70

We can see from Table II that the presented maintenance policy with opportunistic replacement performs better than the optimal maintenance policy without opportunistic replacement.

IV. CONCLUSION

In this paper, we have developed a model to determine the optimal maintenance policy for a two unit system, where unit 1 is subject to condition monitoring and only age information of unit 2 is available. Unit 1 deterioration is described by a continuous-time Markov process and it is considered as a failed unit when its deterioration level exceeds level N at an inspection time. Failure of unit 2 is observable and if it fails in an inspection interval, it is correctively replaced by the next inspection epoch. Unit 1 is preventively replaced when its deterioration level exceeds N_1 and unit 2 is preventively replaced when its age exceeds M_1. At the time of replacement of unit 2, there is an opportunity to replace unit 1 if its deterioration level reaches or exceeds N_2.

SMDP framework is applied to find the optimal preventive and opportunistic replacement limits for unit 1 and preventive replacement age limit for unit 2, that minimize the long run expected average cost per unit time. A numerical example is provided to illustrate the proposed maintenance policy and a comparison is given with an optimal policy which does not consider opportunistic replacement level for unit 1. The results show that the model with opportunistic replacement for unit 1 when economic dependency exists among the units outperforms the model without opportunistic replacement.

ACKNOWLEDGMENT

The authors would like to thank the Natural Sciences and Engineering Research Council of Canada and the Ontario Centers of Excellence (OCE) for the financial support under grants number RGPIN 121384-11 and 201461, respectively.

REFERENCES

[1] HZ. Wang and H. Pham, *Reliability and Optimal Maintenance,*. Springer, 2006.
[2] V. Legat, AH. Zaludova, V. Cervenka, and V. Jurca, *Contribution to optimization of preventive maintenance.* Reliability Engineering System, 51:25966, 1996.
[3] C.T. Lam, and R.H. Yeh, *Comparison of sequential and continuous inspection strategies for deterioration systems.* Advances in Applied Probability, 26, 423-435, 1994.
[4] A. Lehmann, *Joint modeling of degradation and failure time data.* Journal of Statistical Planning and Inference, 139(5), 1693-1706, 2009.
[5] W. Li and H. Pham, *An inspection maintenance model for systems with multiple competing processes.* IEEE Transactions on Reliability, 54(2), 318-327, 2005.
[6] K.T. Huynh, A. Barros, C. Brenguer, and I.T. Castro, *A periodic inspection and replacement policy for systems subject to competing failure modes due to degradation and traumatic events* Reliability and System Safety, 96(4), 497-508, 2011.
[7] D. Cho and M. Parlar, *A survey of maintenance models for multi-unit systems.* European Journal of Operational Research, 51 (1), 123, 1991.
[8] H. Shi and J. Zeng, *Real-time prediction of remaining useful life and preventive opportunistic maintenance strategy for multi-component systems considering stochastic dependence.* Computers & Industrial Engineering 93, 192204, 2016.
[9] B. Castanier, A. Grall and C. Brenguer *A condition-based maintenance policy with non-periodic inspections for a two-unit series system.* Reliability Engineering & System Safety 87(1): 109 - 120, 2005.
[10] Q. Zhu, H. Peng and GJ. Houtum,*A condition-based maintenance policy for multi-component systems with a high maintenance setup cost.* OR Spectrum, 37, 1007-1035, 2015.
[11] M. Lai and Y. Chen *Optimal periodic replacement policy for a two-unit system with failure rate interaction.* The International Journal of Advanced Manufacturing and Technology, 29, 367-371, 2006.
[12] R. Laggoune, A. Chateauneu and A. Djamil *Opportunistic policy for optimal preventive maintenance of a multi-component system in continuous operating units*, Computers & Chemical Engineering, Volume 33, Issue 9, Pages 14991510, 2009.
[13] G.J. Wang and Y.L. Zhang *An optimal replacement policy for a two - component series system assuming geometric process repair.* Computers & Mathematics with Applications 54(2): 192 - 202, 2007.
[14] N. Salari and V. Makis, *Optimal preventive and opportunistic maintenance policy for a two-unit system*, International Journal of Advanced Manufacturing Technology, DOI 10.1007/s00170-016-9127-x, 2016.
[15] H.C Tijms, *Stochastic Modeling and Analysis: A computational Approach.* John Wiley & Sons, New York, 1994.