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Experiments of a Free Surface Flow in a Hydraulic Channel over an Uneven Bottom

M. Bouinoun, M. Bouhadef

***Abstract*—**The present study is concerned with the problem of determining the shape of the free surface flow in a hydraulic channel which has an uneven bottom. For the mathematical formulation of the problem, the fluid of the two-dimensional irrotational steady flow in water is assumed inviscid and incompressible. The solutions of the nonlinear problem are obtained by using the usual conformal mapping theory and Hilbert's technique. An experimental study, for comparing the obtained results, has been conducted in a hydraulic channel (subcritical regime and supercritical regime).

***Keywords*—**Free-surface flow, experiments, numerical method, uneven bottom, supercritical regime, subcritical regime.

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I. Introduction

THE hydraulics in pipes which are completely filled with fluid, differs from that of open channels by the existence of a free-surface. In the case of a free surface flow, there is an air-fluid interface on which is exerted the atmospheric pressure. The difficulty of the mathematical resolution of this type of flow is the fact that the free surface is generally unknown on which, in addition, boundary conditions should be satisfied.

Analytical and numerical methods are used to obtain the basic mechanism of such flow situations. The work of Thomson [22] in 1886 is considered as a basis for research in this area. Linear solutions for flows over obstacles have been studied by [1]-[5], [7], [9]-[11], [13]-[15], [23], to name only a few. In addition, the equivalent nonlinear problem has also been studied by several authors, and references to a part of their work can be found in [16].

In this study, we determine the free surface location in a hydraulic channel for a flow of an inviscid fluid over an irregular bottom. To do this, we use the nonlinear theory and the solution is based on conformal transformations, specifically the transformation of Schwartz-Christoffel and the Hilbert's method for a mixed problem [1]. For the mathematical problem, reference may be made to the research works of [12], [17]-[21], [24].

II. Mathematical Formulation

A. Position of the Problem

We resume below most of the formulation described in [8]. The fluid is assumed inviscid and incompressible. The flow is steady, 2D and irrotational. The streamfunction ψ(x,y) and the velocity potential φ(x,y) are therefore harmonic functions of the (x-y) coordinates in the physical plane defined by the Cauchy-Riemann relations:

$$u = \frac{\partial \varphi}{\partial x} = \frac{\partial \psi}{\partial y} \quad (1)$$

$$v = \frac{\partial \varphi}{\partial y} = -\frac{\partial \psi}{\partial x} \quad (2)$$

where u and v are the velocity components. Thus, φ and ψ are, respectively, the real and imaginary parts of the complex potential w.

The uneven bottom consists of a horizontal plane AB, a plane BC inclined at an angle α with a length L and a horizontal semi-infinite plane CD. The point A is at the abscissa - ∞, while the point D is at the abscissa + ∞ (see Fig. 1). The flow is from left to right. For convenience, we choose the origin of axes at B. In the plane, the horizontal axis is in the flow direction and the y-axis directed upwards. (U_1, h_1) and (U_2, h_2) are the velocity and the water depth at A and D respectively.

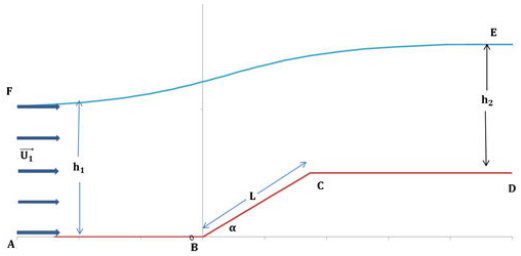


Fig. 1 Schematic of the flow

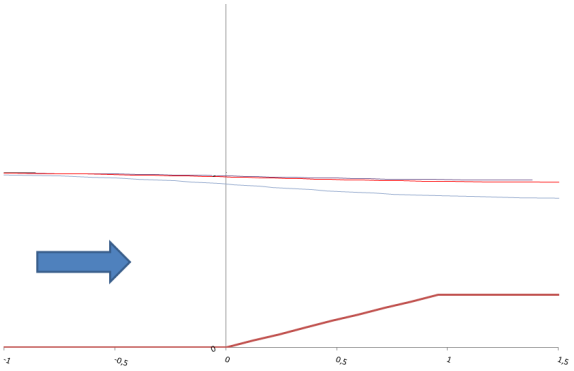


Fig. 2 Subcritical regime

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B. Numerical Results

Fig. 2 shows the obtained free-surfaces, corresponding to the inclination $\alpha = \pi/6$, for three Froude numbers which are, from top to bottom F = 0.4, F = 0.7 and F = 0.8.

Fig. 3 is relative to an inclination $\alpha = \pi/4$ for a supercritical regime configuration.

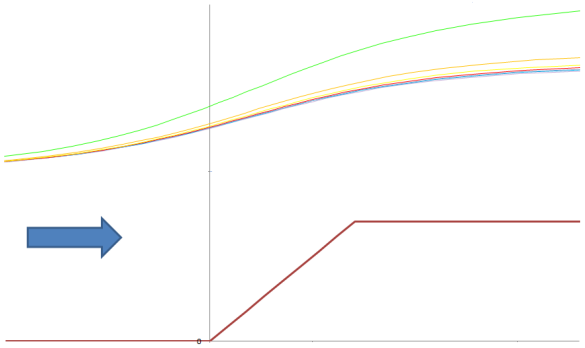


Fig. 3 Supercritical regime

The different values of the Froude number are F = 10, F = 7, F = 6, F = 5, F = 4, F = 3 and F = 2 (top to bottom respectively).

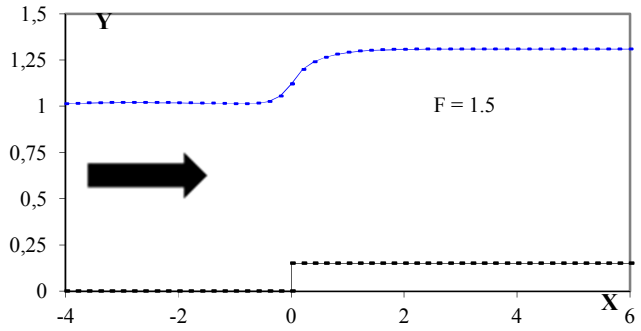


Fig. 4 Supercritical flow over a step (from [10])

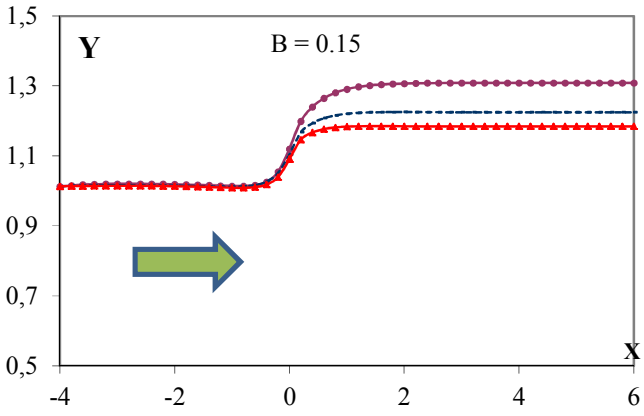


Fig. 5 Evolution of the free-surface over a step (from [10])

In a previous work using the finite volume method, [10] examined the supercritical flow over a step and obtained the configuration showed, as an example, in Fig. 4. Fig. 5 gives, for the same height of the step, the decreasing evolution of the free-surface jump with the Froude number.

In Fig. 5 is shown the theoretical evolution of the free-surface with the Froude number. The values of the Froude number are F = 1.5, F = 2.0 and F = 3.0 respectively from top to bottom.

III. Experimentation

A. Description

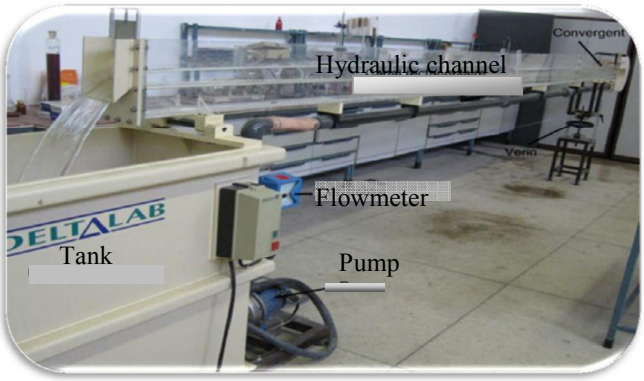


Fig. 6 View of the hydraulic channel

Our experiments were conducted in a horizontal rectangular Plexiglas channel of width b = 75mm and length l = 6m. This channel is connected by means of a special assembly which ensures parallel walls with a height of 160mm. The perfect rigidity of the channel is provided by a box girder made of Plexiglas. At its upstream end, it is based on a manual jack screw with an adjustable slope by means of a wheel provided with graduations. It is fed from a PVC supply tank which is fixed on a frame of tubular steel. The flow is supplied by a pump (flow rate: 1.6 m^3/h - 16 m^3/h), fixed to the frame. The flow control is done by a manual valve (Fig. 6).

To determine the Froude number, we use the upstream average velocity U which is given by the discharge Q = UbH. We have previously calibrated the flow meter and the relationship between the real flow rate and the indicated one is given by:

$$Q = 1.0522Q_r - 0.0001$$

Q_r (m^3/s) is the value read on the flow meter and Q the effective discharge.

We give in Table I the main characteristics of the flow for two inclination angles.

B. Subcritical Regime

For low velocities corresponding to a waveless subcritical regime, the free surface aspect is given hereafter (Fig. 7).

We must recall that the upstream water depth H is equal to the length of the inclined plane (H = 2.62 cm).

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TABLE I
CHARACTERISTICS OF THE FLOW

α	Q(l/s)	U(m/s)	Fr
π/4	0.37	0.19	0.37
	0.48	0.25	0.49
π/6	0.37	0.12	0.19
	0.48	0.16	0.25
	1.07	0.34	0.50
π/4	3.12	1.59	3.13
	3.41	1.74	3.43
	3.70	1.89	3.72
	4.14	2.11	4.16
π/6	3.70	1.19	1.87
	4.14	1.33	2.09

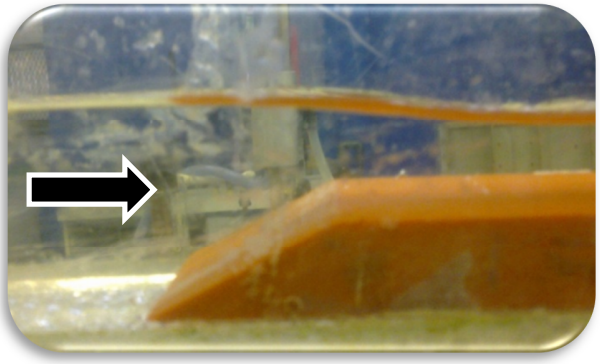


Fig. 7 Aspect of the free-surface for F = 0.19 and α = π/6

To determine the free surface profiles, we use a camera and we treat the obtained photographs by using the Auto-Cad software. We did not use the level meter for the determination of the free surface profiles because the rod itself disturbs the flow and distorts the free-surface.

The evolution of the free surface, with the Froude number, is represented in Fig. 8 for an inclination angle α = π/6 and Froude numbers respectively F = 0.19, F = 0.25 and F = 0.5, respectively from top to the bottom.

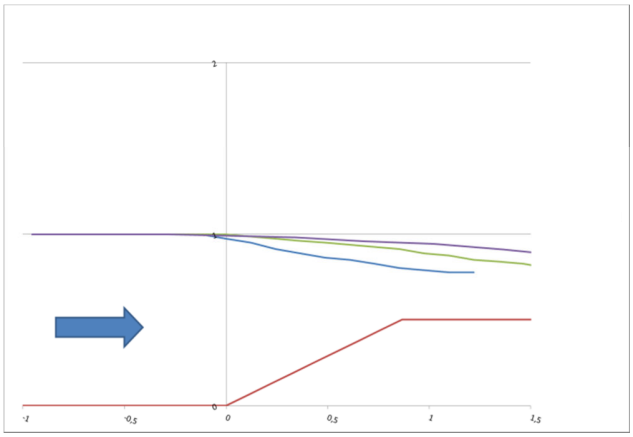


Fig. 8 Measured free-surface for various Froude numbers (α = π/6)

C. Supercritical Regime

The upstream supercritical regime is generated by the addition of a convergent (nozzle). It consists on a flexible PVC sheet which thickness is 3 mm and 80 cm length; it is placed so as it gradually directs the flow towards a narrowed exit (Fig. 9). The distance between the entry of the channel and the end of the convergent is selected so as to reach a fully supercritical regime slightly disturbed. Indeed, we reduce the cross section of the flow to a narrowed exit opening H = 2.5 cm to allow great velocities.

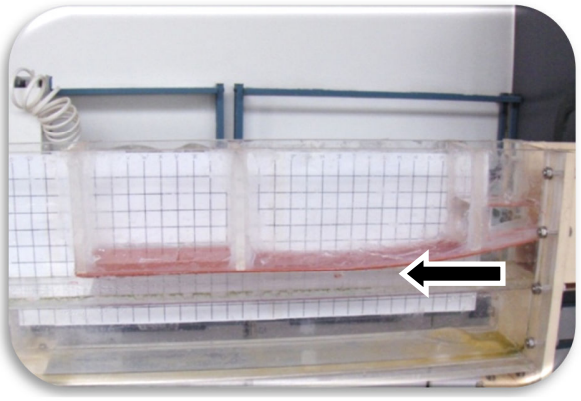


Fig. 9 The upstream convergent

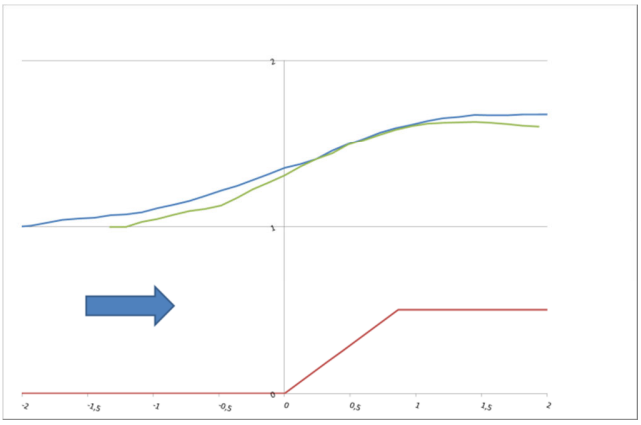


Fig. 10 Free-surfaces for F = 1.87 and F = 2.09 (α = π/6)

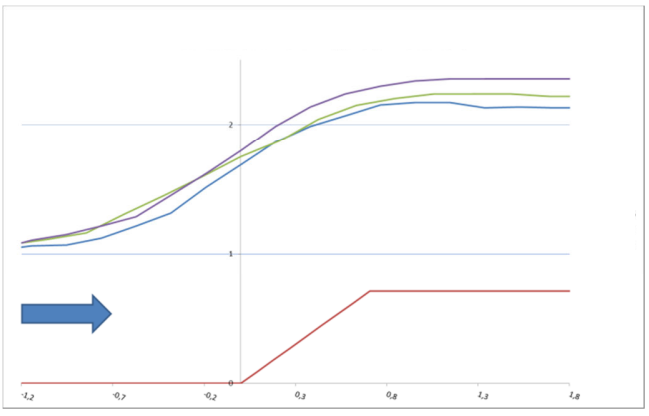


Fig. 11 Free-surfaces for F = 3.43, F = 3.72 and F = 4.16 (α = π/4)

The experimental free surface profiles, obtained for an inclined angle of α = π/6, are given at Fig. 10 for Froude numbers F = 1.87 and F = 2.09. For another inclination angle

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$\alpha = \pi/4$, the experimental free-surfaces are shown in Fig. 11 for Froude numbers F = 3.43, F = 3.72 and F = 4.16.

The same equivalent results have been obtained by [11] for the step as shown on the photos and figures hereafter.

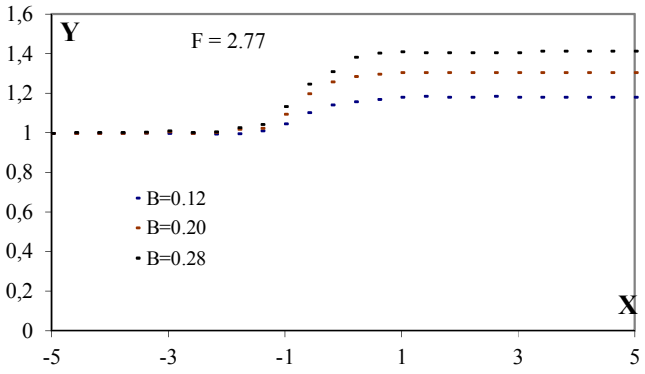


Fig. 12 Free-surface for F = 2.77 with different heights of the step

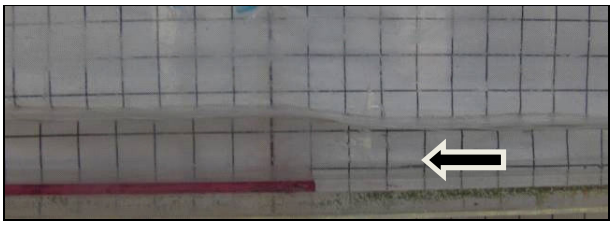


Fig. 13 Free-surface for F = 2.77 and B=0 .2

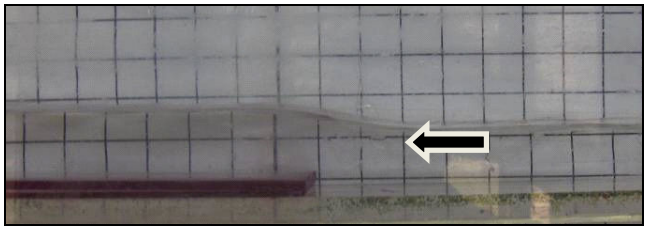


Fig. 14 Free-surface for F = 2.77 and B = 0 .28

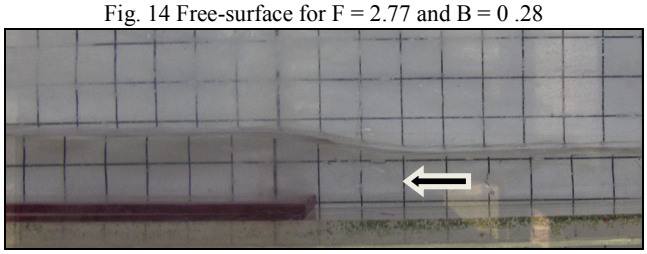


Fig. 15 Free-surface for F = 2.77 and B = 0 .28

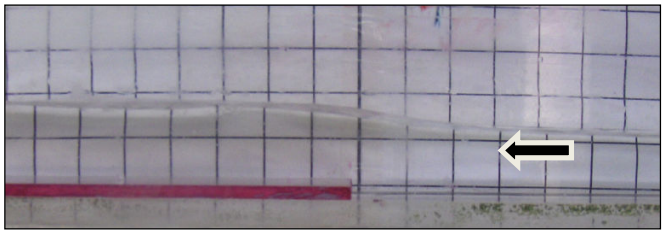


Fig. 16 Free-surface for F = 2.3 and B = 0 .2

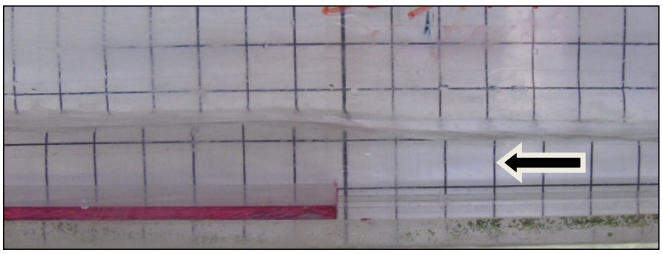


Fig. 17 Free-surface for F = 3.19 and B = 0 .2

IV. Theory - Experiments

Before proceeding to a comparison between the numerical results with those relating to the experiments, we must remember that the main hypothesis of the mathematical formulation is that the fluid is inviscid. As the velocity at the bottom is therefore nonzero, the Froude number is easy to calculate for a uniform velocity profile. In the real case, the fluid velocity at the wall is zero and the profile is sheared. Should we then consider the Froude number calculated using the average fluid velocity or the velocity at the free surface? (see [6]). We recall that, in our case, we used the average velocity since we read directly the volumetric flow rate. We can notice that the calculated free surface is in a relatively good agreement with that which has been measured experimentally (Fig. 18). Let us remember, however, that the theoretical model is based on the assumption of an inviscid fluid which means that all the effects of turbulence are omitted.

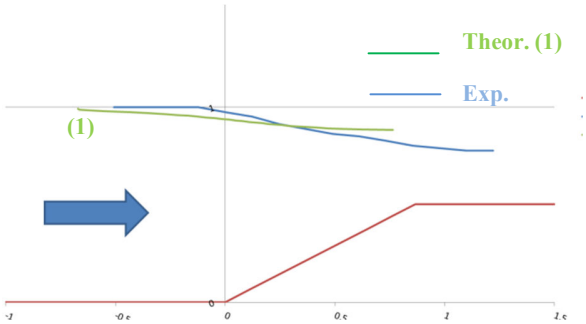


Fig. 18 Calculated and measured free-surface for F = 0.19 ($\alpha = \pi/6$)

We give in Fig. 19, another example with a step and a supercritical regime.

V. Conclusion

As it was already mentioned [8], we think that to get a good agreement, between the numerical results and those obtained experimentally, requires a match between the experimentation and the mathematical assumptions with the adequate boundary conditions. To do this, if you want to stay in the case of an inviscid fluid, we suggest that experiments should be done in water at rest with a moving bottom. Indeed, the relative movement of the water will be at a constant velocity, from the bottom to the free surface.

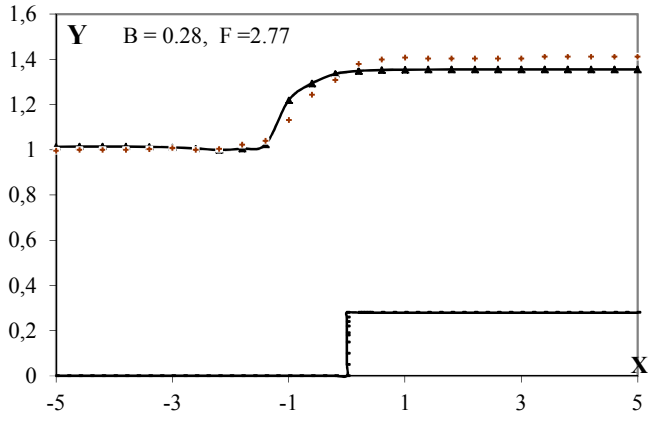


Fig. 19 Calculated (+) and measured (Δ) free-surface for a step

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